Probing Scalar-Tensor Theories of Gravity

: parametric method on both geometric and dynamical observables

Seokcheon (sky) Lee

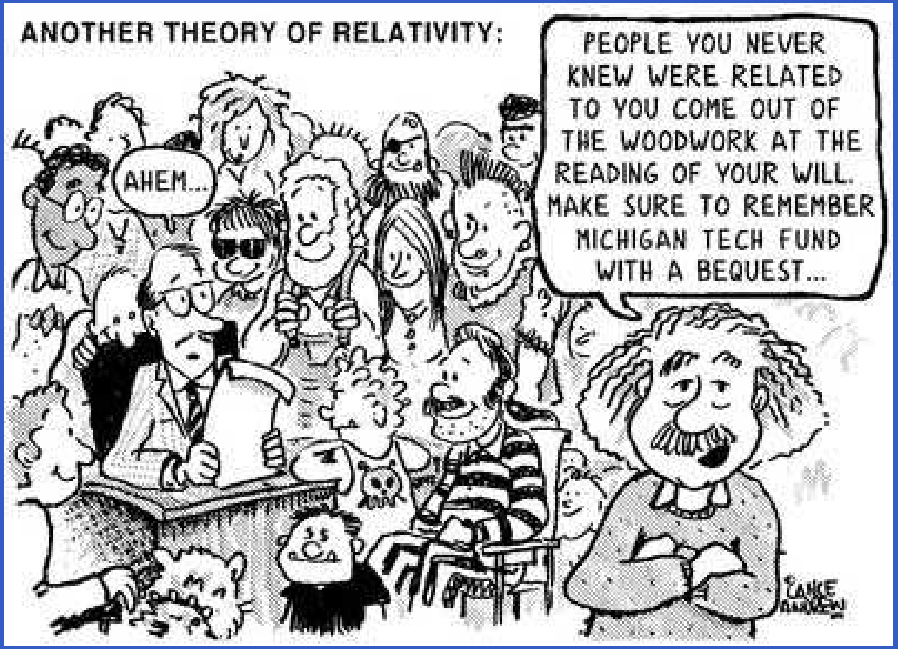
skylee@phys.sinica.edu.tw

Institute of Physics, Academia Sinica

Outline

- Alternative Theories of Gravity
  - Alternative to Dark Matter
    - Adding vector field : Einstein-æther Theories
    - Adding tensor field : Bimetric Theories
    - Adding both vector and tensor fields : TeVeS
  - Alternative to Dark Energy
    - Adding scalar field : Scalar-Tensor Theories
    - Changing HE action : f(R), Hořava-Lifschitz gravity, Galileons
    - Adding space dimension : KK, Brane, DGP, EGB gravity
- Scalar-Tensor Theories of Gravity
  - Background evolution
  - Perturbed evolution
  - Comparison with other models
- Future Work

Alternative to General Relativity ?



Alternative Theories of Gravity

Alternative to Dark Matter

- Einstein-æther Theories :

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \mathcal{L}_{EA}(g_{\mu\nu}, A_\mu)\right] + S_m(g_{\mu\nu}, \Psi)$$

- non-linear Lagrangian for æther field, A_μ :

$$\mathcal{L}_{EA}(g_{\mu\nu}, A_\mu) = \frac{1}{16\pi G}\left(M^2F(K) + \lambda(A^\mu A_\mu + 1)\right)$$

where $K^{\mu\nu}_{\alpha\beta} \equiv c_1 g^{\mu\nu}_{\alpha\beta} + c_2\delta^\mu_\alpha\delta^\nu_\beta + c_3\delta^\mu_\beta\delta^\nu_\alpha - c_4A^\mu A^\nu g_{\alpha\beta}$,

$K = K^{\mu\nu}_{\alpha\beta}\nabla_\mu A^\alpha\nabla_\nu A^\beta$, λ : Lagrange multiplier

- FLRW solutions with $\alpha \equiv c_1 + 3c_2 + c_3$:

$$\left[1 - \alpha\sqrt{K}\frac{d}{dK}\left(\frac{F}{\sqrt{K}}\right)\right]H^2 = \frac{8\pi G}{3}\rho$$

$$\frac{d}{dt}(-2H + \frac{dF}{dK}\alpha H) = 8\pi G(\rho + P)$$

- Perturbation Eqs with notation $ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\vec{x}^2\right]$:

$$k^2\Phi = -4\pi Ga^2\sum_\alpha\left[\rho^0_\alpha\delta_\alpha + 3(\rho^0_\alpha + P^0_\alpha)\mathcal{H}\theta_\alpha\right] - \frac{1}{2}\frac{dF}{dK}c_1k^2\left[V' + \Psi + (3+2c_3)\mathcal{H}V\right]$$

$$\Psi - \Phi = (c_1 + c_3)\left[\frac{dF}{dK}(2\mathcal{H}V + V') + \frac{d^2F}{dK^2}K'_0V\right]$$

Alternative to Dark Matter

- Bimetric Theories : massive gravity or bigravity

$$S_{bg} = \frac{1}{16\pi G}\int d^4x\left[\sqrt{-g}(R-2\Lambda)+\sqrt{-\tilde{g}}(\tilde{R}-2\tilde{\Lambda})-\frac{\sqrt{-\tilde{g}}}{l^2}(\tilde{g}^{-1})^{\alpha\beta}g_{\alpha\beta}\right] \quad \text{where } \tilde{\Lambda}=\frac{\alpha}{l^2}$$

- FLRW solution : $g_{\alpha\beta}=(-1,a^2,a^2,a^2), \;\; \tilde{g}_{\alpha\beta}=(-X^2,Y^2,Y^2,Y^2)$

$$H^2=\frac{8\pi G}{3}(\rho+\tilde{\rho})\equiv\frac{8\pi G}{3}\rho_c \quad \text{where } \tilde{\rho}=\frac{1}{8\pi G l^2}\frac{Y^3}{Xa^3}$$

$$\frac{d\tilde{\omega}}{dt}=2\tilde{\omega}\left[1+3\tilde{\omega}+\sqrt{4(-\tilde{\omega})^{3/2}\tilde{\Omega}\alpha-2(1+3\tilde{\omega})\frac{\rho_l}{\rho_c}}\right] \quad \text{where } \rho_l\equiv\frac{1}{8\pi G l^2}$$

- extra metric mimic the effects of dark matter

- Perturbation Eqs :

$$k^2\Phi=-4\pi G a^2\sum_\alpha\left[\rho_\alpha^0\delta_\alpha+3(\rho_\alpha^0+P_\alpha^0)\mathcal{H}\theta_\alpha\right]$$

$$k^2(\Psi-\Phi)=-8\pi G a^2\sum_\alpha P_\alpha\pi_\alpha$$

refer Hu (1998), Hwang & Noh (2002)

Alternative to Dark Matter

- Tensor-Vector-Scalar Theories : Bekenstein frame

  $$S = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\Big[\tilde{R} - \frac{1}{2}\Big(KF^{\mu\nu}F_{\mu\nu} - 2\lambda(A_\mu A^\mu + 1)\Big) - \Big(\mu\hat{g}^{\mu\nu}\tilde{\nabla}_\mu\phi\tilde{\nabla}_\nu\phi + V(\mu)\Big)\Big]$$

  where μ is a non-dynamical scalar field

- Metric : $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$ and $d\tilde{s}^2 = -d\tilde{t}^2 + b^2(\tilde{t})d\vec{x}^2$

  $a = be^{-\phi}$ and $dt = e^{\phi}d\tilde{t}$

- Hubble parameter : $\tilde{H} = \frac{d\ln b}{d\tilde{t}}e^{\phi}H + \frac{d\phi}{d\tilde{t}}$

- FLRW solution :

  $$\tilde{H}^2 = \frac{8\pi G}{3}e^{-2\phi}(\rho + \rho_\phi)$$

  where $\rho_\phi = \frac{e^{2\phi}}{16\pi G}\left(\mu\frac{dV}{d\mu} + V\right)$ and $P_\phi = \frac{e^{2\phi}}{16\pi G}\left(\mu\frac{dV}{d\mu} - V\right)$

- extra metric mimic the effects of dark matter

- Perturbation Eqs :

  $$k^2\Phi = -4\pi Ga^2\sum_\alpha\left[\rho^0_\alpha\delta_\alpha + 3(\rho^0_\alpha + P^0_\alpha)\mathcal{H}\theta_\alpha\right] - e^{4\phi}\tilde{\mathcal{H}}(3\tilde{\Phi}' + k^2\tilde{\zeta} + 3\tilde{H}\tilde{\Phi})$$

  $$k^2(\Psi - \Phi) = -8\pi Ga^2\sum_\alpha P_\alpha\pi_\alpha - e^{4\phi}\left(\tilde{\zeta}' + 2(\tilde{\mathcal{H}} + \tilde{\Phi})\tilde{\zeta}\right)$$

Alternative to Dark Energy

- Scalar-Tensor Gravities of Theory :

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left[F(\phi)R - Z(\phi)\nabla^\mu\phi\nabla_\mu\phi - 2U(\phi)\right] + S_m(g_{\mu\nu},\psi_m)$$

where $F(\phi) > 0$ is required to ensure that the gravity is attractive

- FLRW solutions :

$$FH^2 = 8\pi G_*(\rho_m + \rho_{rad}) + \frac{1}{2}Z\dot{\phi}^2 - 3H\dot{F} + U$$

$$2F\dot{H} = -8\pi G_*(\rho_m + \frac{4}{3}\rho_{rad}) - Z\dot{\phi}^2 - \ddot{F} + H\dot{F}$$

$$Z\left(\ddot{\phi} + 3H\dot{\phi}\right) = \frac{1}{2}F_{,\phi}R - \frac{1}{2}Z_{,\phi}\dot{\phi}^2 - U_{,\phi}$$

- Perturbation Eqs :

$$\delta''_m + \left(2 + \frac{H'}{H}\right)\delta'_m - \frac{4\pi G_{eff}\rho_m}{H^2}\delta_m \simeq 0$$

$$\frac{k^2}{a^2}\Phi \simeq -4\pi G_{eff}\rho_m\delta_m$$

$$\eta = \frac{(\Phi-\Psi)}{\Psi} \simeq -\frac{F^2_{,\phi}}{ZF+3F^2_{,\phi}}$$

where $G_{eff} = \frac{G_*}{F}\left(\frac{2Z(\phi)F+4F^2_{,\phi}}{2Z(\phi)F+3F^2_{,\phi}}\right)$

Alternative to Dark Energy

- f(R) :

  Replace Hilbert-Einstein action term R with $f(R)$

  metric formalism : $\omega_{BD} = 0$ Palatini formalism : $\omega_{BD} = -fr32$ if $\mathcal{L}_m$ is independent of Γ

  since there is no equivalence between BD theory and true metric affine f(R) gravity, one cannot derive a conclusion about whether certain models will pass the solar system tests, by using a PPN expansion of Brans.Dicke theories

- Hořava-Lifschitz :

  non-renormalisability arises due to coupling constant M_{pl}^{-2}

  HL is non-relativistic and relies on anisotropic scaling btw t and $\vec{x}$

  GR recovered in the IR by including additional relevant operators

- Galileons :

  GR on perturbed Minkowski space is modified by an additional single scalar field, the galileon, with derivative self interactions

Alternative to Dark Energy

- Kaluza-Klein Theories :

  To unify gravity with EM

  size of compacttification is $10^{-19}m$

  How to shrink to 4D ?

- Brane models :

  To solve hierarchy problem $M_{pl}^2 \sim M_D^{2+n} L^n$

  SM fields are not universal

- Dvali-Gabadadze-Porrati Gravity :

  graviton is a resonance of finite width $1/r_c$

  normal branch and self-accelerating branch

  ghost-like instabilities of self-accelerating branch

  at $r \ll r_c$: branch dynamics do not feel the width of the resonance $\rightarrow$ 4D GR

  at $r \gg r_c$: resonance decays into continuum KK modes $\rightarrow$ 4D GR

Scalar-Tensor Theories of Gravity

- Motivations :
  - existence a ubiquitous fundamental scalar coupled to gravity in unified theories
  - dynamical equivalence between f(R) theories and a particular class of STG
  - the lithium problem in BBN might be solved in STG due to the slower expansion than in general relativity before BBN, but faster during BBN
  - WL shear power spectrum in STG is different from GR
  - ISW effect
  - phantom crossing

Scalar-Tensor Theories of Gravity

- Action :

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left[F(\phi)R - Z(\phi)\nabla^\mu\phi\nabla_\mu\phi - 2U(\phi)\right] + S_m(g_{\mu\nu},\psi_m)$$

where $F(\phi) > 0$: gravity is attractive

matter fields ψ_m is universally coupled to the metric $g_{\mu\nu}$

- FLRW solutions :

$$3F_0H^2 = 8\pi G_*\rho_m + \frac{1}{2}H^2\phi'^2 - 3H^2F' + 3H^2(F_0 - F) + U$$

$$2F_0HH' = -8\pi G_*\rho_m - H^2\phi'^2 - H^2F'' + (H^2 - HH')F' + 2HH'(F_0 - F)$$

$$\phi'' + \left(3 + \frac{H'}{H}\right)\phi' = 3\left(2 + \frac{H'}{H}\right)\frac{F'}{\phi'} - \frac{1}{H^2}\frac{U'}{\phi'}$$

where primes mean differentiate w.r.t $n = \ln a$

$R = 6(2H^2 + \dot{H})$ and $G_{eff} = \frac{G_*}{F}\left(\frac{2Z(\phi)F + 4F_{,\phi}^2}{2Z(\phi)F + 3F_{,\phi}^2}\right)$

- BD theory is obtained from $F = \phi$ and $Z = \frac{\omega_{BD}}{\phi}$
- current limit $G_{eff}(z=0) - G_N(z=0) = 0.02\%$ and $\left|\dot{G}_{eff}/G_{eff}\right| < 6\times10^{-12}yr^{-1}$
- positive energy does not imply that $\dot{\phi}^2 > 0$ either $U_{,\phi\phi} \neq m_\phi^2$ due to the mixture of tensor and scalar degrees of freedom in the JF

Scalar-Tensor Theories of Gravity

- Perturbations :

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)d\vec{x}^2$$

$$\delta_m \equiv \frac{\delta\rho_m}{\rho_m} + 3Hv$$

$$3F'\Phi' + \left(2\lambda^{-2}F - Z\phi'^2 + 3F'\right)\Phi =$$

$$-\left[\frac{8\pi G_*\rho_m}{H^2}\delta_m + \left(\lambda^{-2} - 6 - 3\frac{F'^2}{F^2}\right)\delta F + \frac{\delta U}{H^2} + 3\frac{F'}{F}\delta F' + Z\phi'\delta\phi' + 3Z\phi'\delta\phi + \frac{1}{2}\delta Z\phi'^2\right]$$

$$2F(\Psi' + \Phi) + F'\Phi = 8\pi G_*\rho_m\frac{v}{H} + Z\phi'\delta\phi + \delta F' - \delta F$$

$\Psi - \Phi = \frac{\delta F}{F}$ where $\lambda^2 = \frac{a^2H^2}{k^2}$

$$\delta\phi'' + \left(3 + \frac{H'}{H} + \frac{Z_{,\phi}}{Z}\phi'\right)\delta\phi' + \left[\lambda^{-2} - 3(2 + \frac{H'}{H})\left(\frac{F_{,\phi}}{Z}\right)_{,\phi} + \frac{1}{H^2}\left(\frac{U_{,\phi}}{Z}\right)_{,\phi} + \left(\frac{Z_{,\phi}}{Z}\right)_{,\phi}\frac{\phi'^2}{2}\right]\delta\phi$$

$$= \left[\lambda^{-2}(\Phi - 2\Psi) - 3\left(\Psi'' + (4 + \frac{H'}{H})\Psi' + \Phi'\right)\right]\frac{F_{,\phi}}{Z} + (3\Psi' + \Phi')\phi' - 2\frac{\Phi}{Z}\frac{U_{,\phi}}{H^2}$$

$$\delta_m'' + \left(2 + \frac{H'}{H}\right)\delta_m' + \lambda^{-2}\Phi = 3(\Psi + Hv)'' + \left(6 + 3\frac{H'}{H}\right)(\Psi + Hv)'$$

subhorizon limit (ignoring time derivatives) holds at scales $k \gtrsim aH \lesssim 10^{-3}h/Mpc$

$$\frac{k^2}{a^2}\Phi \simeq -4\pi G_{eff}\rho_m\delta_m\,, \quad \eta \equiv (\Phi - \Psi)/\Psi \simeq -\frac{F^2_{,\phi}}{ZF + 3F^2_{,\phi}}$$

$$\delta_m'' + \left(2 + \frac{H'}{H}\right)\delta_m' - \frac{4\pi G_{eff}\rho_m}{H^2}\delta_m \simeq 0$$

Scalar-Tensor Theories of Gravity

- Constraints on $F(\phi)$ and $U(\phi)$:

using CPL $\omega = \omega_0 + \omega_a(1 - e^n)$ and growth index parameter $f = \frac{d\ln\delta_m(n)}{dn} \equiv \Omega(n)^\gamma$ with

$$\gamma = \gamma_0 + \gamma_a(1 - e^n)$$

$$\frac{H^2}{H_0^2} = \Omega_{m0}e^{-3n} + (1 - \Omega_{m0})e^{-3(1+\omega_0+\omega_a)n}e^{-3\omega_a(1-e^n)}$$

$$\frac{H'}{H} = -\frac{3}{2}\left[1 + \omega\frac{(1-\Omega_{m0})e^{-3(\omega_0+\omega_a)n}e^{-3\omega_a(1-e^n)}}{\Omega_{m0}+(1-\Omega_{m0})e^{-3(\omega_0+\omega_a)n}e^{-3\omega_a(1-e^n)}}\right]$$

$$\frac{F(n)}{F_0} = \frac{3}{2}\frac{\Omega_m(n)}{P(n)}, \text{ where} P(n) = \Omega_m(n)^\gamma\left(\Omega_m(n)^\gamma + \gamma'\ln\Omega_m(n) - \gamma\left[3 + 2\frac{H'}{H}\right] + 2 + \frac{H'}{H}\right)$$

$$\frac{U(n)}{F_0H_0^2} = \frac{1}{2}\frac{H^2}{H_0^2}\left(\frac{F''}{F_0} + \left[5 + \frac{H'}{H}\right]\frac{F'}{F_0} + 2\left[3 + \frac{H'}{H}\right]\frac{F}{F_0} - 3\Omega_m(n)\right)$$

$$\omega = \frac{\phi'^2+2F''+2(2+H'/H)F'+4(F-F_0)H'/H+6(F-F_0)-2U/H^2}{\phi'^2-6F'-6(F-F_0)+2U/H^2}$$

$$\phi' = \sqrt{-F'' + \left(1 - \frac{H'}{H}\right)F' - 2\frac{H'}{H}F - 3F_0\Omega_m}$$

$$\frac{F'}{F_0} = -\left(3 + 2\frac{H'}{H} + \frac{P'}{P}\right)\frac{F}{F_0}$$

$$\frac{F''}{F_0} = \left(\left[3 + 2\frac{H'}{H} + \frac{P'}{P}\right]^2 - 2\left[\frac{H'}{H}\right]' - \left[\frac{P'}{P}\right]'\right)\frac{F}{F_0}$$, Solar System Test (SST)

$$|F_{,\phi}/\sqrt{F}|_0 < 0.02$$

Scalar-Tensor Theories of Gravity

- Comparison with DE models :

Models	ω_0	ω_a	γ_0	γ_a	$F(n)/F_0$	$F_{,\phi}/\sqrt{F}\vert_0$	ϕ'	viable
$V(\phi) \propto \phi^{-1}$	-0.74	0.07	0.56	-0.018	$\bigcup$ min 0.986	0.018	fine	yes
			0.57	0	$\bigcap$ max 1.015	-0.09	fine	no
			0.6	0.08	$\bigcap$ max 1.150	-0.48	fine	no
SUGRA	-0.92	-0.08	0.56	-0.016	$\nearrow$ max 1.01	0.00	$z < 1.3$	yes
			0.563	0	$\bigcap$ max 1.02	-0.18	fine	no
			0.6	0.11	$\bigcap$ max 1.14	-0.70	fine	no
Phantom Crossing I	-1.1	0.3	0.53	-0.09	$\bigcup$ min 0.93	none	imaginary	no
			0.557	0	$\bigcap$ max 1.01	-0.24	fine	no
			0.6	0.143	$\bigcap$ max 1.15	-0.68	fine	no
Phantom Crossing II	-0.8	-0.3	0.55	-0.049	$\bigcup$ min 0.97	none	imaginary	no
			0.568	0	$\bigcap$ max 1.03	-0.18	$z < 3$	no
			0.6	0.09	$\bigcap$ max 1.13	-0.69	$z < 7$	no

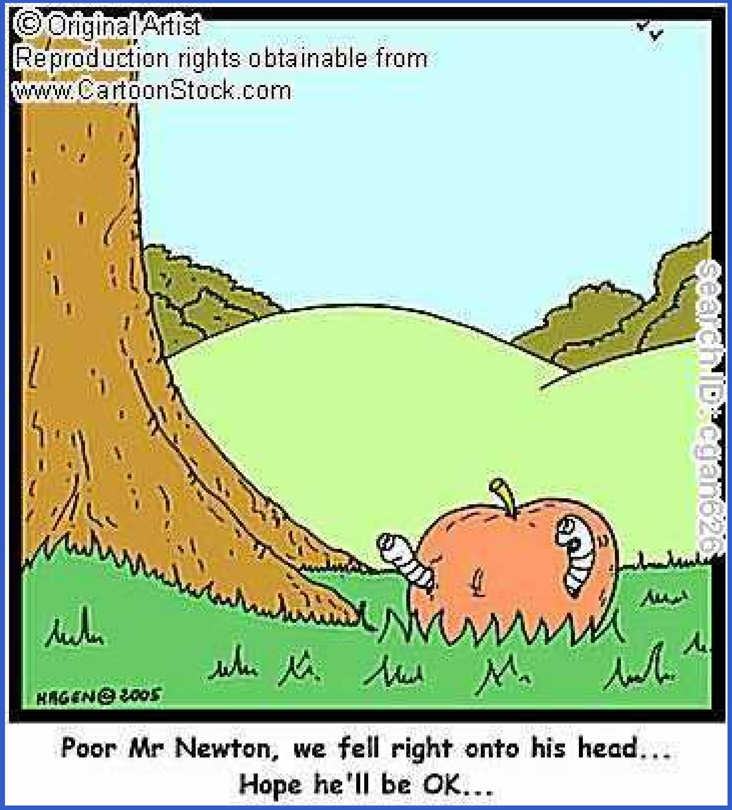
- Comparison with DGP $(\omega_0, \omega_a) = (-0.78, -0.32)$ with $(\gamma_0, \gamma_a) = (11/16, 0.32)$: fails in the SST

Scalar-Tensor Theories of Gravity

- Current status :

z_*	n_*	Ω_{m0}	f^{obs}	γ^{obs}	Ref
0.15	-0.14	0.3	0.51 ± 0.11	$0.72^{+0.26}_{-0.21}$	2dFGRS [?, ?]
0.32	-0.28	0.26	$0.654^{+0.185}_{-0.132}$	$0.52^{+0.28}_{-0.31}$	SDSS R$=10-50h^{-1}$ Mpc [?]
		± 0.02	$0.641^{+0.191}_{-0.134}$	$0.55^{+0.29}_{-0.32}$	R$=2-50h^{-1}$ Mpc
0.35	-0.3	0.3	0.70 ± 0.18	$0.54^{+0.45}_{-0.34}$	SDSS [?]
0.55	-0.44	0.3	0.75 ± 0.18	$0.59^{+0.60}_{-0.44}$	2dF-SDSS [?]
1.4	-0.88	0.3	0.90 ± 0.24	$0.68^{+2.0}_{-1.5}$	2dF-SDSS [?]
3.0	-1.39	0.3	1.46 ± 0.29	$-10.6^{+6.2}_{-5.1}$	Ly-α (SDSS) [?]

Future work



- Searching for a rotten apple ?
- We might not be able to distinguish MG from DE
- Need to investigate general $Z(\phi)$
- Need to consider the solar system test