Polynomials and their monodromy groups

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(1) Given an irreducible integer polynomial $f(t, X)$ in two variables, what can we say about the set of $t_0 \in \mathbb{Z}$ such that $f(t_0, X)$ is reducible?
(2) Given two polynomials f and g, can the two-variable polynomial $f(X) - g(Y)$ factor in any non-trivial way?
(3) What do the value sets $\{f(t) \bmod p \mid t \in \mathbb{Z}\}$ (with p a prime) tell about the polynomial f itself?
(4) How "far" can a polynomial map $x \mapsto f(x)$ on the rational numbers be from being injective?

All of these are (more or less) classical arithmetic-geometric problems. Their common feature is that they lead to interesting questions in group theory. I will motivate this connection by introducing monodromy groups of polynomials and explaining how they can be used to attack the above problems. Along the way, I will try to show how group theory can explain some astonishing examples occurring around the above questions.