

WKYC 2011 - Future of Large Scale Structure Formation

2011/6/27-7/1, Korea Institute for Advanced Study, Seoul, Korea

## Exploring parameter constraints on dark energy models

( $w$ -fluid, quintessence, Chaplygin gas, and  $f(R)$  gravity)

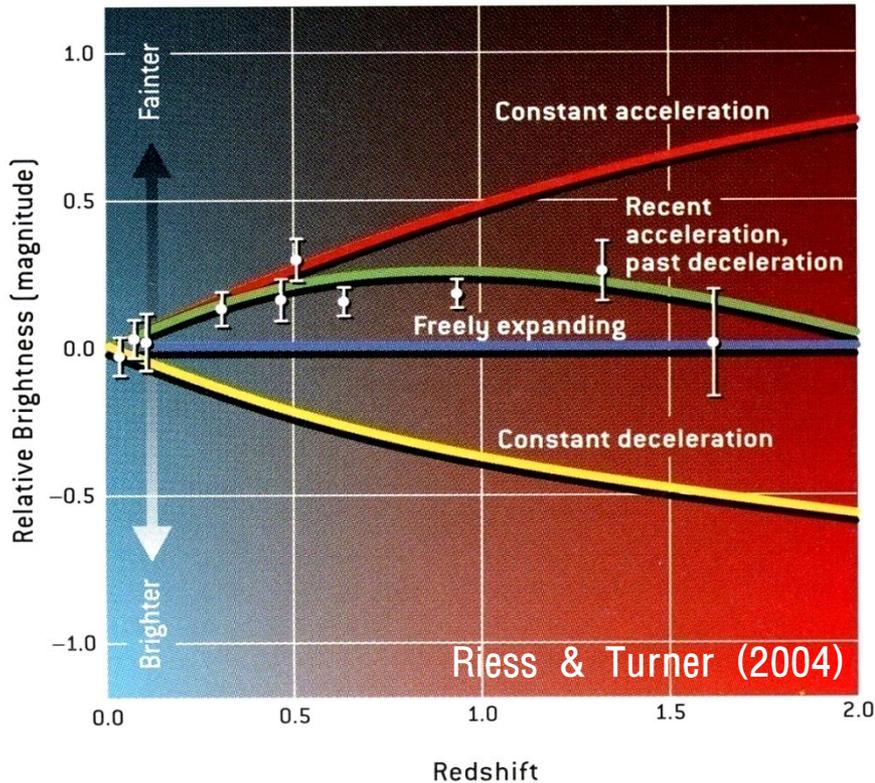
Chan-Gyung Park

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# The acceleration of the Universe

The high- $z$  type Ia supernova (SNIa) luminosity-distance relation, large-scale structures and CMB observations suggest that the expansion rate of our universe is currently under acceleration.

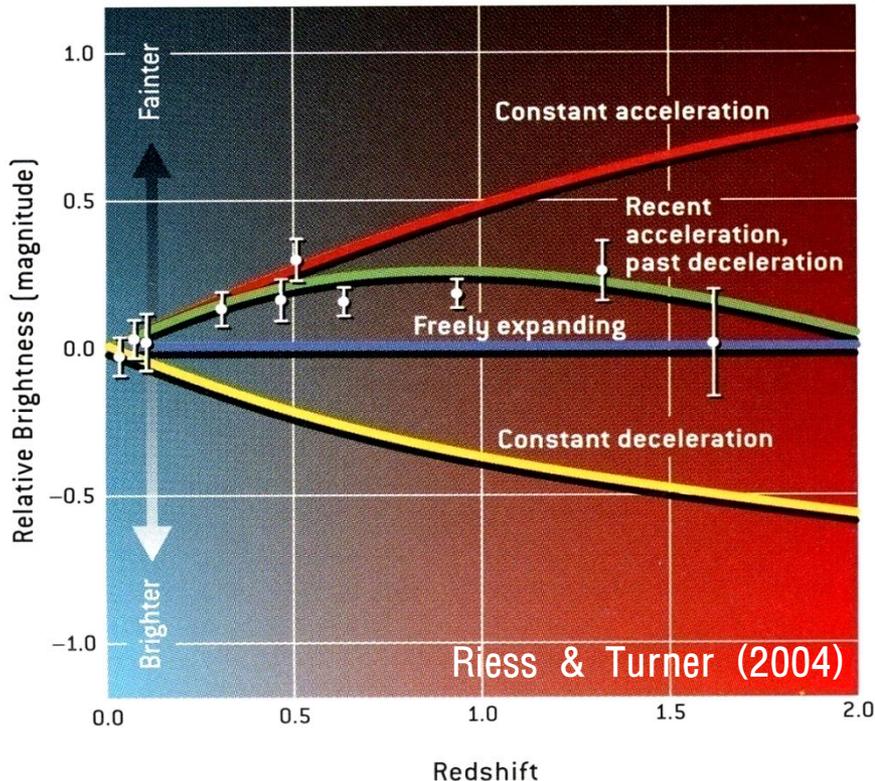
## Type Ia supernova (SNIa)



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## Type Ia supernova (SNIa)



Introduction of cosmological constant  $\Lambda$  to explain the late-time acceleration:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \underbrace{(\mu_m + 3p_m)}_{\substack{\propto a^{-4}(R) \\ a^{-3}(M)}} + \underbrace{\frac{\Lambda}{3}}_{= \text{const.}}$$

$\Lambda$  equation of state:

$$w = p_\Lambda / \mu_\Lambda = -1$$

The nature of the agent causing the acceleration is still unknown, and it is one of the fundamental mysteries in the present day theoretical cosmology.

→ dark energy

# Many dark energy models

## Cosmological constant $\Lambda$

The simplest.  $w = -1$ .

## Modified matter models

$w$ -fluid, quintessence,  $k$ -essence, coupled dark energy, unified models of dark energy and dark matter (Chaplygin gas).

## Modified gravity models

$f(R)$  gravity, Gauss-Bonnet dark energy models, scalar-tensor theories, DGP models.

## Cosmic acceleration without dark energy

Lemaître-Tolman-Bondi model, backreaction of cosmological perturbations.

# Basic equations for scalar-type perturbations

Metric

$$ds^2 = -(1 + 2\alpha)dt^2 - 2a\beta_{,\alpha}dt dx^\alpha + a^2[g_{\alpha\beta}^{(3)}(1 + 2\varphi) + 2\gamma_{,\alpha|\beta}]dx^\alpha dx^\beta$$

Energy-momentum tensor

$$T_0^0 = -(\mu + \delta\mu), \quad T_\alpha^0 = -\frac{1}{k}(\mu + p)v_{,\alpha}$$
$$T_\beta^\alpha = (p + \delta p)\delta_\beta^\alpha + \left( \frac{1}{k^2} \nabla^\alpha \nabla_\beta + \frac{1}{3} \delta_\beta^\alpha \right) \pi^{(s)}$$

Energy-momentum conservation equations [  $i=R(\gamma+v), M(b+c), X$  ]

$$\delta\dot{\mu}_i + 3H(\delta\mu_i + \delta p_i) = (\mu_i + p_i) \left( \kappa - 3H\alpha - \frac{k}{a} v_i \right)$$
$$\frac{[a^4(\mu_i + p_i)v_i]^\bullet}{a^4(\mu_i + p_i)} = \frac{k}{a} \left( \alpha + \frac{\delta p_i}{\mu_i + p_i} - \frac{2}{3} \frac{k^2 - 3K}{k^2} \frac{\pi_i^{(s)}}{\mu_i + p_i} \right)$$

## Background evolution

$$H^2 = \frac{8\pi G}{3} \mu - \frac{K}{a^2},$$

$$\dot{\mu}_i + 3H(\mu_i + p_i) = 0$$

Einstein equations in gauge-ready form (Bardeen 1988, Hwang 1991)

$$\chi \equiv a(\beta + a\dot{\gamma}) \text{ [shear]} \quad \kappa \equiv 3(-\dot{\phi} + H\alpha) + \frac{k^2}{a^2} \chi \text{ [perturbed expansion]}$$

$$-\frac{k^2 - 3K}{a^2} \phi + H\kappa = -4\pi G \delta\mu$$

$$\kappa - \frac{k^2 - 3K}{a^2} \chi = 12\pi G \frac{a}{k} (\mu + p)v$$

$$\dot{\chi} + H\chi - \alpha - \phi = 8\pi G \frac{a^2}{k^2} \pi^{(s)}$$

$$\dot{\kappa} + 2H\kappa + \left( 3\dot{H} - \frac{k^2}{a^2} \right) \alpha = 4\pi G (\delta\mu + 3\delta p)$$

For energy density, pressure, velocity, we use collective quantities including radiation (R), matter (M), dark energy (X). For example,

$$\mu = \sum_{i=R,M,X} \mu_i, \quad \delta\mu = \sum_{i=R,M,X} \delta\mu_i$$

$$(\mu + p)v = \sum_{i=R,M,X} (\mu_i + p_i)v_i$$

# Gauge choice

We choose a gauge to fix the temporal gauge (hypersurface) condition.

## CDM comoving gauge (CCG)

$v_{\text{CDM}} \equiv 0 \rightarrow \alpha = 0$ . Equivalent to synchronous gauge without gauge modes.

## Uniform curvature gauge (UCG)

$\varphi \equiv 0$  (perturbed part of intrinsic scalar curvature)

## Uniform expansion gauge (UEG)

$\kappa \equiv 0$  (perturbed expansion of normal frame vector)

## Zero shear gauge (ZSG)

$\chi \equiv 0$ . Equivalent to conformal Newtonian gauge or longitudinal gauge.

# Quintessence (minimally coupled scalar field)

Scalar field

$$\tilde{\phi}(t, \vec{x}) = \phi(t) + \delta\phi(t, \vec{x})$$

Energy density and pressure:

$$\mu_\phi \equiv \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi \equiv \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

Perturbed quantities:

$$\delta\mu_\phi = \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\alpha + V_{,\phi}\delta\phi, \quad \delta p_\phi = \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\alpha - V_{,\phi}\delta\phi$$

$$\text{velocity: } v_\phi = \frac{k}{a} \frac{\delta\phi}{\dot{\phi}} \quad \text{anisotropic stress: } \pi_\phi^{(s)} = 0$$

Equation of motion of the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left( \frac{k^2}{a^2} + V_{,\phi\phi} \right) \delta\phi = \dot{\phi}(\kappa + \dot{\alpha}) + (2\ddot{\phi} + 3H\dot{\phi})\alpha$$

# Importance of dark energy perturbation

In the presence of a dynamical dark energy it is not guaranteed to use the following conventionally known equation

$$\ddot{\delta}_b + 2H\dot{\delta}_b - 4\pi G\delta\mu_b = 0$$

which is true only for the cosmological constant as the dark energy.

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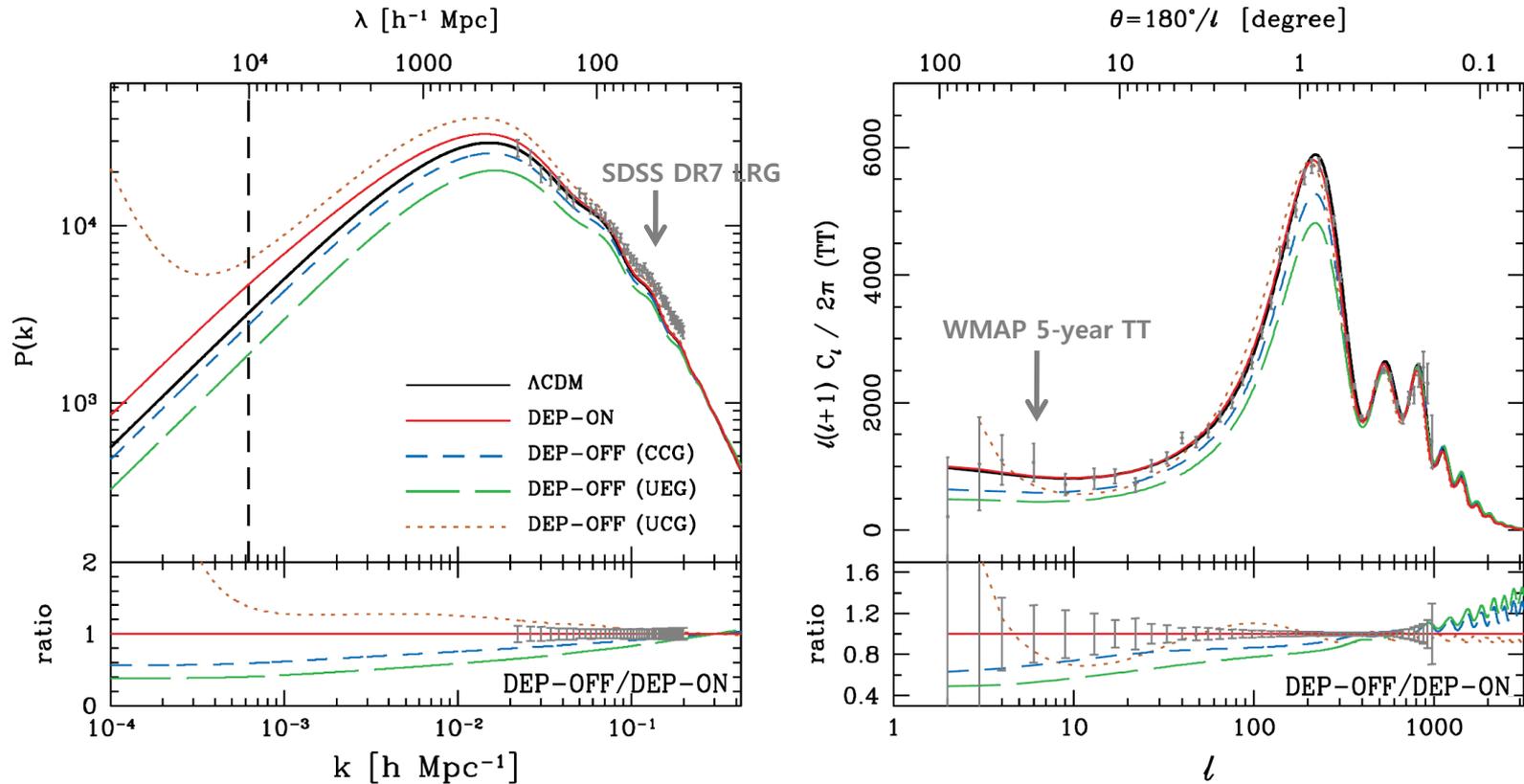
In the presence of dynamical dark energy (e.g., quintessence), the dark energy perturbation (DEP) affect evolutions of radiation and matter density fluctuations:

$$\begin{aligned}\ddot{\delta}_b + 2H\dot{\delta}_b &= 4\pi G \sum_j (\delta\mu_j + 3\delta p_j) \\ &= 4\pi G(\mu_b\delta_b + \mu_c\delta_c + 2\mu_r\delta_r + 4\dot{\phi}\delta\phi - 2V_{,\phi}\delta\phi) \quad (\text{CCG})\end{aligned}$$

# What happens if dark energy perturbation (DEP) is ignored?

C.-G. Park, J. Hwang, J. Lee, H. Noh, Phys. Rev. Lett. 103, 151303 (2009) [arXiv:0904.4007]

Quintessence with  $V(\phi) = V_1 e^{-\lambda_1 \phi} + V_2 e^{-\lambda_2 \phi}$  (scaling initial conditions for  $\lambda_1=9.43$ ;  $\lambda_2=1.0$ )

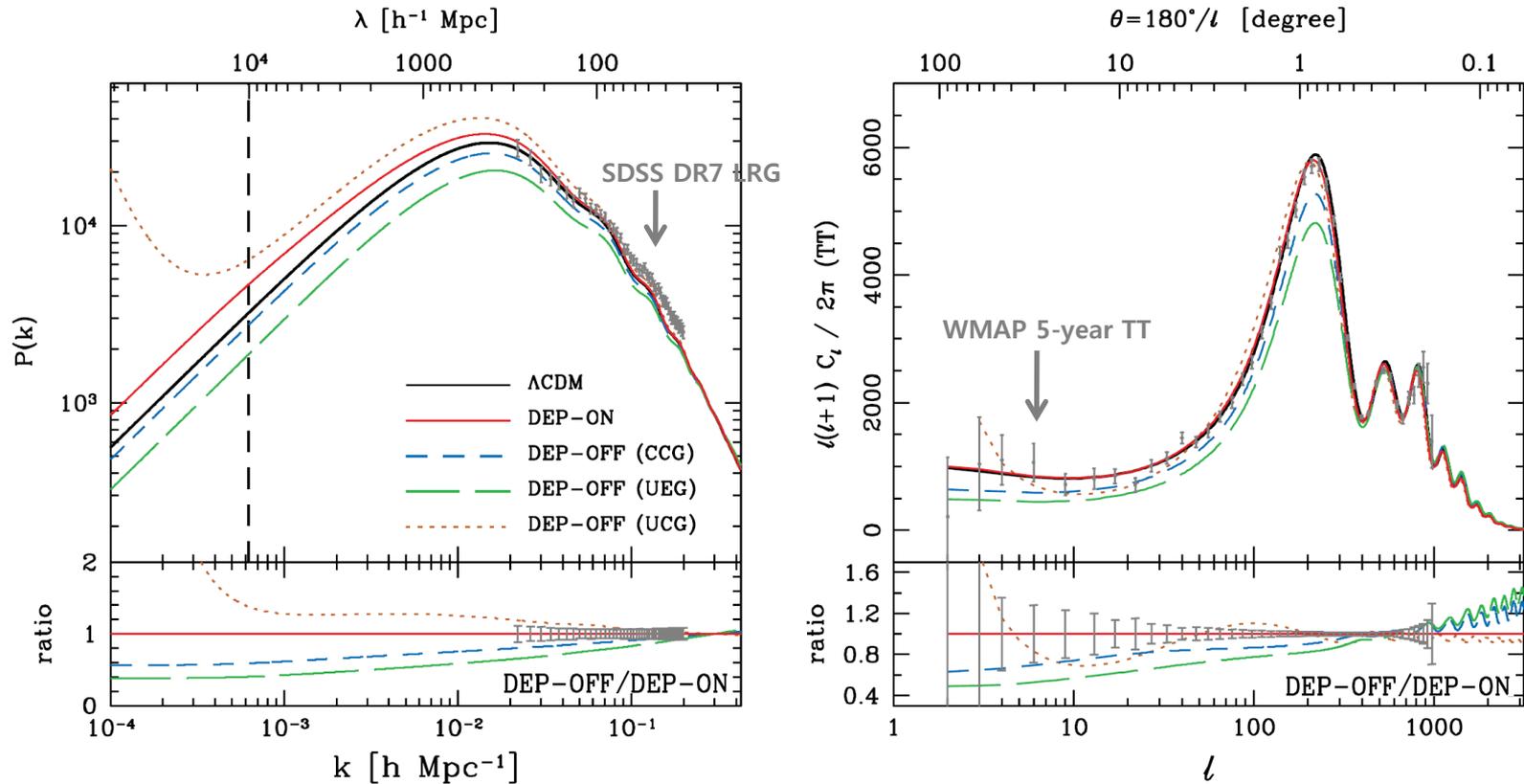


DEP-ON: All calculations are made in three different gauge conditions (CCG, UEG, and UCG). The results in the three gauges coincide exactly (red curves).

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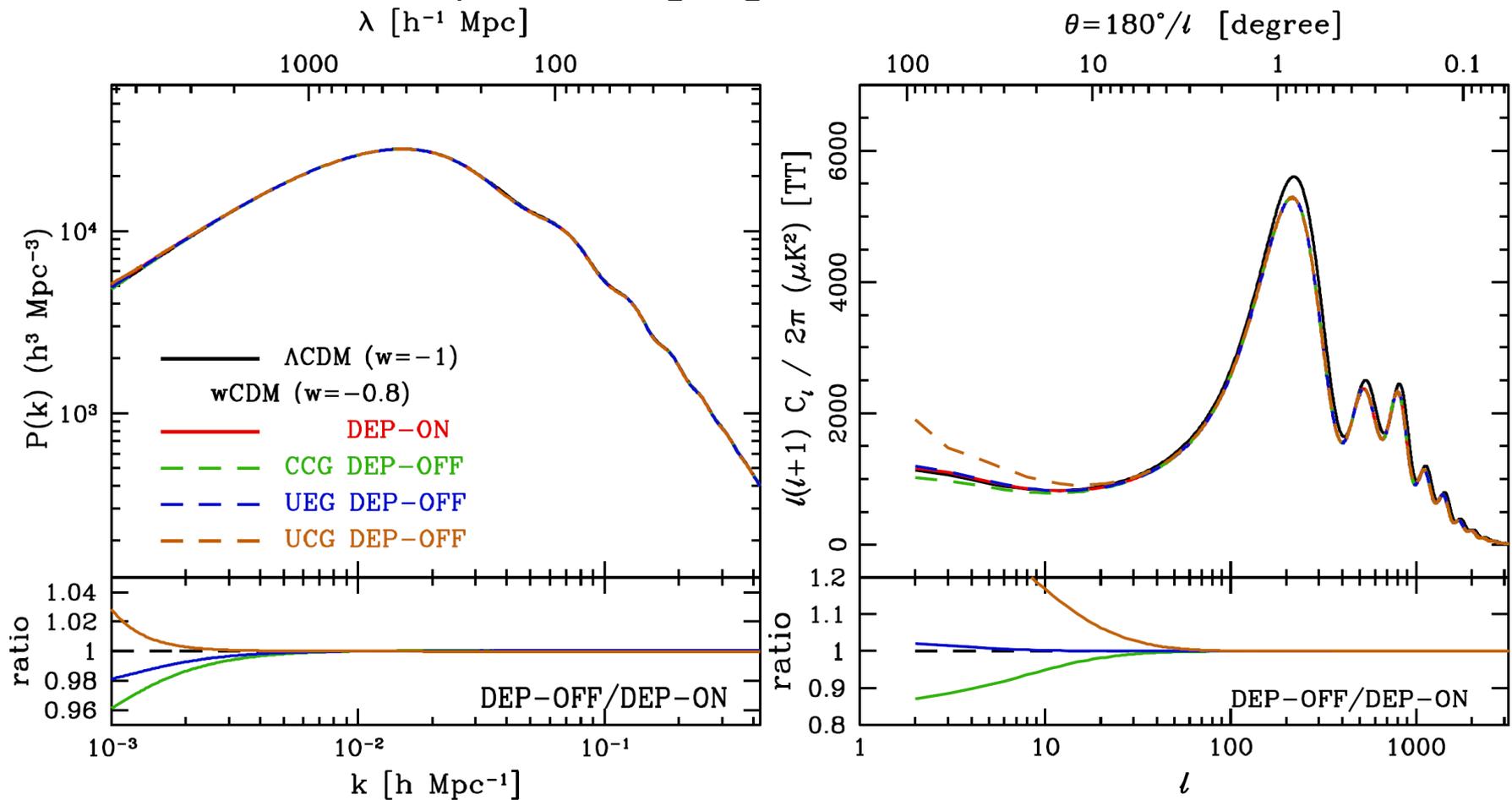
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DEP-OFF: Cases when **ignoring** DE perturbation in the **CCG**, **UEG**, and **UCG**. Observationally distinguishable substantial differences appear by ignoring DEP. By ignoring it the perturbed system of equations becomes inconsistent and deviations in (gauge-invariant) power spectra depend on the gauge choice.

# Is it safe to ignore DEP in $\Lambda$ CDM model with $w \approx -1$ ?

➔ The effect of DEP is weak near  $\Lambda$ CDM model, but it still depends on gauge choice!



# Observational constraints on dark energy models

# Parameter estimation methods

## Direct $\chi^2$ estimation method

explores the gridded parameter space to obtain the likelihood. Accurate but slow.

## Markov Chain Monte Carlo (MCMC) method

explores the parameter space by random walk based on specific criteria. Approximate but fast.

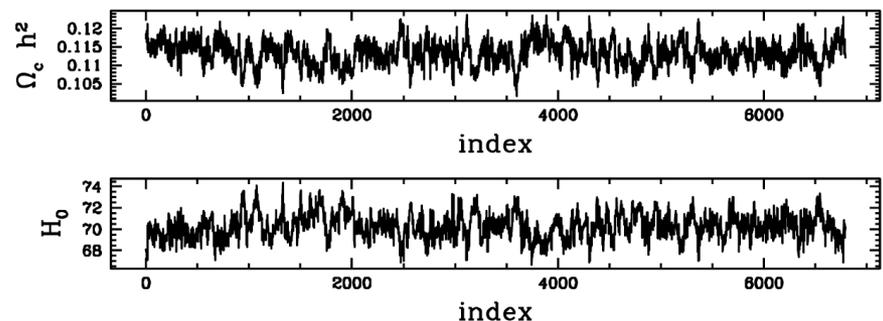
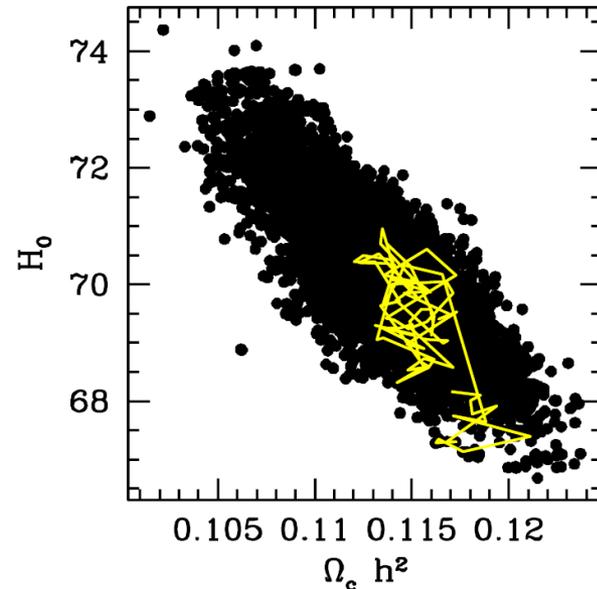
$$P(\boldsymbol{\theta}|\mathbf{D}) \propto \exp\left(-\frac{\chi^2}{2}\right), \quad \chi^2 = \sum_{i=1}^N \frac{[X_i(\boldsymbol{\theta}) - X_{\text{obs},i}]^2}{\sigma_{\text{obs},i}^2}$$

where  $\mathbf{D}$  denotes data,  $X_i(\boldsymbol{\theta})$  represents the model prediction for the  $i$ th observed data point  $X_{\text{obs},i}$  with measurement error  $\sigma_{\text{obs},i}$ , and  $N$  is the total number of data points.

# MCMC method based on Metropolis algorithm

Metropolis, et al. 1953, "Equations of State Calculations by Fast Computing Machines", J. Chem. Phys. 21, 1087-1092 (1953).

- (1) Starts from the initial parameter  $\theta_i$  .
- (2) Calculates probability  $p(\theta_i)$  .
- (3) Proposes a new parameter  $\theta_{\text{trial}}$  by random walk from the position  $\theta_i$  based on a jump distribution (usually Gaussian).
- (4) Calculates probability  $p(\theta_{\text{trial}})$  .
- (5) Makes decision whether or not to accept the new parameter with a probability  $p(\theta_{\text{trial}})/p(\theta_i)$ . If accepted,  $\theta_{i+1} = \theta_{\text{trial}}$  . Otherwise,  $\theta_{i+1} = \theta_i$  .
- (6) Repeats (3)—(5) .



# Analysis of recent type Ia supernova data based on evolving dark energy models

Park J., Park, C.-G., Hwang J., PRD [arxiv:1011.1723v2]

Friedmann equation for general  $w$ -fluid model:

$$E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{DE0}f(z) + \Omega_{K0}(1+z)^2$$

$$f(z) \equiv e^{3 \int_0^z [1+w(z)] d \ln(1+z)}$$

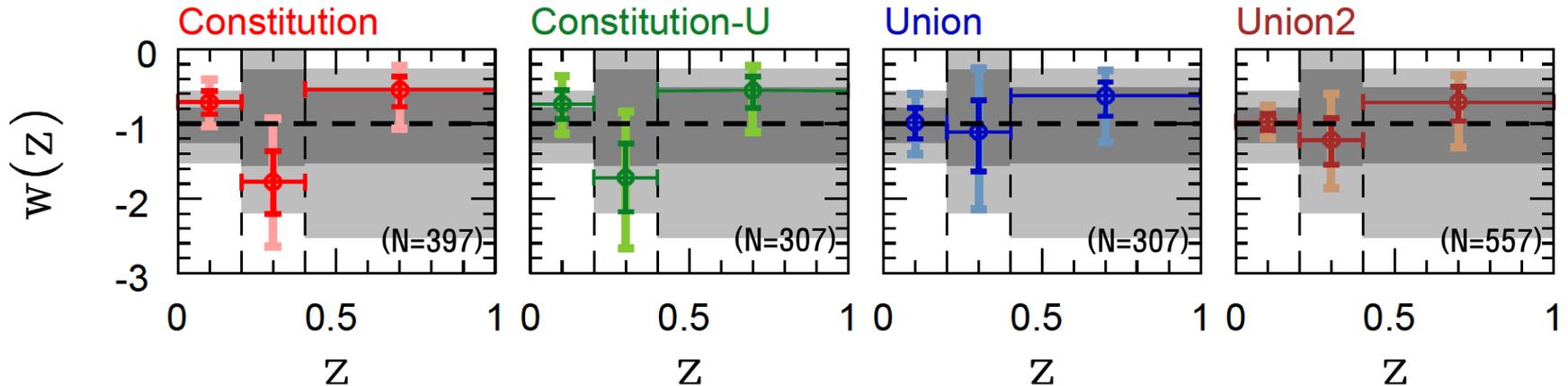
We use the piecewise constant  $w$  parameterization with sudden transitions (where  $a$  and  $\dot{a}$  are continuous)

For double transition model

$$f(z) = \begin{cases} (1+z)^{3(1+w_0)} & z < z_{\text{tr1}} \\ (1+z)^{3(1+w_1)} (1+z_{\text{tr1}})^{3(w_0-w_1)} & z_{\text{tr1}} \leq z < z_{\text{tr2}} \\ (1+z)^{3(1+w_2)} (1+z_{\text{tr1}})^{3(w_0-w_1)} (1+z_{\text{tr2}})^{3(w_1-w_2)} & z \geq z_{\text{tr2}}, \end{cases}$$

# MCMC Analysis with SNIa + BAO A+ CMB R

(Eisenstein et al. 2005)(Komatsu et al. 2009)



**Constitution-U** (a subset of Constitution) and **Union** have the same SNIa members, originating from exactly the same light-curve fit parameters (SALT; Kowalski et al. 2008).

**Noticeable differences** between Constitution-U and Union at  $w_0$  and  $w_1$  are **purely due to the different calibration** experienced during the production of distance modulus.

# Observational constraints on the quintessence with inverse power law potential

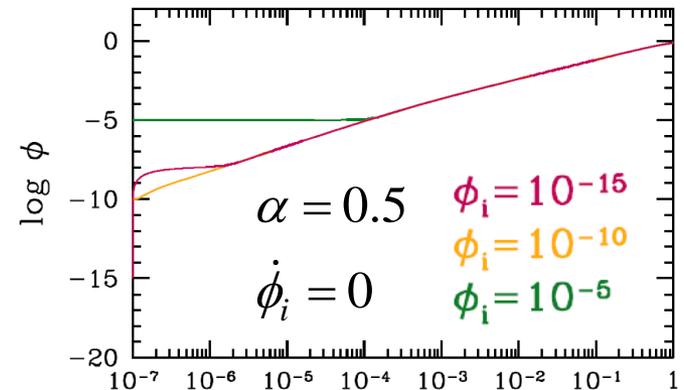
Inverse power law (IPL) model, introduced by Ratra and Peebles (1988), is the one of the simplest, and most widely investigated scalar field quintessence model.

$$V = \frac{V_0}{\phi^\alpha}$$

IPL allows the late-time cosmic acceleration. ( $\alpha=0 \rightarrow \Lambda\text{CDM}$ )

IPL exhibits **tracking behavior** where many different solutions (after some initial transient period) lock on the same attractor solution.

→ The initial conditions for  $\phi$  is irrelevant for predicting cosmological observations. No need for tuning of initial conditions which is generally seen in many other scalar field models.



# Modification of CAMB+COSMOMC

## Basic parameters of COSMOMC (for scalar-type perturbations)

$\Omega_b h^2$  : physical baryon density

Lewis and Bridle PRD 66, 103511 (2002)

$\Omega_c h^2$  : physical dark matter density

$H_0$  : Hubble constant [km/s/Mpc]

$\tau$  : the reionization optical depth

$\Omega_k$  : curvature density parameter

$w$  : the constant equation of state of the dark energy (based on quintessence)

$n_s$  : the spectral index of scalar-type perturbation

$n_{run}$  : the running of the scalar spectral index

$\log A$  :  $\ln[10^{10} A_s]$  where  $A_s$  is the primordial super-horizon power in the curvature perturbation on 0.02/Mpc scales (i.e., an amplitude parameter)

## Quintessence parameters

$\phi_i, \dot{\phi}_i$   
(background)

$\delta\phi_i, \delta\dot{\phi}_i$   
(perturbation)

$V_0, \alpha$   
(Potential parameters)

# IPL tracking-parameter space explored by COSMOMC

Tracking regime  $\log \phi_i = [-20, -5]$  &  $\phi_i' = 0$  at  $a_i = 10^{-7}$

Data used : WMAP7 + BAO +  $H_0$

Parameters varied:  $\log A$ ,  $H_0$ ,  $\Omega_c h^2$ ,  $\alpha$ ,  $V_0$ ,  $\phi_i$ . Others fixed with WMAP best-fit values

WMAP 7-year best-fit parameters (flat  $\Lambda$ CDM model, WMAP7+BAO+ $H_0$ )

$$H_0 = 70.4_{-1.4}^{+1.3} \text{ km/s/Mpc} \quad \Omega_b h^2 = 0.00226 \pm 0.00053$$

Komatsu et al.

arXiv:1001.4538v2

$$\Omega_c h^2 = 0.1123 \pm 0.0035$$

$$\Omega_\Lambda = 0.728_{-0.016}^{+0.015}$$

$$n_s = 0.963 \pm 0.012 \quad \tau = 0.087 \pm 0.014 \quad \Delta_R^2(k_0 = 0.002 \text{ Mpc}^{-1}) = (2.44_{-0.092}^{+0.088}) \times 10^{-9}$$

BAO parameters measured from 2dFGRS+SDSS DR7 (Percival et al. 2009)

$$r_s(z_d)/D_V(z=0.2) = 0.1905 \pm 0.0061$$

$$r_s(z_d)/D_V(z=0.35) = 0.1097 \pm 0.0036$$

Hubble constant data (Riess et al. 2009)

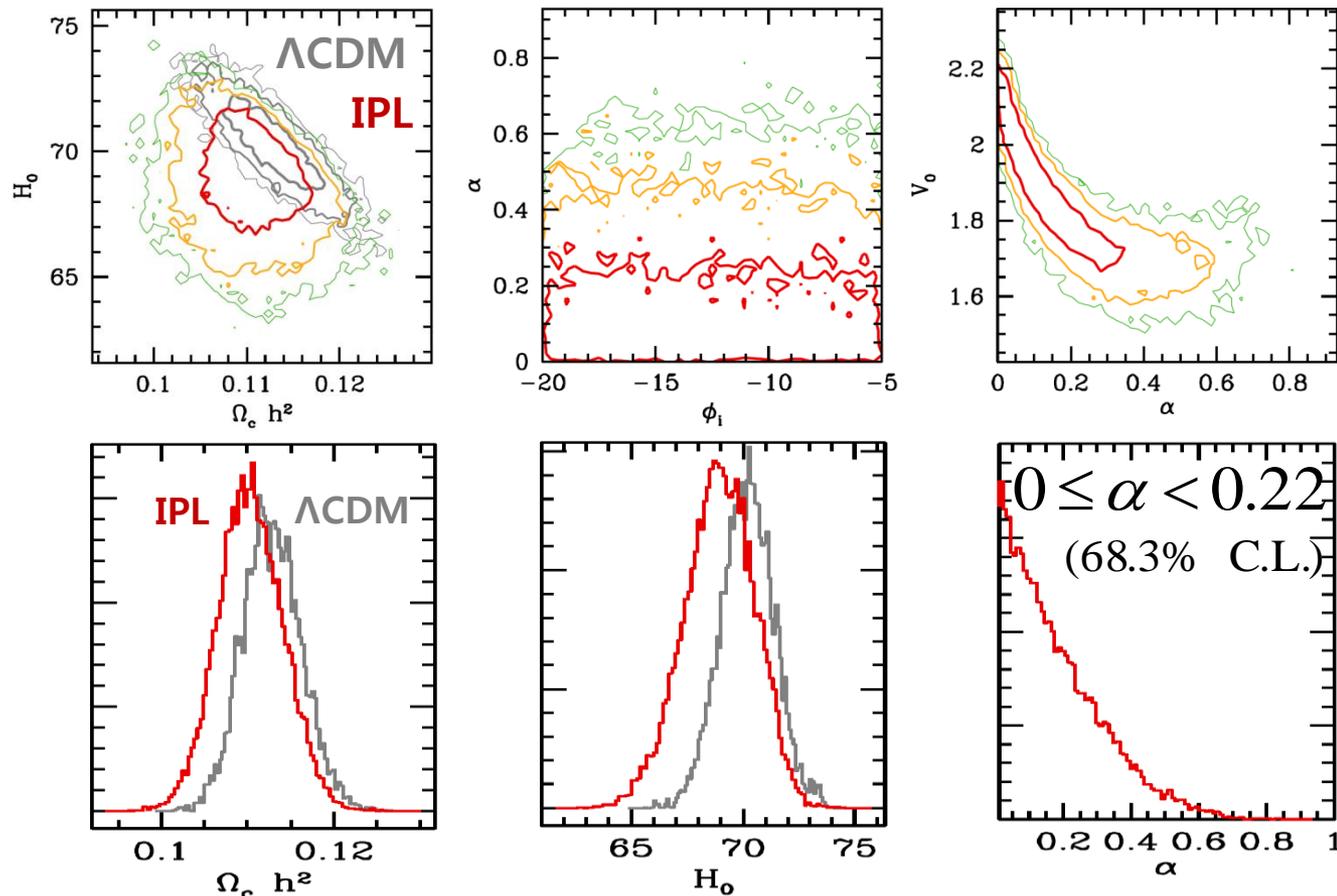
$$H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

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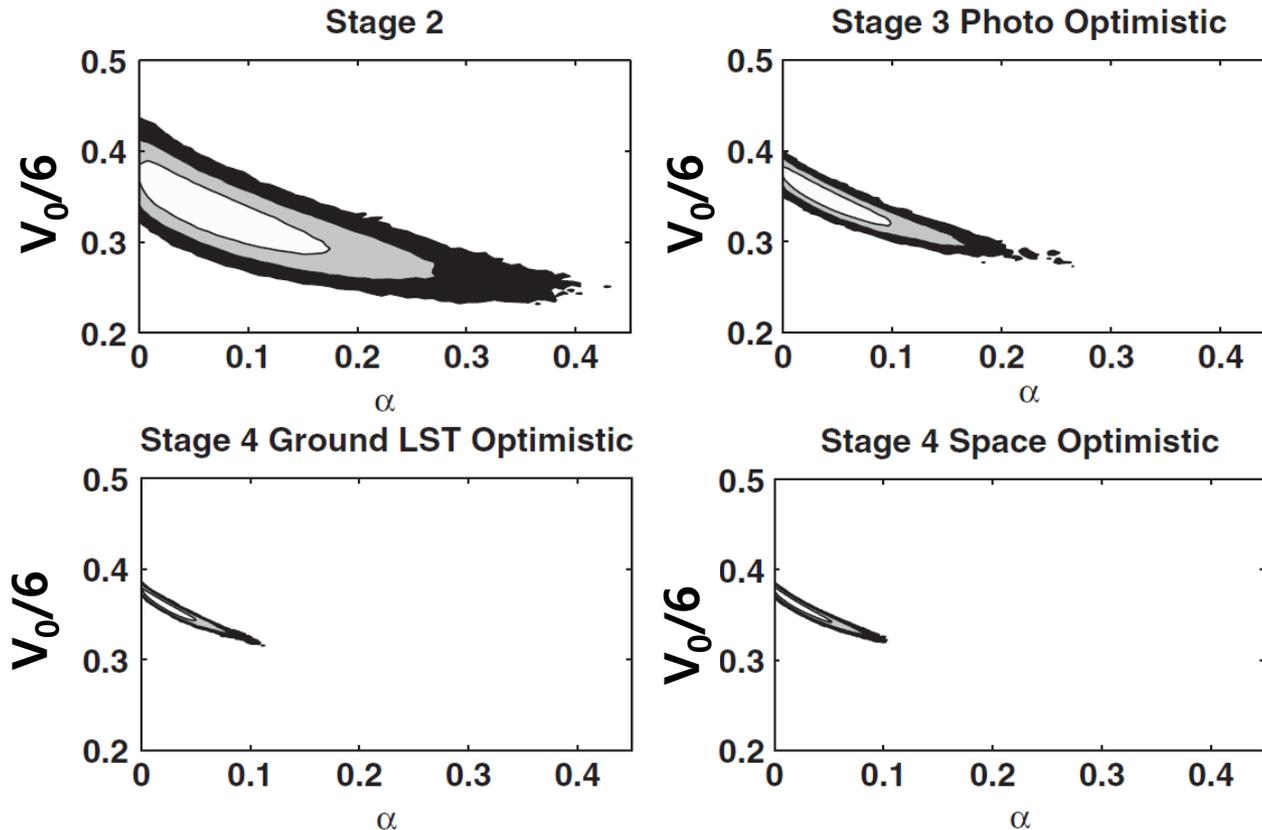
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Parameters varied:  $\log A$ ,  $H_0$ ,  $\Omega_c h^2$ ,  $\alpha$ ,  $V_0$ ,  $\phi_i$ . Others fixed with WMAP best-fit values



# Dark Energy Task Force (DETF) prediction for the tracking regime

Yashar et al. Phys. Rev. D 79, 103004 (2009).



Stage 2 : ongoing projects

Stage 3 : medium-cost (4 m class telescope), currently proposed projects

Stage 4 : Joint Dark Energy (Space) Mission or Large Survey Telescope (LST)

# Generalized Chaplygin gas (GCG) model

C.-G. Park, J. Hwang, J. Park, H. Noh, Phys. Rev. D 81, 063532 (2010).

A simple single fluid unified model of dark energy and dark matter with pressure given by

$$p_X = -A\mu_X^{-\alpha}$$

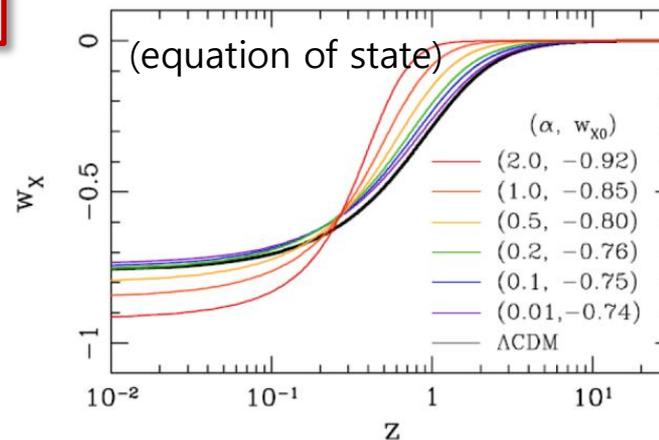
$\alpha=1$  : Chaplygin gas  
 $\alpha=0$  :  $\Lambda$ CDM

Background evolution:

$$\mu_X = \left( A + \frac{\mu_{X0}^{1+\alpha} - A}{a^{3(1+\alpha)}} \right)^{1/(1+\alpha)}$$

$$w_X \equiv \frac{p_X}{\mu_X} = - \left( 1 + \frac{\mu_{X0}^{1+\alpha}/A - 1}{a^{3(1+\alpha)}} \right)^{-1}$$

$$c_X^2 \equiv \frac{\dot{p}_X}{\dot{\mu}_X} = -\alpha w_X$$

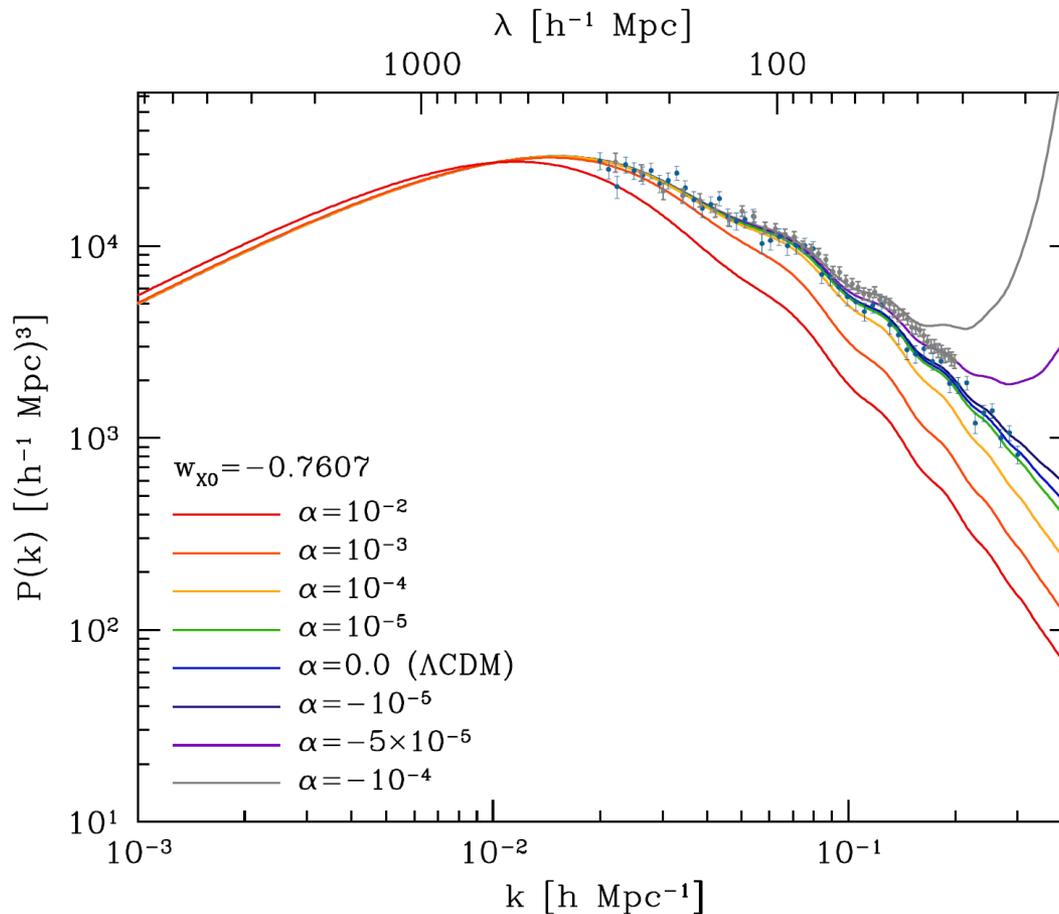


We consider a flat background with radiation, baryon, and GCG.

two free parameters  $w_{X0}$  and  $\alpha$   $\left[ A = -w_{X0}\mu_{X0}^{1+\alpha} \right]$

The  $\Lambda$ CDM limit ( $\alpha = 0$ )  $w_{X0} = -\frac{\Omega_{\Lambda 0}}{\Omega_{X0}} = -\frac{\Omega_{\Lambda 0}}{\Omega_{\Lambda 0} + \Omega_{c0}} \approx -0.76$

# Baryonic matter power spectra of GCG models near $\alpha=0$



## Data points:

SDSS DR7 LRG PS  
(window-convolved)  
Reid et al.  
arXiv:0907.1659v2

$\Lambda$ CDM-motivated mock  
PS (50 data points at  
 $k=0.02-0.3$  h/Mpc)

For negative  $\alpha$  (imaginary sound speed), power spectrum diverges at small scales due to instability. ( $\alpha < 0 \Rightarrow c_x^2 < 0$ )

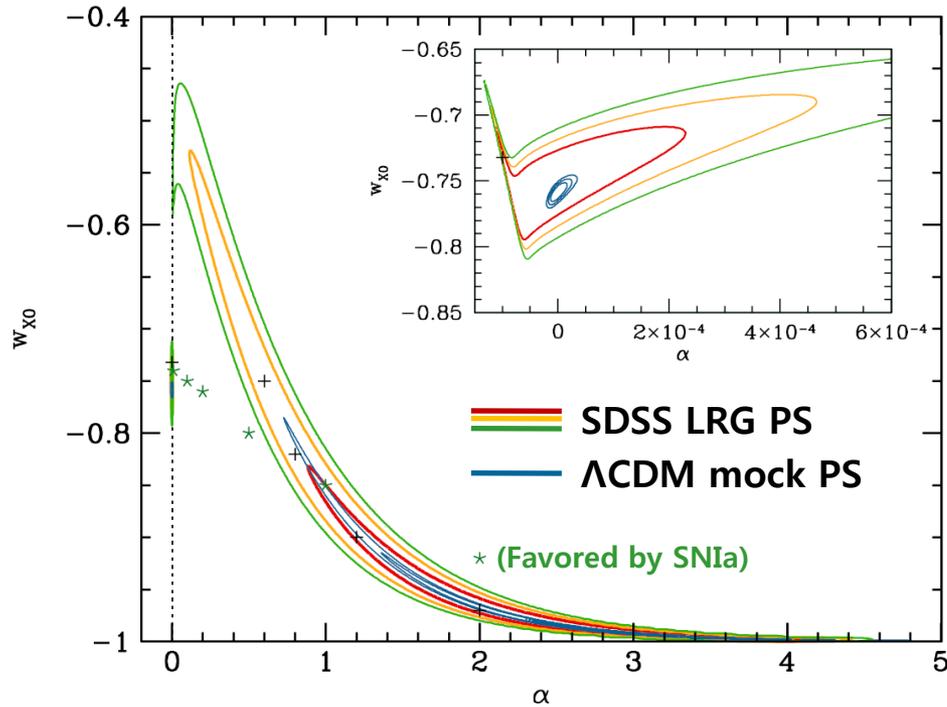
GCG model probability distribution  $\mathcal{L} \propto e^{-\chi^2/2}$

$$\chi^2 = \mathbf{d}^T \mathbf{C}^{-1} \mathbf{d}$$

**d**: vector containing GCG powers relative to LRG measurement

**C**: covariance matrix between measurement errors

# GCG model probability distribution $\mathcal{L} \propto e^{-\chi^2/2}$



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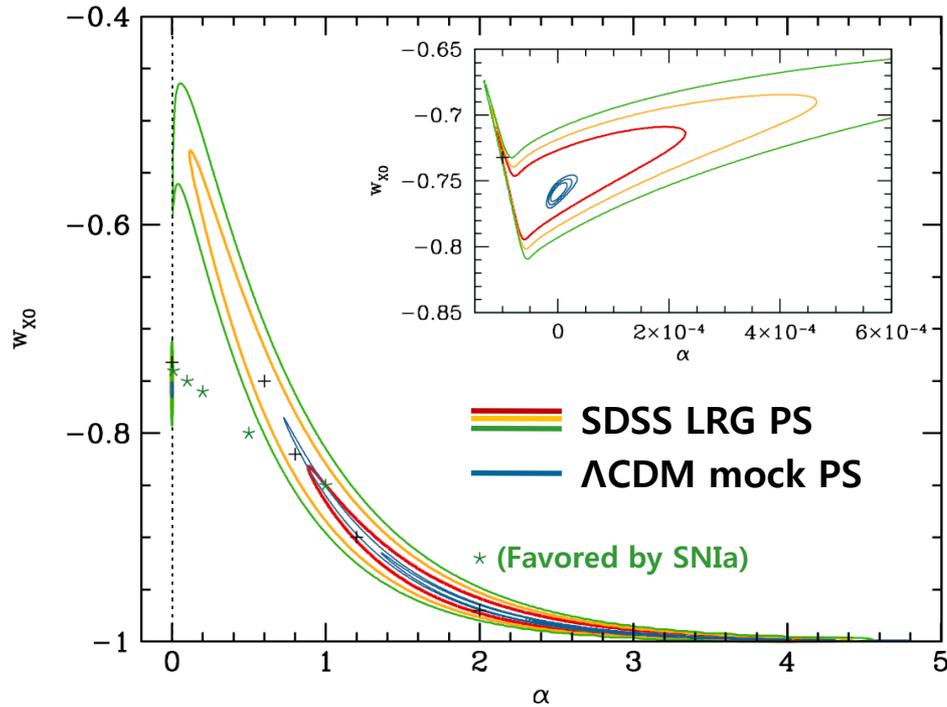
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Notice that besides the region around  $\Lambda$ CDM **near  $\alpha=0$**  (inner panel), the matter power spectrum favors another island with **positive  $\alpha$** .

This island is excluded by the CMB observation (next slide).

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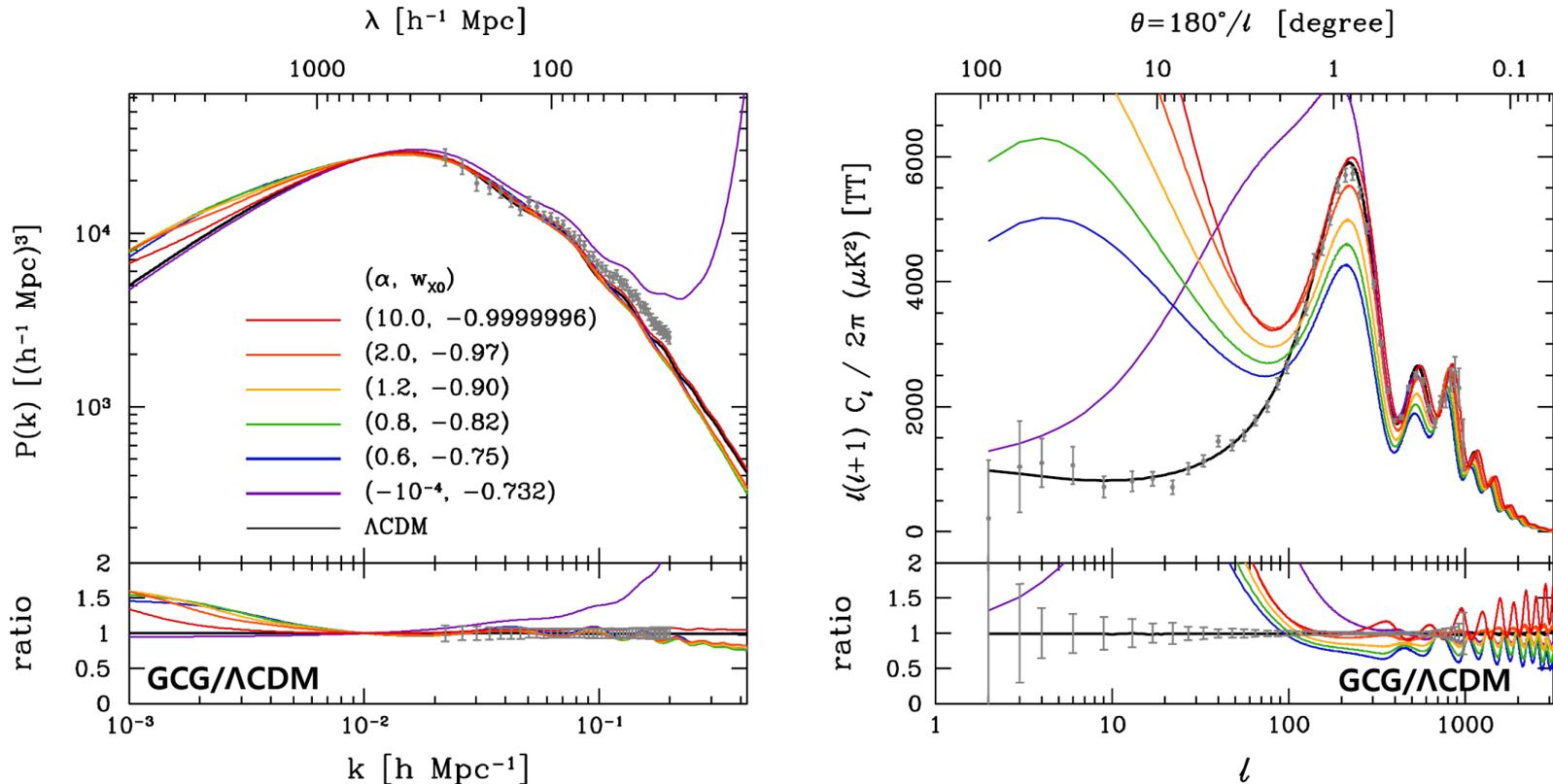
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Notice that besides the region around  $\Lambda$ CDM **near  $\alpha=0$**  (inner panel), the matter power spectrum favors another island with **positive  $\alpha$** . This island is excluded by the CMB observation (next slide).

GCG model parameter constraints (68.3% CL) around  $\alpha=0$  (near  $\Lambda$ CDM model)

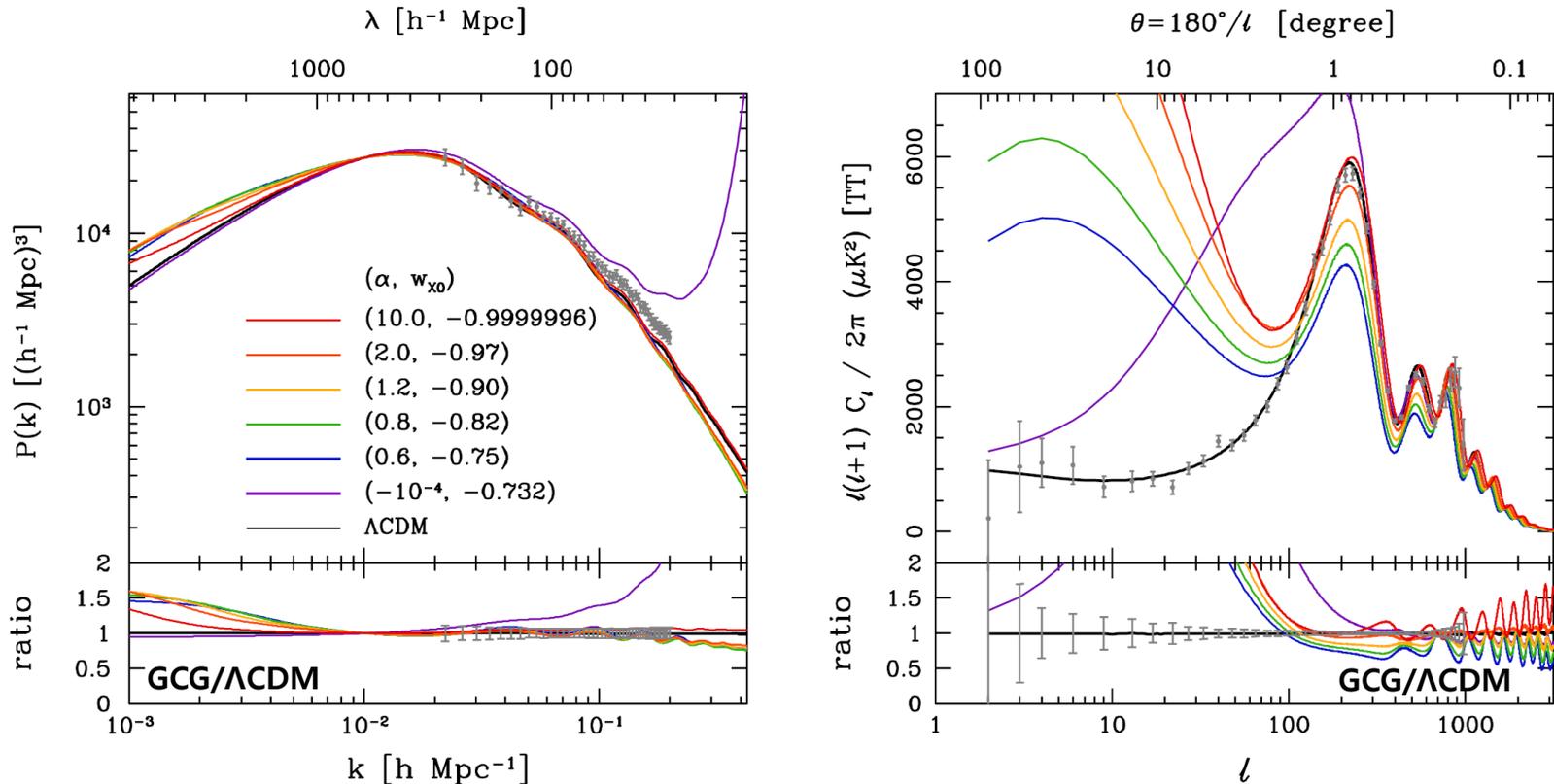
	$\alpha$	$w_{X0}$
LRG	$-5.98^{+11.3}_{-2.19} \times 10^{-5}$	$-0.756^{+0.023}_{-0.016}$
$\Lambda$ CDM	$-0.25^{+5.78}_{-5.76} \times 10^{-6}$	$-0.7585^{+0.0035}_{-0.0030}$

# Power spectra of GCG models favored by baryonic matter PS



Power spectra of GCG models with parameters indicated by "+" in the previous slide. Most of GCG parameter space is excluded by CMB observation.

# Power spectra of GCG models favored by baryonic matter PS



Power spectra of GCG models with parameters indicated by "+" in the previous slide. Most of GCG parameter space is excluded by CMB observation.

Therefore, the only parameter space extremely close to the  $\Lambda\text{CDM}$  model is allowed in the generalized Chaplygin gas model.

# f(R) gravity

Reviews:

de Felice & Tsujikawa (2010)

Sotiriou & Faraoni (2010)

Nojiri & Odintsov (2010)

Action:

$$S = \int \left[ \frac{1}{2} f(R) + L_m \right] \sqrt{-g} d^4x$$

(We use the Planck unit  
with  $8\pi G \equiv 1 \equiv c$ )

Modified Einstein equations:

$$F(R)R_{ik} - \frac{1}{2}g_{ik}f(R) + (g_{ik}\square - \nabla_i\nabla_k)F(R) = T_{ik}$$

[ $F(R) \equiv df/dR$ ]

Trace:  $FR - 2f + 3\square F = -\mu_m + 3p_m$

→ Differential equations for dynamics of modified gravity sector

$$[\text{BG}] \quad \ddot{F} + 3H\dot{F} + \frac{1}{3}(2f - FR) = \frac{1}{3}(\mu_m - 3p_m)$$

$$[\text{Pert.}] \quad \delta\ddot{F} + 3H\delta\dot{F} + \left( \frac{k^2}{a^2} - \frac{R}{3} \right) \delta F$$
$$= \dot{F}\dot{\alpha} + (2\ddot{F} + 3H\dot{F})\alpha + \dot{F}\kappa - \frac{1}{3}F\delta R + \frac{1}{3}(\delta\mu_m - 3\delta p_m)$$

# $f(R)$ gravity dark energy model with early scaling evolution

Double power-law  $f(R)$  gravity model:

Park, C.-G. Hwang, J. Noh, H.  
arXiv:1012.1662

$$f(R) = R^{1+\varepsilon} + qR^{-n} \quad (\varepsilon > 0, \quad -1 < n \leq 0)$$

$$q < 0$$

exact scaling during  
the radiation and  
matter dominated eras

late time  
acceleration

$$\varepsilon=0, n=0 \rightarrow \Lambda\text{CDM}$$

It is known that the first term  $R^{1+\varepsilon}$  which is dominant in the early epoch allows the density of gravity sector to follow that of dominant fluid (**scaling evolution**). (Amendola et al. 2007; Tsujikawa 2007)

We have derived initial conditions of background and perturbation variables during the scaling evolution regime in this modified gravity. (Details are omitted)

The value of  $\varepsilon$  is tightly constrained by the solar system test.

$$\varepsilon \lesssim 10^{-17} \text{ for } R/H_0^2 = 10^5$$

# f(R) gravity dark energy model with early scaling evolution

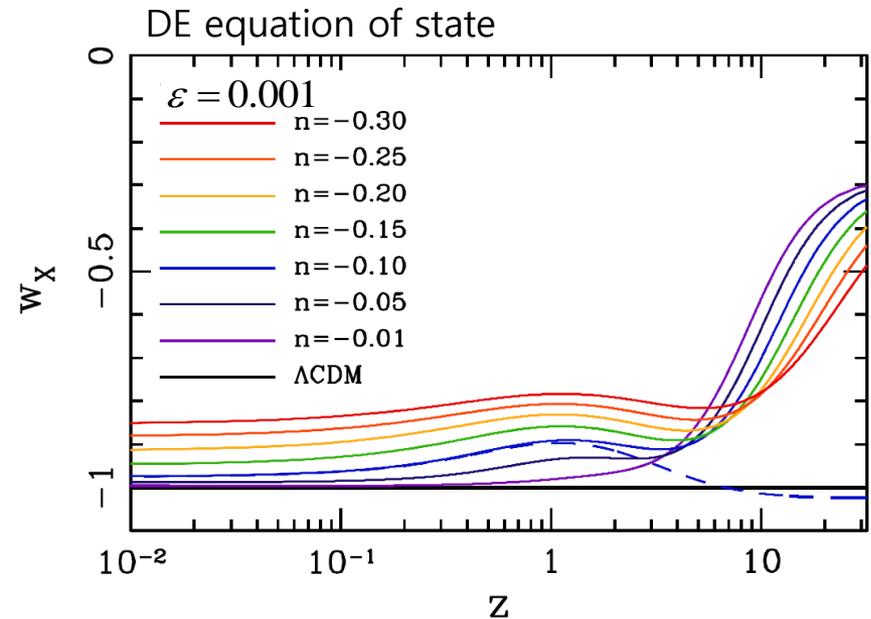
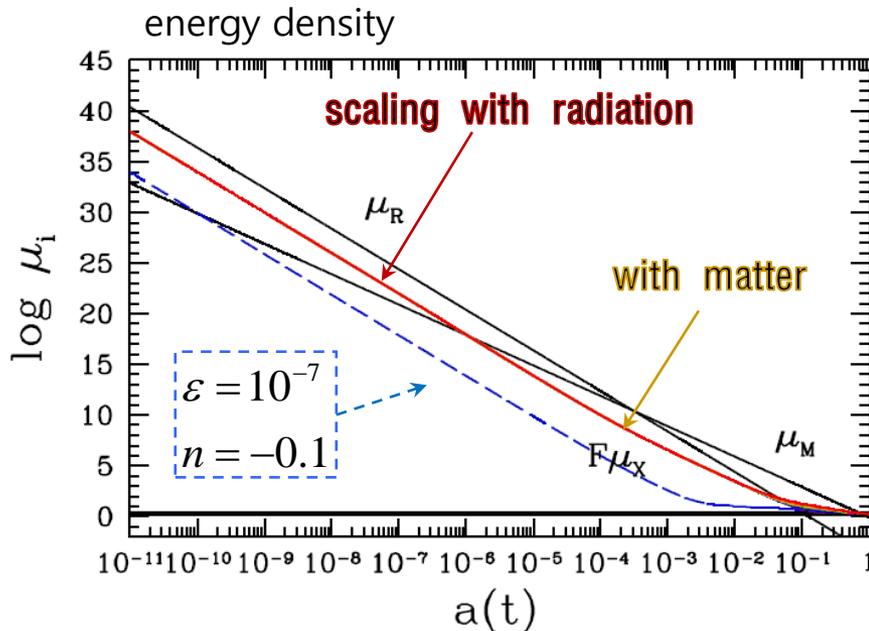
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arXiv:1012.1662

Double power-law f(R) gravity model:

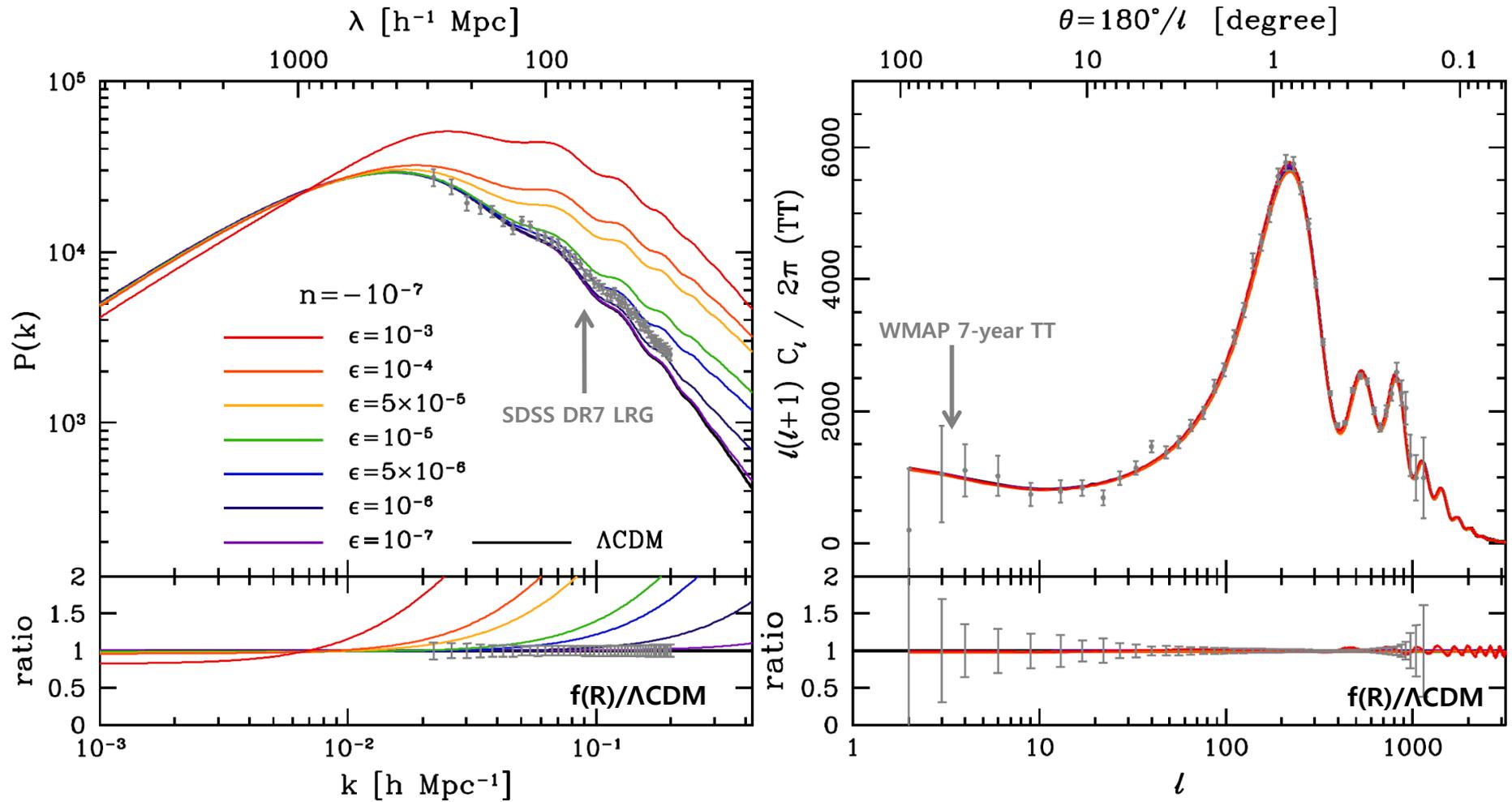
$$f(R) = R^{1+\varepsilon} + qR^{-n} \quad (\varepsilon > 0, -1 < n \leq 0) \\ q < 0$$

exact scaling during  
the radiation and  
matter dominated eras

late time  
acceleration

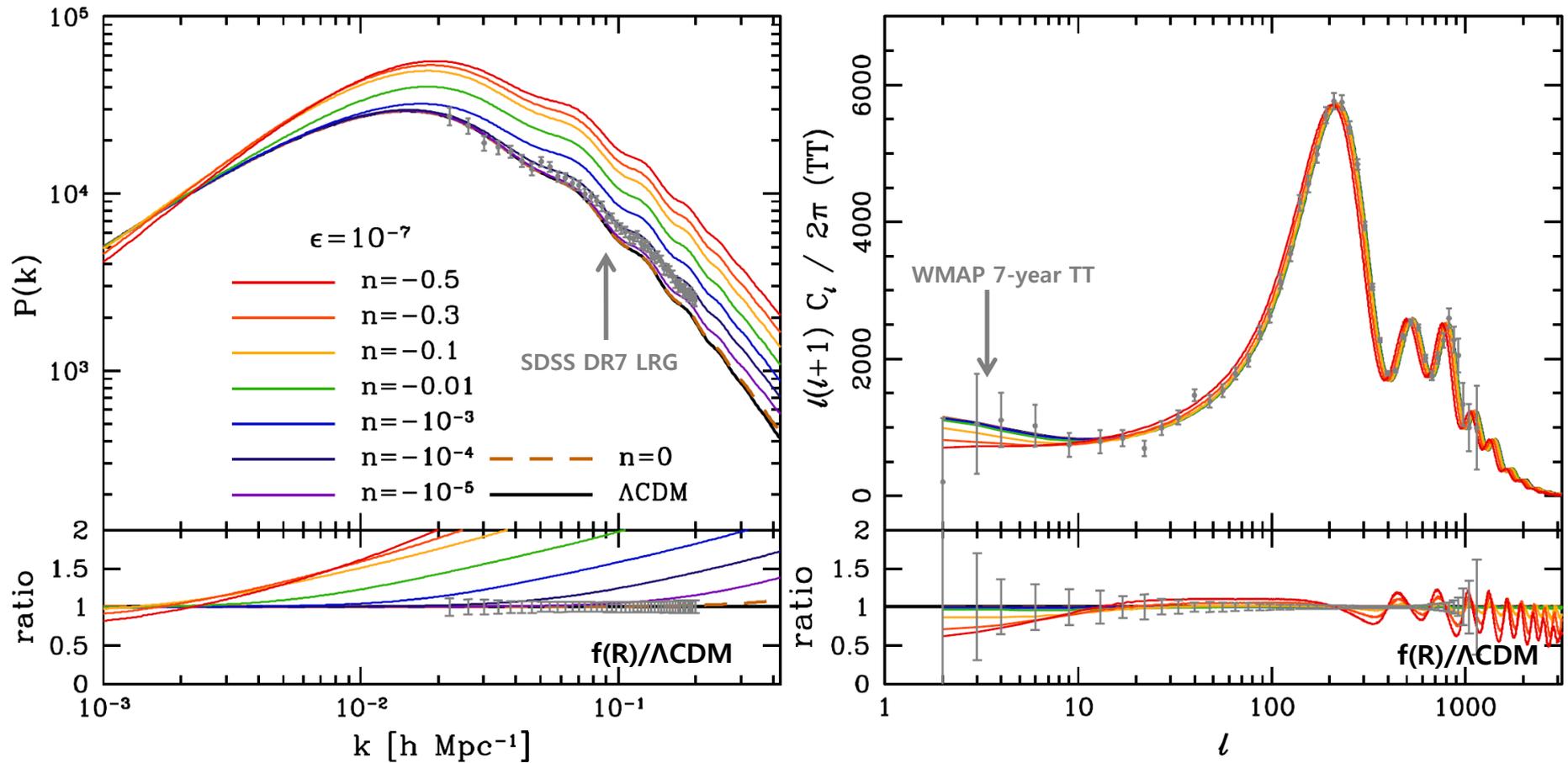


# Power spectra of $f(R)$ gravity models for varying $\epsilon$ (with $n=-10^{-7}$ )



Unlike baryonic matter power spectrum (PS), the CMB PS is not sensitive to  $\epsilon$ .

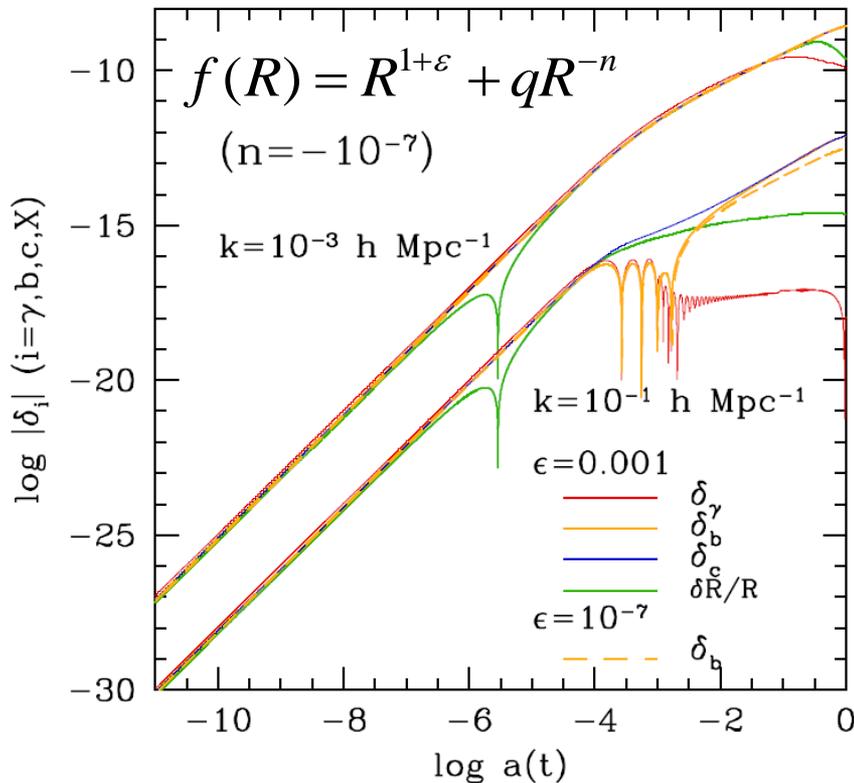
# Power spectra of $f(R)$ gravity models for varying $n$ (with $\epsilon=10^{-7}$ )



The sensitivity of CMB PS to parameter  $n$  is weak compared to baryonic matter PS.

# Perturbation growth in f(R) gravity models for varying $\epsilon$ (with $n=-10^{-7}$ )

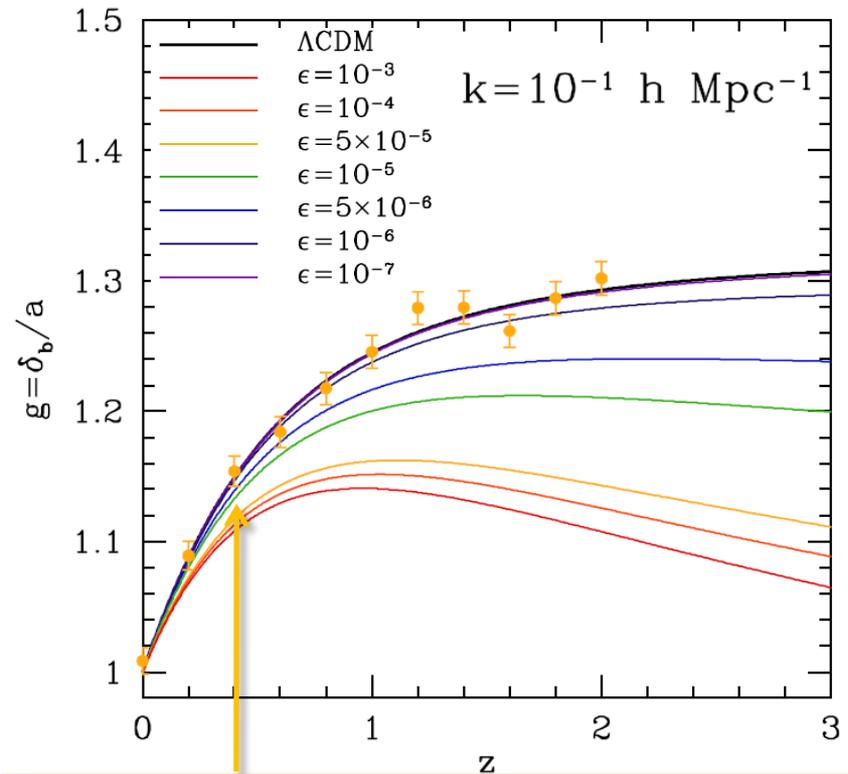
Evolution of perturbation variables



Growth factor deviations from  $\Lambda$ CDM are particularly significant at small scale.

**Perturbation growth factor:**

$$g \equiv \delta_b / a$$

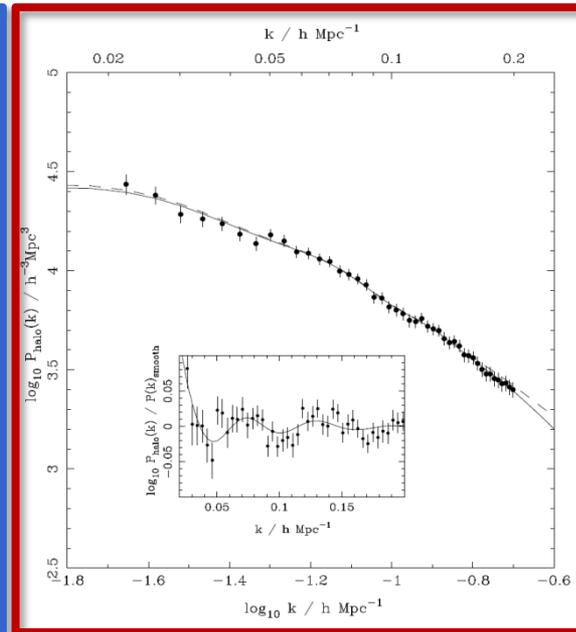
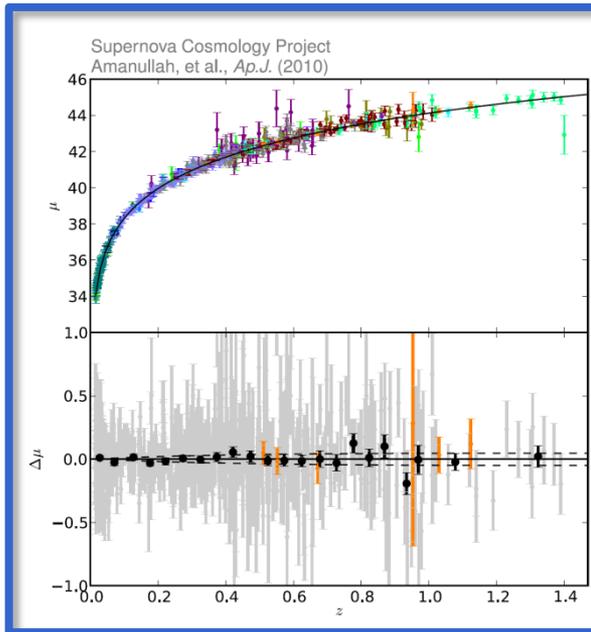


$\Lambda$ CDM-motivated mock growth factor data expected in future X-ray and weak-lensing observations (1% precision; 11 data points between  $z=0-2$ ). [Vikhlinin et al. 2009]

# Likelihood distribution of f(R) gravity parameters

We explore the  $(\epsilon, n)$ -parameter space to estimate the likelihood using SNIa, matter PS, and perturbation growth factor data.

(Other cosmological parameters are fixed with WMAP 7-yr best-fit values.)



## Data used:

SNIa (Union2)

Amanullah et al. 2010

SDSS DR7 LRG PS ( $k < 0.1 h/\text{Mpc}$ )  
(window-convolved)

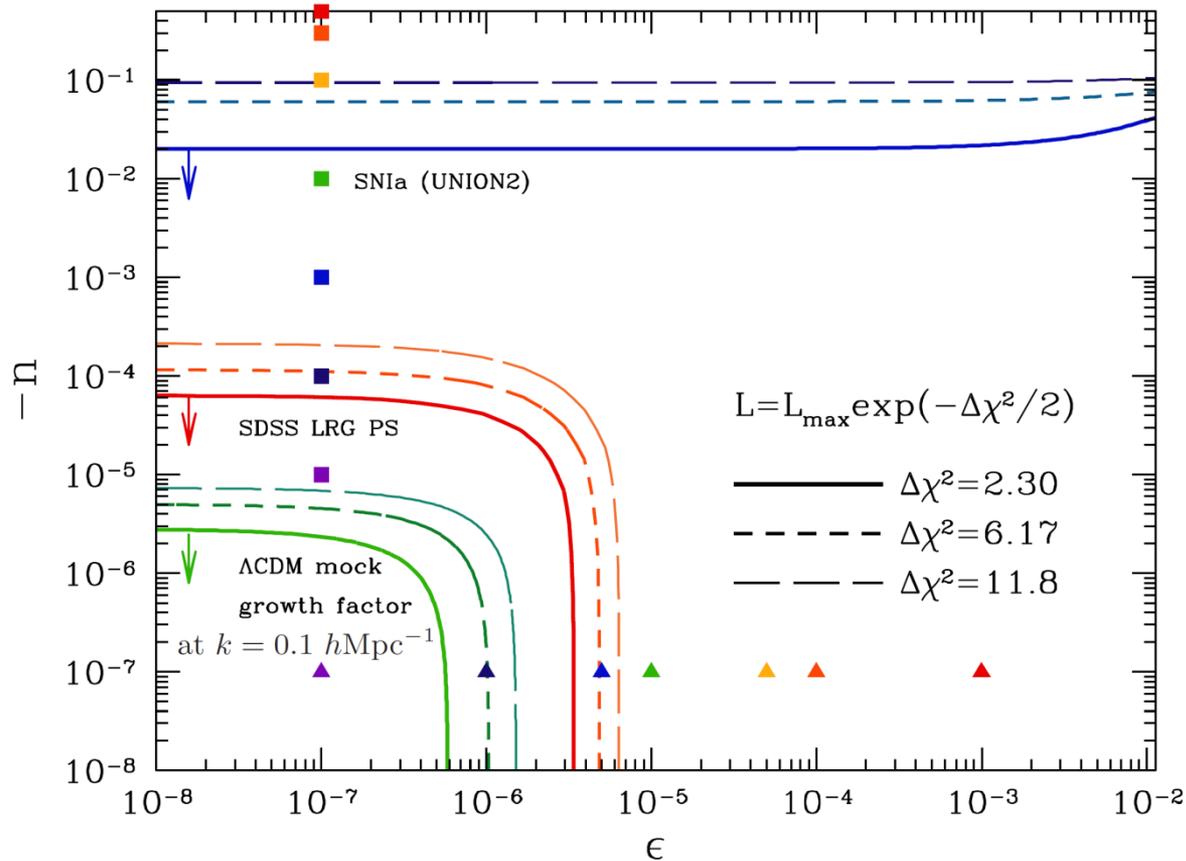
Reid et al. arXiv:0907.1659v2

$\Lambda$ CDM-motivated mock  
growth factor  
at small-scale  $k=0.1 h/\text{Mpc}$

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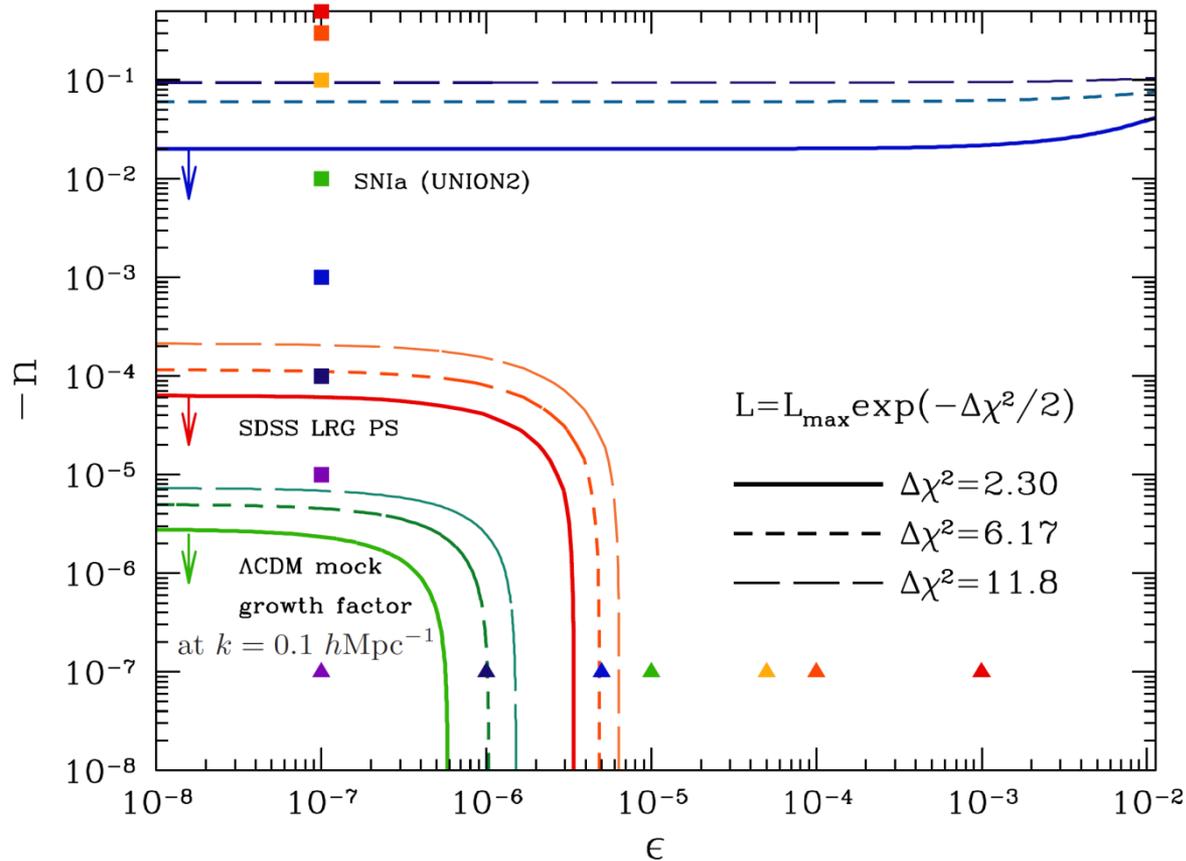
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f(R) gravity parameters,  $\epsilon$  and  $n$ , are very sensitive to the growth factor at small scales, and are already tightly constrained by the current measurement of galaxy power spectrum.

# Summary

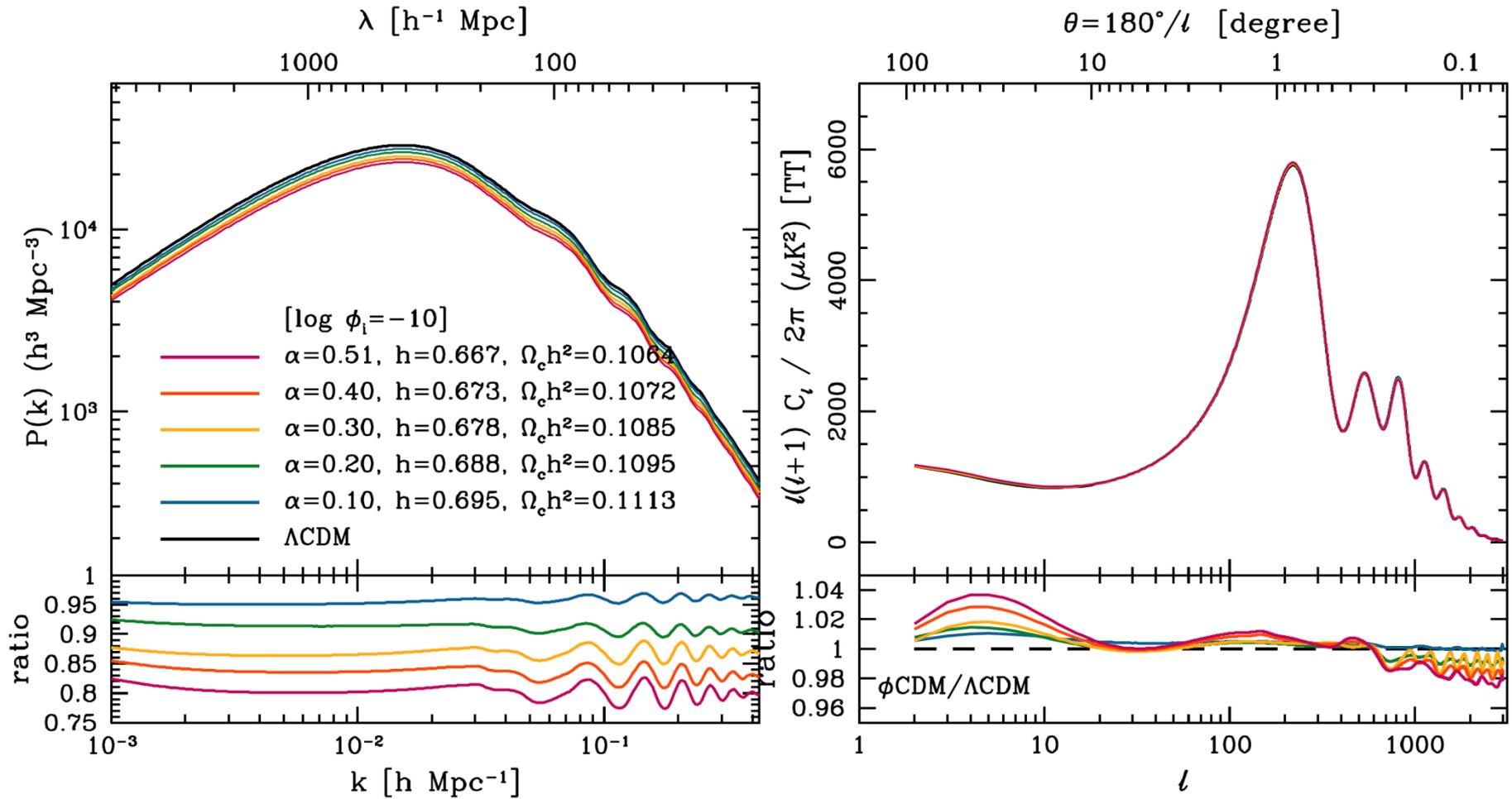
We obtained observational constraints on some dark energy models including  $w$ -fluid, quintessence, generalized Chaplygin gas, and  $f(R)$  gravity.

It is crucially important to include the dark energy perturbation. Otherwise, the system of equations becomes inconsistent, and the consequent results are not reliable compared with currently available observations.

At the current observational precision, all the dark energy models (considered in this talk) are consistent with the simplest  $\Lambda$ CDM world model.

Thank You

# Power spectra for IPL parameters favored by current observations



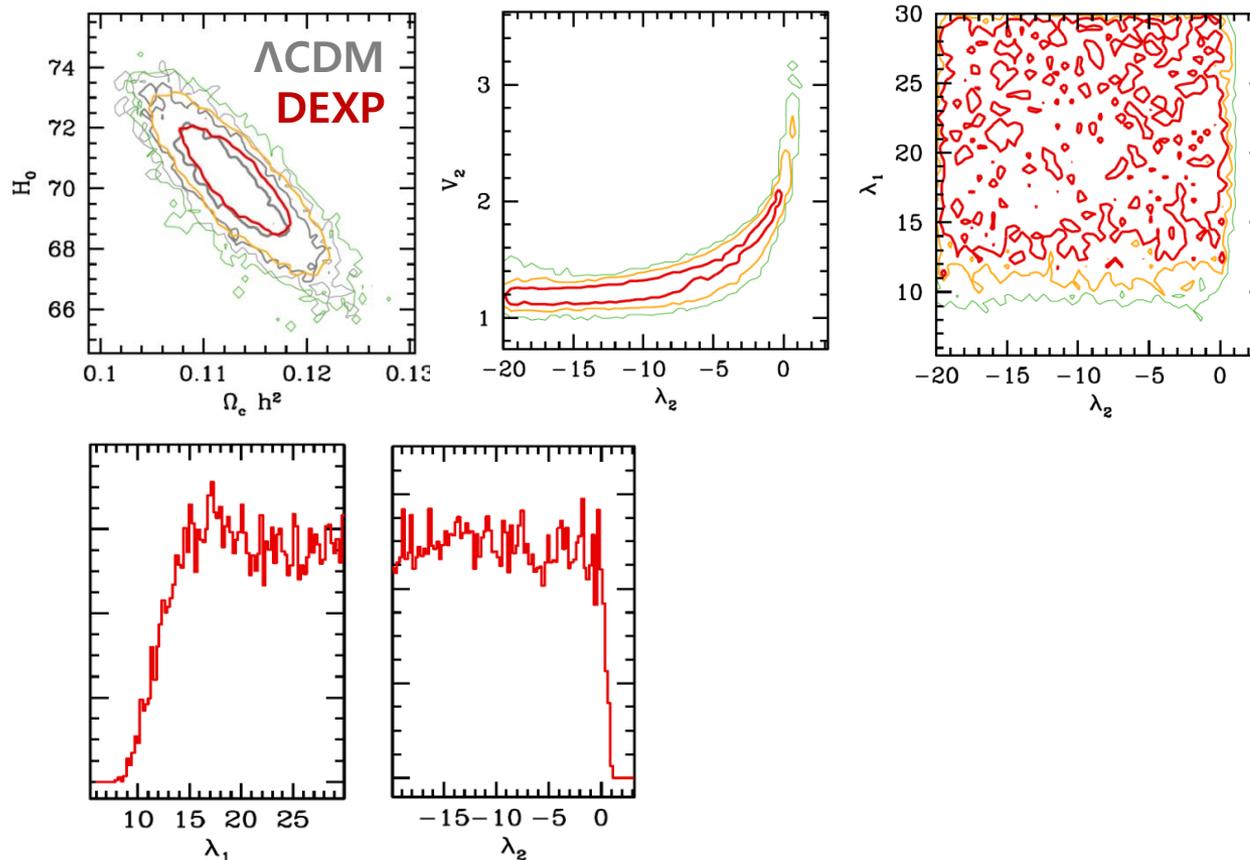
# Quintessence with double exponential potential

Early scaling regime ( $V_1=1$  with scaling initial conditions)

Data used : WMAP7 + BAO +  $H_0$

$$V(\phi) = V_1 e^{-\lambda_1 \phi} + V_2 e^{-\lambda_2 \phi}$$

Parameters varied:  $\log A$ ,  $H_0$ ,  $\Omega_c h^2$ ,  $\lambda_1$ ,  $\lambda_2$ ,  $V_2$ . Others fixed with WMAP best-fit values



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