A G-BRICKSET FOR THE TYPE $\frac{1}{39}(1,5,11)$

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Consider the group G of type $\frac{1}{39}(1,5,11)$. Then the star subdivision at $v = \frac{1}{39}(1,5,11)$ gives:

(i) $\sigma_2 := \mathrm{Cone}(e_1, v, e_3)$ corresponds to the quotient singularity of type $\frac{1}{5}(1,1,1)$;

(ii) $\sigma_3 := \mathrm{Cone}(e_1, e_2, v)$ corresponds to the quotient singularity of type $\frac{1}{11}(1,5,5)$.

The relative canonical model X_{can} of $X = \mathbb{C}^3/G$ is Gorenstein, but not $\mathbb{Q}$-factorial.

Let v_i denote the lattice point $\frac{1}{r}(\bar{i}, \overline{5i}, \overline{11i})$ where $^{-}$ denotes the residue modulo r. In particular, $v_1 = v$ and $v_8 = w$. Note that there exists a plane Π containing e_1, e_2, v_1 and v_8. Observe that the lattice points v_4, v_{11}, v_{18}, v_{25}, and v_{32} lie on the plane Π. Thus subdividing the cone σ_5 into smooth cones only using these points defines a crepant resolution of the toric singularity given by σ_5 where

$$\sigma_5 = \mathrm{Cone}(e_1, e_2, v_1, v_8).$$

Consider the relative minimal model Z in Figure 0.1. The variety Z is a relative minimal model having no morphism to X_v. At this moment, we do not know that whether Z is isomorphic to Y_θ for some θ. In this section, although we cannot see that Z is isomorphic to the birational component Y_θ of $\mathcal{M}_\theta$, we show that there exists a G-brickset for $Z \to X = \mathbb{C}^3/G$.

Although there is no morphism $Z \to X_v$, there exists a morphism $\overline{\varphi}\colon Z \to X_u$ where X_u is the toric variety given by the star subdivision at $u = v_4 = \frac{1}{39}(4,20,5)$. Using the star subdivision at u and the round down functions, we can find the following G-brickset.

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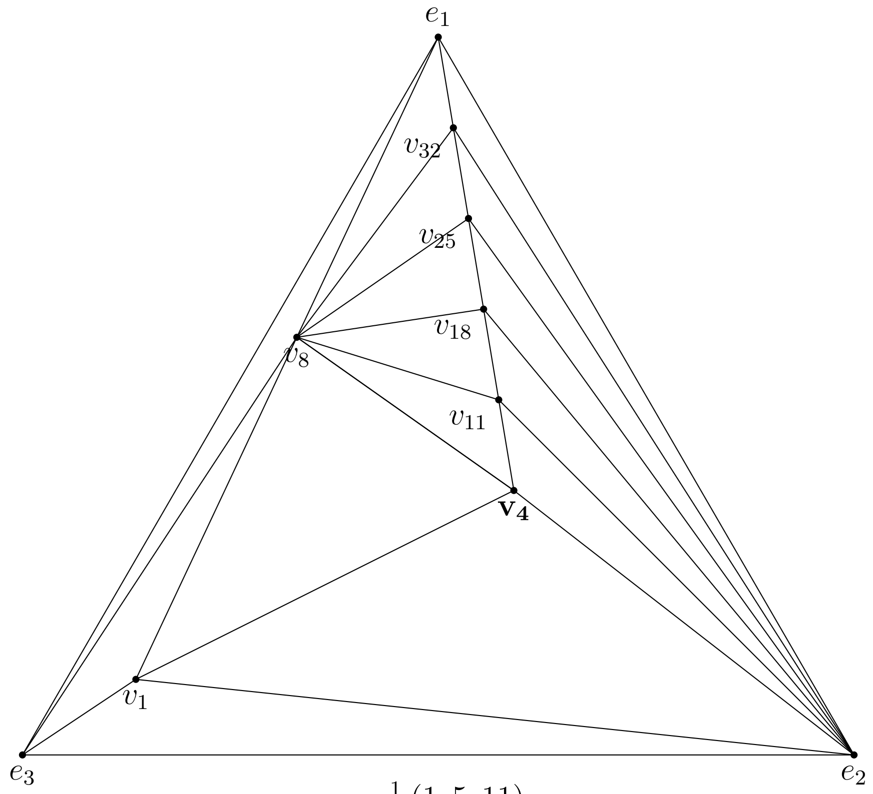

$\frac{1}{39}(1,5,11)$
Z : a relative minimal model

FIGURE 0.1. Toric fan for Z

Cone(v_1, e_2, e_3). Only x's.

Cone(v_4, e_2, v_1).

$$\left\{\begin{array}{lllllllllll}
x^{-3}z^3 & x^{-2}z^3 & x^{-1}z^3 & z^3 & xz^3 & \dots & x^5z^3 & & & & \\
 & x^{-2}z^2 & x^{-1}z^2 & z^2 & xz^2 & \dots & x^5z^2 & x^6z^2 & x^7z^2 & & \\
 & & x^{-1}z & z & xz & \dots & x^5z & x^6z & x^7z & x^8z & \\
 & & & 1 & x & \dots & x^5 & x^6 & x^7 & x^8 & x^9
\end{array}\right\}.$$

Cone(e_1, e_2, v_{32}). Only z's.

Cone(v_{32}, e_2, v_{25}).

$$\left\{\begin{array}{llllll}
z^8 & z^9 & \dots & z^{31} & & \\
 & & & & & \\
x & xz & xz^2 & \dots & xz^6 & \\
1 & z & z^2 & \dots & z^6 & z^7
\end{array}\right\}.$$

Cone(v_{25}, e_2, v_{18}).

$$\left\{\begin{array}{ccccccc}
z^8 & z^9 & \dots & z^{23} & & & \\
 & & & & & & \\
x^2z^{-1} & x^2 & x^2z & x^2z^2 & \dots & x^2z^6 & \\
 & x & xz & xz^2 & \dots & xz^6 & \\
 & 1 & z & z^2 & \dots & z^6 & z^7
\end{array}\right\}.$$

Cone(v_{18}, e_2, v_{11}).

$$\left\{\begin{array}{ccccccccc}
z^8 & z^9 & \dots & z^{15} & & & & & \\
 & & & & & & & & \\
x^3z^{-2} & x^3z^{-1} & x^3 & x^3z & x^3z^2 & \dots & x^3z^5 & & \\
 & x^2z^{-1} & x^2 & x^2z & x^2z^2 & \dots & x^2z^5 & x^2z^6 & \\
 & & x & xz & xz^2 & \dots & xz^5 & xz^6 & \\
 & & 1 & z & z^2 & \dots & z^5 & z^6 & z^7
\end{array}\right\}.$$

Cone(v_{11}, e_2, v_4).

$$\left\{\begin{array}{ccccccccccc}
x^4z^{-3} & x^4z^{-2} & x^4z^{-1} & x^4 & x^4z & x^4z^2 & x^4z^3 & x^4z^4 & & & \\
 & x^3z^{-2} & x^3z^{-1} & x^3 & x^3z & x^3z^2 & x^3z^3 & x^3z^4 & x^3z^5 & & \\
 & & x^2z^{-1} & x^2 & x^2z & x^2z^2 & x^2z^3 & x^2z^4 & x^2z^5 & x^2z^6 & \\
 & & & x & xz & xz^2 & xz^3 & xz^4 & xz^5 & xz^6 & \\
 & & & 1 & z & z^2 & z^3 & z^4 & z^5 & z^6 & z^7
\end{array}\right\}.$$

Cone(e_1, v_{32}, v_8).

$$\left\{\begin{array}{cccccccccc}
z^3 & yz^3 & y^2z^3 & y^3z^3 & y^4z^3 & y^5z^3 & y^6z^3 & y^7z^3 & y^8z^3 & \\
z^2 & yz^2 & y^2z^2 & y^3z^2 & y^4z^2 & y^5z^2 & y^6z^2 & y^7z^2 & y^8z^2 & y^9z^2 \\
z & yz & y^2z & y^3z & y^4z & y^5z & y^6z & y^7z & y^8z & y^9z \\
1 & y & y^2 & y^3 & y^4 & y^5 & y^6 & y^7 & y^8 & y^9
\end{array}\right\}.$$

Cone(v_{32}, v_{25}, v_8).

$$\left\{\begin{array}{cccccccc}
xz^3 & & & & & & & \\
xz^2 & xyz^2 & & & & & & \\
xz & xyz & & & & & & \\
x & xy & & & & & & \\
 & & & & & & & \\
z^3 & yz^3 & y^2z^3 & y^3z^3 & y^4z^3 & y^5z^3 & y^6z^3 & y^7z^3 \\
z^2 & yz^2 & y^2z^2 & y^3z^2 & y^4z^2 & y^5z^2 & y^6z^2 & y^7z^2 \\
z & yz & y^2z & y^3z & y^4z & y^5z & y^6z & y^7z \\
1 & y & y^2 & y^3 & y^4 & y^5 & y^6 & y^7
\end{array}\right\}.$$

Cone(v_{25}, v_{18}, v_8).

$$\left\{\begin{array}{cccccc}
x^2z^2 & x^2yz^2 & & & & \\
x^2z & x^2yz & & & & \\
x^2 & x^2y & & & & \\
x^2z^{-1} & x^2yz^{-1} & & & & \\
& & & & & \\
xz^3 & & & & & \\
xz^2 & xyz^2 & & & & \\
xz & xyz & & & & \\
x & xy & & & & \\
& & & & & \\
z^3 & yz^3 & y^2z^3 & y^3z^3 & y^4z^3 & y^5z^3 \\
z^2 & yz^2 & y^2z^2 & y^3z^2 & y^4z^2 & y^5z^2 \\
z & yz & y^2z & y^3z & y^4z & y^5z \\
1 & y & y^2 & y^3 & y^4 & y^5
\end{array}\right\}.$$

Cone(v_{18}, v_{11}, v_8).

$$\left\{\begin{array}{ccccccc}
x^4yz^{-3} & x^4yz^{-2} & x^4yz^{-1} & x^4y & & & \\
& x^3yz^{-2} & x^3yz^{-1} & x^3y & x^3yz & & \\
& & x^2yz^{-1} & x^2y & x^2yz & x^2yz^2 & \\
& & & xy & xyz & xyz^2 & \\
& & & y & yz & yz^2 & yz^3 \\
& & & & & & \\
x^4z^{-3} & x^4z^{-2} & x^4z^{-1} & x^4 & & & \\
& x^3z^{-2} & x^3z^{-1} & x^3 & x^3z & & \\
& & x^2z^{-1} & x^2 & x^2z & x^2z^2 & \\
& & & x & xz & xz^2 & xz^3 \\
& & & 1 & z & z^2 & z^3
\end{array}\right\}.$$

Cone(v_{11}, v_4, v_8).

$$\left\{\begin{array}{ccccccc}
x^4yz^{-3} & x^4yz^{-2} & x^4yz^{-1} & x^4y & & & \\
& x^3yz^{-2} & x^3yz^{-1} & x^3y & x^3yz & & \\
& & x^2yz^{-1} & x^2y & x^2yz & x^2yz^2 & \\
& & & xy & xyz & xyz^2 & \\
& & & y & yz & yz^2 & yz^3 \\
& & & & & & \\
x^4z^{-3} & x^4z^{-2} & x^4z^{-1} & x^4 & & & \\
& x^3z^{-2} & x^3z^{-1} & x^3 & x^3z & & \\
& & x^2z^{-1} & x^2 & x^2z & x^2z^2 & \\
& & & x & xz & xz^2 & xz^3 \\
& & & 1 & z & z^2 & z^3
\end{array}\right\}.$$

Cone(e_1, v_8, e_3). Only y's.

Cone(v_8, v_4, v_1).

$$\left\{\begin{matrix} x^{-3}yz^3 & x^{-2}yz^3 & x^{-1}yz^3 & yz^3 & & & & \\ x^{-3}z^3 & x^{-2}z^3 & x^{-1}z^3 & z^3 & xz^3 & & & \\ \\ & x^{-2}yz^2 & x^{-1}yz & yz^2 & xyz^2 & x^2yz^2 & & \\ & x^{-2}z^2 & x^{-1}z^2 & z^2 & xz^2 & x^2z^2 & & \\ \\ & & x^{-1}yz & yz & xyz & x^2yz & x^3yz & \\ & & x^{-1}z & z & xz & x^2z & x^3z & \\ \\ & & & y & xy & x^2y & x^3y & x^4y \\ & & & 1 & x & x^2 & x^3 & x^4 \end{matrix}\right\}.$$

Cone(v_8, v_1, e_3).

$$\left\{\begin{matrix} y^7 & xy^7 & x^2y^7 & x^3y^7 & \\ y^6 & xy^6 & x^2y^6 & x^3y^6 & x^4y^6 \\ y^5 & xy^5 & x^2y^5 & x^3y^5 & x^4y^5 \\ y^4 & xy^4 & x^2y^4 & x^3y^4 & x^4y^4 \\ y^3 & xy^3 & x^2y^3 & x^3y^3 & x^4y^3 \\ y^2 & xy^2 & x^2y^2 & x^3y^2 & x^4y^2 \\ y & xy & x^2y & x^3y & x^4y \\ 1 & x & x^2 & x^3 & x^4 \end{matrix}\right\}.$$