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Large and strong scale dependent bispectrum from a sharp mass step

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FA, A. E. Romano and M. Sasaki, arXiv:1106.5384 [astro-ph.CO]

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Outline

- Motivations
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- The background analytical solution
- Linear perturbations
  - ✓ Analytical approximation to the mode function
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- The bispectrum
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- Summary and Conclusion

Motivations

- The inflaton's potential might not be a smooth function.
  - Models with features in the potential (Lagrangian) have been shown to provide better fits to the power spectrum of the CMB. Covi *et al.* '06 Joy *et al.* '08

  Introduction of new parameters (scales) that are fine-tuned to coincide with the CMB "glitches" at $\ell \sim 20-40$

  But there might be many features so the tuning can be alleviated

- Once we fix these parameters to get a better fit to the power spectrum the bispectrum signal is completely fixed: predictable

- Interesting bispectrum signatures: scale-dependence

  (e.g. "ringing" and localization of f_{NL})

  Might be due to: - particle production

  Chen '10

  - duality cascade during brane inflation
  - periodic features (instantons in axion monodromy inflation)
  - phase transitions

- These are more realistic scenarios. One can learn about the microscopic theory of inflation.

- ✓PLANCK is out there taking data, its precision is higher so the current constraints on nG will improve considerably.

⇨ It's time for theorists to get the predictions in!

The model

A toy model of a sharp transition

$$V(\phi) = \begin{cases} V_0 + \frac{1}{2} m_\phi^2 \phi^2 & : \phi > \phi_0 \\ V_{0a} + \frac{1}{2} m_\phi^2 (1 + A) \phi^2 & : \phi < \phi_0 \end{cases}$$

Continuity implies $V_{0a} = V_0 - 1/2 m_\phi^2 A \phi_0^2 \approx V_0$

FLRW background

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + m_\phi^2 \phi(t) \left[1 + A\theta(\phi_0 - \phi)\right] = 0$$

Vacuum domination assumption $H^2 \approx \dfrac{V_0}{3M_{Pl}^2}$

The background analytical solution

Before t_0

$$\phi_b(t) = \phi_{0b} e^{\lambda_b^+ H t}$$

Growing mode

$$\lambda_b^+ \approx -\mu^2/3$$
$$\lambda_a^+ \approx -\mu^2/3(1+A)$$

After t_0

Decaying solution

Decaying mode

$$\phi_a(t) = \phi_{0a} e^{\lambda_a^+ H t} + \tilde{\phi}_{0a} e^{\lambda_a^- H t}$$

$$\lambda_b^- \approx \lambda_a^- \approx -3$$

Determined by the continuity of $\phi,\ \dot{\phi}$

$$\mu^2 = \frac{m_\phi^2}{H^2}$$

Parameters in the plots

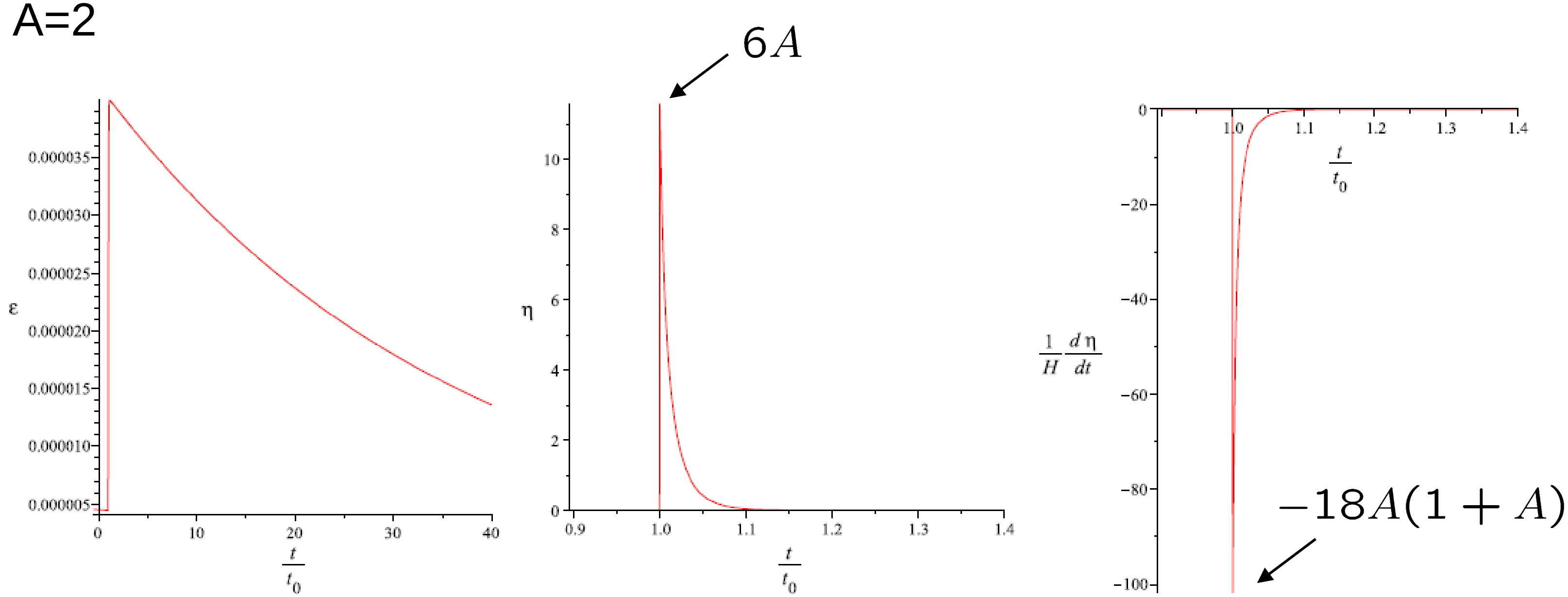
$$m_\phi = 6 \times 10^{-9} M_{Pl}, \quad H = 2 \times 10^{-7} M_{Pl}, \quad \phi_{0b} = 10 M_{Pl}$$

$$t_0 = -1/H \ln H$$

The slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2M_{Pl}^2}, \qquad \eta = \frac{\dot{\epsilon}}{\epsilon H} = 2\left(\frac{\ddot{\phi}}{\dot{\phi}H} + \epsilon\right)$$

A=2



Always small, inflation does not stop

Violate the slow-roll conditions temporarily

- Will introduce oscillations in the power spectrum and bispectrum and may produce non-Gaussian perturbations.

Linear perturbations

Comoving gauge: $\delta\phi = 0$

$$h_{ij} = a^2 e^{2\mathcal{R}} \delta_{ij}$$

Mukhanov-Sasaki eq:

$$\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + k^2\mathcal{R}_k = 0 \qquad (1)$$

$$z = a\sqrt{2\epsilon}$$

$u \equiv a\delta\phi = -\mathrm{sign}(\dot{\phi}) M_{Pl} z \mathcal{R}$

For the field pert.

$$u'' + \left(k^2 - \frac{z''}{z}\right) u = 0$$

$\mathcal{R}, \mathcal{R}', u$ are continuous

u' is not continuous

Initial conditions for numerical integration (integrate eq. (1)):

$$u_b = v \equiv \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right)$$

Mode function: analytical approximation

Starobinsky '92

Romano and Sasaki '08

$$\mathcal{R}(\tau,k) = \frac{1}{M_{Pl}a(\tau)} \begin{cases} \frac{v(\tau,k)}{\sqrt{2\epsilon(\tau)}} & : k \leq k_0 \text{ and } \tau \leq \tau_k \\ \frac{v(\tau,k)}{\sqrt{2\epsilon(\tau_k)}} & : k \leq k_0 \text{ and } \tau > \tau_k \\ \frac{v(\tau,k)}{\sqrt{2\epsilon(\tau)}} & : k > k_0 \text{ and } \tau \leq \tau_0 \\ \frac{\alpha(k)v(\tau,k)+\beta(k)v^*(\tau,k)}{\sqrt{2\epsilon(\tau)}} & : k > k_0 \text{ and } \tau_0 < \tau \leq \tau_k \\ \frac{\alpha(k)v(\tau,k)+\beta(k)v^*(\tau,k)}{\sqrt{2\epsilon(\tau_k)}} & : k > k_0 \text{ and } \tau > \tau_k \end{cases}$$

Good approximation for any time but for scales sufficiently different from the step scale k_0

$\alpha(k), \beta(k)$ Bogoliubov coefficients are determined by the matching conditions

$\Rightarrow$ $\mathcal{R}, \mathcal{R}'$ are continuous across the transition

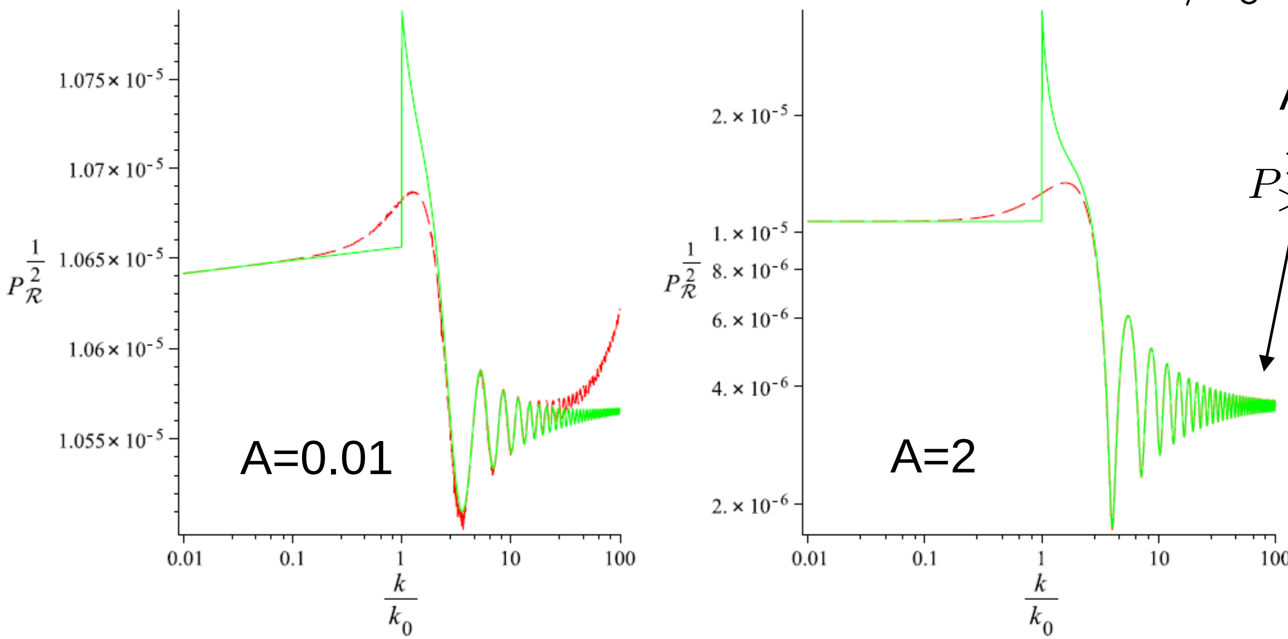
Power spectrum

$$P_{\mathcal{R}}^{1/2}(k) = \frac{H^2}{2\pi|\dot{\phi}(t_k)|} = \frac{H}{2\pi\lambda_b^+\phi_{0b}}\left(\frac{H}{k}\right)^{\lambda_b^+} \equiv P_<^{1/2}(k) \qquad k \ll k_0$$

$$P_{\mathcal{R}}^{1/2}(k) = \frac{H^2}{2\pi|\dot{\phi}(t_k)|}|\alpha_k - \beta_k| \qquad k \gg k_0$$

$$\approx \frac{H^2}{2\pi|\dot{\phi}(t_k)|}\left[1 + \frac{D_0}{k}\sin(2k\tau_0) + \frac{D_0^2}{2k^2}(1+\cos(2k\tau_0))\right]^{1/2}$$

Small scales damped oscillations with “angular frequency”: $2/k_0$



Amplitude

$$P_>^{1/2}(k) \approx \frac{P_<^{1/2}(k)}{1+A}$$

$$n_s - 1 \approx \frac{2}{3}(m_\phi/H)^2(1+A) > 0$$

Need to be small

The bispectrum

The third order interaction Hamiltonian

Maldacena '02 Collins '11

$$S_3 \supset M_{Pl}^2 \int dt d^3x [\frac{a^3\epsilon}{2}\frac{d\eta}{dt}\mathcal{R}^2\dot{\mathcal{R}} + 2\frac{\eta}{4}\mathcal{R}^2\frac{\delta L}{\delta \mathcal{R}}|_1] + M_{Pl}^2 \int dt d^3x \frac{d}{dt}[-\frac{\eta a}{2}\mathcal{R}^2\partial^2\chi]$$

These boundary terms are necessary to erase terms from the action that contain higher-derivatives in time that were generated by integrations by parts.

One integration by parts in time

$$H_{int}(\tau) = M_{Pl}^2 \int d^3x [\epsilon\eta a \mathcal{R}\mathcal{R}'^2 + \frac{\epsilon\eta}{2} a \mathcal{R}^2\partial^2\mathcal{R}]$$

One can ignore all the boundary terms that appear when one simplifies the action but then one has to perform a field redefinition to eliminate terms in the action that are proportional to the first order equation of motion.
On the other hand, one might choose to keep all the boundary terms and calculate the bispectrum using the usual method without the need to do the field redefinition.
We have shown that in the end the bispectrum of the curvature perturbation is the same in both procedures.

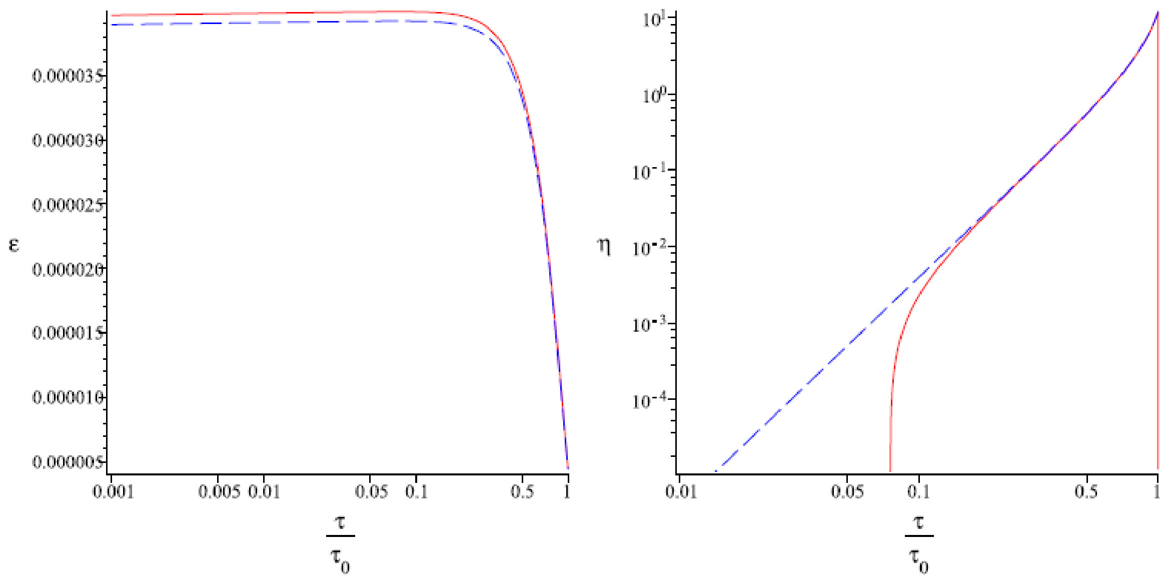
The in-in formalism formula

$$\langle\Omega|\widehat{\mathcal{R}}(\tau_e,\mathbf{k}_1)\widehat{\mathcal{R}}(\tau_e,\mathbf{k}_2)\widehat{\mathcal{R}}(\tau_e,\mathbf{k}_3)|\Omega\rangle = -i\int_{-\infty}^{\tau_e} d\tau a \langle 0|[\widehat{\mathcal{R}}(\tau_e,\mathbf{k}_1)\widehat{\mathcal{R}}(\tau_e,\mathbf{k}_2)\widehat{\mathcal{R}}(\tau_e,\mathbf{k}_3), \widehat{H}_{int}(\tau)]|0\rangle$$

Arroja and Tanaka '11

The approximate slow-roll parameters

$$\epsilon(\tau) \approx \frac{\phi_{0b}^2(\lambda_a^+)^2}{2M_{Pl}^2} a(\tau)^{2\lambda_a^+} \left[1 - \frac{A}{1+A}\left(\frac{\tau}{\tau_0}\right)^3\right]^2, \qquad \eta(\tau) \approx \frac{-6}{1 - \frac{1+A}{A}\left(\frac{\tau_0}{\tau}\right)^3}$$



Good approximation for $\tau > \tau_0$

From now, in the plots:

A=2

Definition of $\mathcal{G}/k^3$ and F_{NL}

$$\frac{\mathcal{G}(k_1,k_2,k_3;k_*)}{k_1k_2k_3} = \frac{1}{\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)}\frac{(k_1k_2k_3)^2}{(2\pi)^7 P_{\mathcal{R}}^2(k_*)}\langle\Omega|\mathcal{R}(\tau_e,\mathbf{k}_1)\mathcal{R}(\tau_e,\mathbf{k}_2)\mathcal{R}(\tau_e,\mathbf{k}_3)|\Omega\rangle$$

Smooth power spectrum amplitude

If it is of order one, PLANCK might detect it

$\mathcal{G}_>$ Small scales

$\mathcal{G}_<$ Large scales

compared with k_0

$$F_{NL}(k_1,k_2,k_3;k_*) \equiv \frac{10k_1k_2k_3}{3\sum_i k_i^3}\frac{\mathcal{G}(k_1,k_2,k_3;k_*)}{k_1k_2k_3}$$

Reduces to f_{NL} for the local model

Local model:

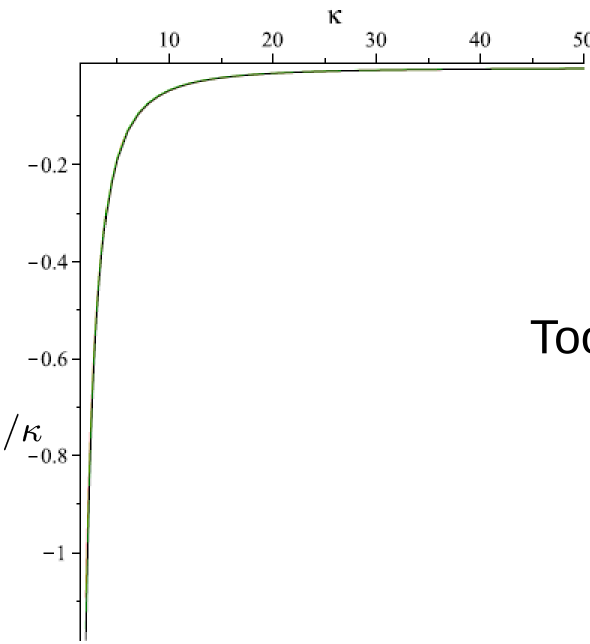
$$\langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\rangle_{local} = (2\pi)^7\delta^{(3)}\left(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3\right)\frac{3}{10}f_{NL}P_{\mathcal{R}}^2\frac{\sum_i k_i^3}{\Pi_i k_i^3}$$

The equilateral limit: large scales

$$k \ll k_0$$

$$\frac{\mathcal{G}_<(k,k,k;k_*)}{k^3} \approx -\frac{27}{160}A(8+3A)\left(\frac{k}{k_0}\right)^2$$

$$\left.\frac{\mathcal{G}_<(k,k,k;k_0)}{k^3}\right|_{k=k_0/\kappa}$$



Too small

Impossible to generate large nG in single field inflation when the scales are sufficiently outside the horizon, this is because the curvature perturbation is constant in this case.

The equilateral limit: small scales

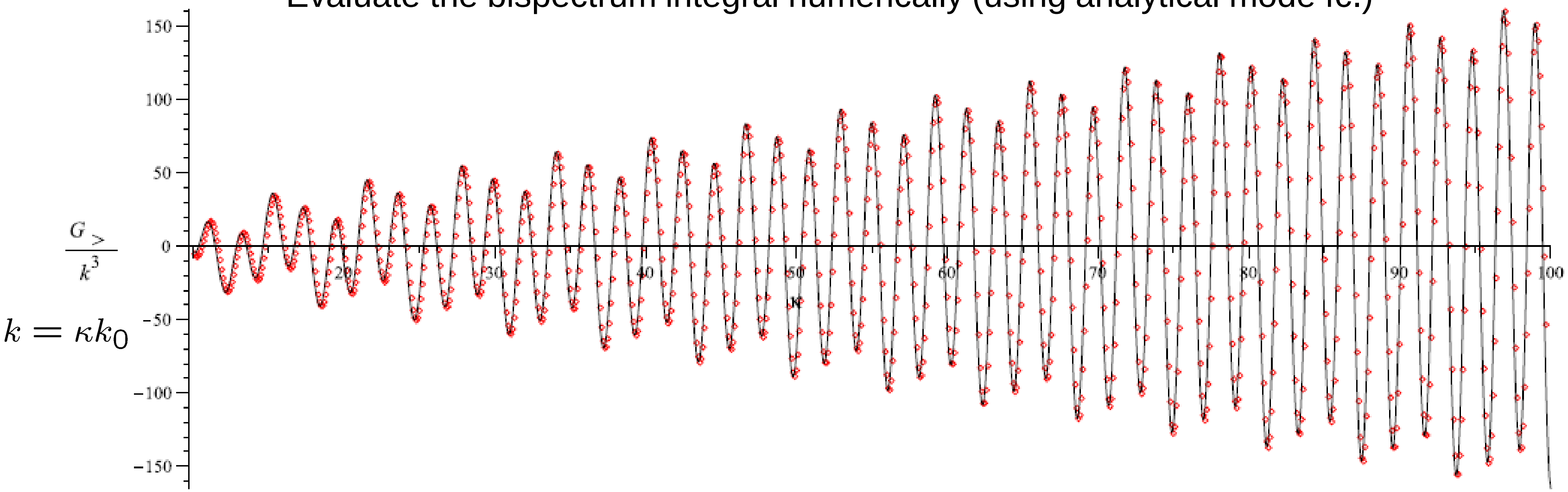
With the extra approximation on the mode function $k \gg k_0$

$$\frac{\mathcal{G}_>(k,k,k;k_*)}{k^3} \approx \frac{9}{4}\frac{A}{(1+A)^4}\left[\frac{k}{k_0}\sin\left(\frac{3k}{k_0}\right) + \frac{6-5A}{2}\cos\left(\frac{3k}{k_0}\right) - \frac{9A}{2}\cos\left(\frac{k}{k_0}\right)\right]$$

Linear growth: large enhancement factor

Unsuppressed by slow-roll

● - Evaluate the bispectrum integral numerically (using analytical mode fc.)



The extra approximation is good to find the right dependence on the wavenumber but not the amplitude. For that, we computed the integral numerically.

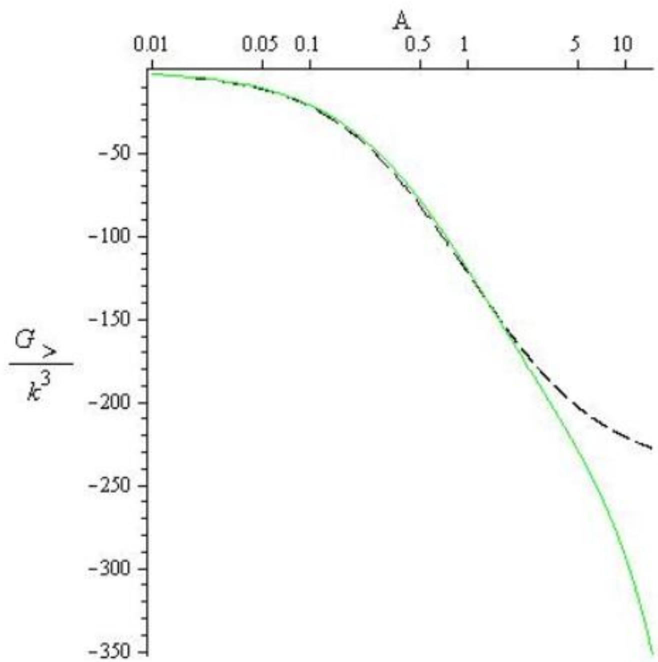
The fitting formula for the amplitude

Amplitude at $k = 100k_0$ as a function of A, without the extra approximation

$$\frac{\mathcal{G}_{>}(k,k,k;k_0)}{k^3}\Big|_{k=100k_0} \approx \frac{9}{200A(1+A)}\tilde{C}_1 A^{\tilde{C}_2} e^{\tilde{C}_3 A}$$

$$\tilde{C}_1 \approx -5157.87,\ \tilde{C}_2 \approx 2.01,\ \tilde{C}_3 \approx 0.03$$

$$\tilde{C}_1 \approx -5399.03,\ \tilde{C}_2 = 2,\ \tilde{C}_3 = 0$$



May be large

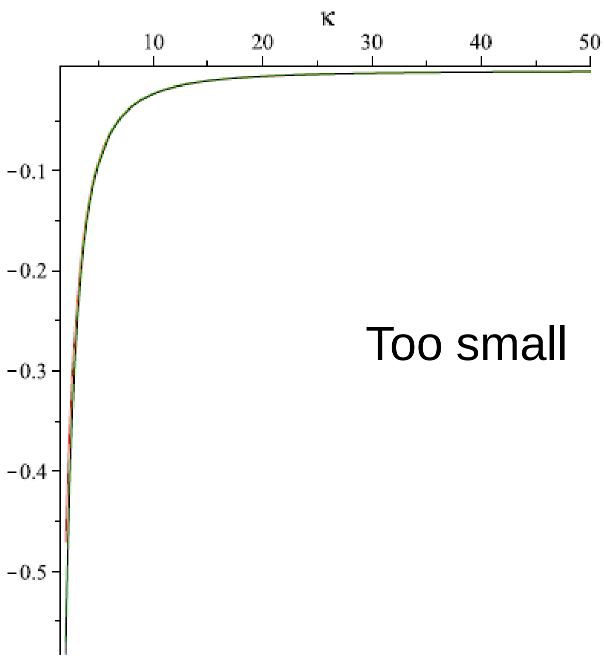
The squeezed limit: large scales

$$k_1 \ll k_2, k_3 \sim k \qquad\qquad k \ll k_0$$

$$F_{NL}^{<}(k_1, k, k; k_*) \approx -\frac{1}{12} A(8 + 3A)\left(\frac{k}{k_0}\right)^2$$

Suppressed by this ratio

$F_{NL}^{<}(k_0/500, k_0/\kappa, k_0/\kappa; k_0)$



Too small

The squeezed limit: small scales

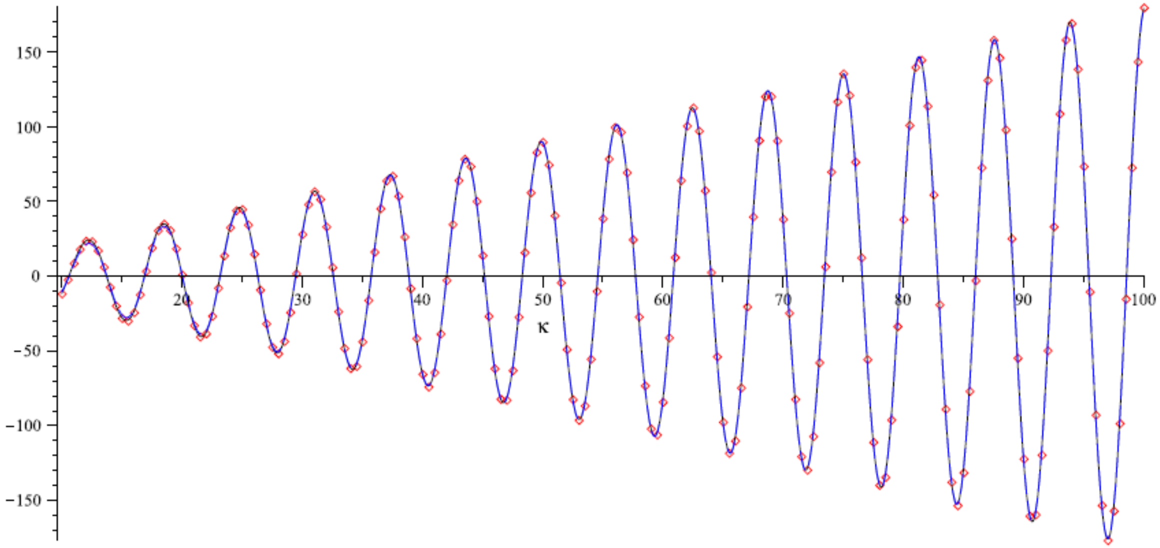
$$k_1 \ll k_2, k_3 \sim k \qquad\qquad k_0 \ll k_1$$

With the extra approximation on the mode function.

$$F_{NL}^{>}(k_1, k, k; k_*) \approx \frac{5}{4}\frac{A}{(1+A)^4}\overbrace{\frac{2k+k_1}{k_0}\frac{k_1}{k}}^{\text{Large enhancement factor}} \sin\left(\frac{2k+k_1}{k_0}\right)$$

Strong scale-dependence of the envelope

$F_{NL}^{>}(\kappa k_0, 1000k_0, 1000k_0; k_0)$



The fitting formula for the amplitude

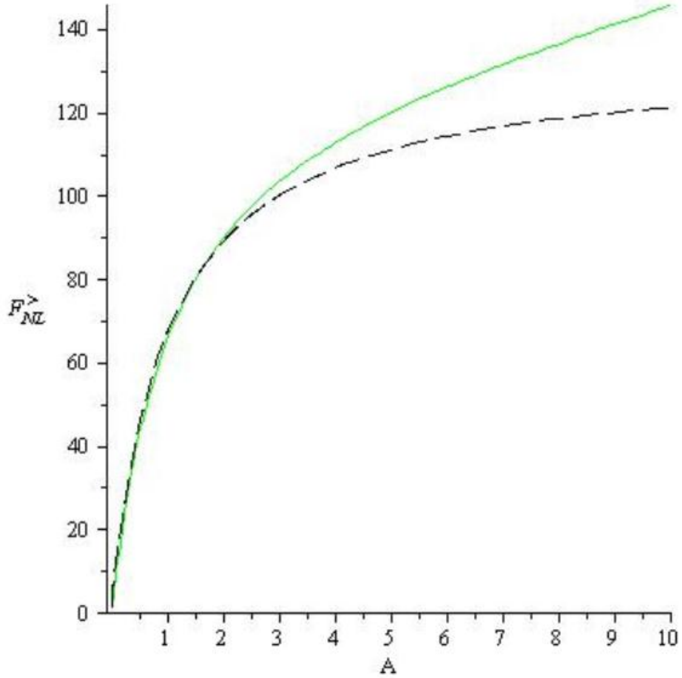
The small scales amplitude as a function of A, without the extra approximation

$$F_{NL}^{>}(50k_0, 1000k_0, 1000k_0; k_0) \approx \frac{9}{200A(1+A)} C_1 A^{C_2} e^{C_3 A}$$

$C_1 \approx 2873.31,\ C_2 \approx 2.00,\ C_3 \approx 0.02$

$C_1 \approx 2961.32,\ C_2 = 2,\ C_3 = 0$

May also be large



Some comments

- Estimate of the range of scales Δk affected by a smooth transition

  If the transition width in field space is $\Delta\phi$

  For our potential the transition happens over a number of e-foldings:

$$\Delta N \sim (3\Delta\phi)/(\mu^2\phi_0)$$

Because $\Delta k/k_0 \sim \tau_0/\Delta\tau \sim 1/(\Delta N)$

then

$$\Delta k \sim k_0/(\Delta N) \sim (\mu^2\phi_0)/(3\Delta\phi)k_0$$

If $\Delta\phi = 0$ then $\Delta k \longrightarrow \infty$ All scales can be affected

e.g. Heaviside approximation was valid for $k < 10^3 k_0$ Elgaroy *et al.* '03 Romano and Sasaki '08

(some particular more realistic model)

- This gives the cut-off scale for the small scales linear growth. For smaller scales the amplitude should go quickly to zero.

Summary and Conclusion

❖ We studied a model of single field inflation. The potential is vacuum dominated and the mass changes abruptly at a point.

➢ Slow-roll is temporarily violated. However there exists a good analytical approximation for the mode function for scales far from the transition scale.

✓ Computed the power spectrum analytically and numerically.
Found good agreement.

The small scale PS contains damped oscillations with “frequency”: $2/k_0$

✓ For the bispectrum, on large scales in both the equilateral and squeezed limits, The amplitude of the NL parameter is suppressed by the ratio $(k/k_0)^2 \ll 1$ and the amplitudes are small.

✓ For small scales, in the equilateral limit, the amplitude can be large due to the enhancement factor $\mathcal{G}_>/k^3 \propto k/k_0 \gg 1$

The “angular frequency” is $3/k_0$

Summary and Conclusion 2

✓ For small scales, in the squeezed limit, the amplitude can be also be large due to an enhancement factor $F_{NL}^{>} \propto k_1/k_0 \gg 1$

The "angular frequency" is $2/k_0$

➢ These are significant enhancements with respect to the single canonical kinetic term slow-roll models. The results are not suppressed by slow-roll.

❖ Non-vacuum initial state modifications also give significant enhancement factor but recently was shown not to be enough to be seen with PLANCK.

Agullo and Parker '10 Ganc '11

❑ These highly oscillatory signals should be orthogonal to most of other known shapes (including astrophysical and systematic effects). We might be able to use information from higher CMB multipoles; Chen '11
A new analysis and forecasts are needed.

Feature models already have started to the constrained by bispectrum observations, see e.g. Fergusson *et al.* '10