

Excitonic Bistabilities, Instabilities and Chaos in Laser-pumped Semiconductors

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Abstract

The Hurwitz criteria are used for a stability analysis of the steady state excitonic optical bistability curves in a semiconductor pumped by an external laser resonant with the exciton level. Besides the middle branch of the bistability curves which is unstable in the sense of the linear stability theory, we have found other domains of instability in the upper and lower branches of the steady state curves. Numerical results show that a possible route to chaos in the photon-exciton system is period-doubling self-oscillation process. The influence of the presence of free carriers that coexist with the excitons is also discussed.

1. Introduction

Stability and instability are two sides of matter which are by no means independent but closely related. For example, in case the system state curve develops three branches, it is the unstable behaviour of the middle branch that causes a jumping up or down to the upper or lower branches thus generating the effect of bistability. Bistable devices themselves may in turn manifest a rich variety of instabilities leading to spontaneous pulsations of both regular and irregular nature. Since 1987 a fully new direction of research dealing with optical self-pulsing and chaos has been oriented by the original works of McCall [1], Bonifacio and Lugiato [2], and Ikeda [3]. These fundamental works justify on the one hand the generality of the Synergetics theory [4] as well as of the self-organization theory in nonlinear dynamical system [5]. On the other hand, they make a strong push on different kinds of investigations in the field of nonlinear optics for controlling chaos towards avoiding it in optical devices like memory elements or towards exploiting it for the purpose of all-optical communication and signal modulation.

This paper is dedicated to a stability analysis of optical bistability in a laser-pumped two-band direct gap semiconductor modelled as a nonlinear dynamical system containing simultaneously excitons, intracrystal photons and free electron-hole pairs. Let us consider a direct-gap semicon-

ductor which is anticipated to contain a number of free electron-hole pairs whose origin of appearance is not specified. Supposing now that the semiconductor is irradiated by an external laser field with given wave-vector \mathbf{k} , complex amplitudes $\mathcal{E}_k^{(\pm)}$ and frequency Ω_k . The laser frequency is assumed to be resonant with the exciton level while the free carriers are in the bands. The exciton system is thus directly driven by the laser field via the exciton-photon transition which can however be screened by free carriers [6] and also by excitons themselves [7]. A Hamiltonian which describes this system has the form (throughout we put for convenience \hbar (Planck constant) = c (light velocity) = 1):

$$H = H_1 + H_2 + H_3 + H_4 + H_5 \quad (1)$$

$$H_1 = \sum_{\mathbf{p}} (E_{c\mathbf{p}} e_{\mathbf{p}}^+ e_{\mathbf{p}} + E_{h\mathbf{p}} h_{\mathbf{p}}^+ h_{\mathbf{p}}) + \frac{1}{2} \sum_{\mathbf{p}, \mathbf{p}', l} V_l \times \{e_{\mathbf{p}}^+ e_{\mathbf{p}'}^+ e_{\mathbf{p}'-l} e_{\mathbf{p}+l} + h_{\mathbf{p}}^+ h_{\mathbf{p}'}^+ h_{\mathbf{p}'-l} h_{\mathbf{p}+l} - 2e_{\mathbf{p}}^+ h_{\mathbf{p}'}^+ h_{\mathbf{p}'-l} e_{\mathbf{p}+l}\} \quad (2)$$

$$H_2 = \sum_{\mathbf{p}} \{E_{a\mathbf{p}} a_{\mathbf{p}}^+ a_{\mathbf{p}} + E_{b\mathbf{p}} b_{\mathbf{p}}^+ b_{\mathbf{p}} - G_1 [a_{\mathbf{p}}^+ b_{\mathbf{p}} + b_{\mathbf{p}}^+ a_{\mathbf{p}}]\} + \frac{F_1}{V} \sum_{\mathbf{p}, \mathbf{p}', l} a_{\mathbf{p}}^+ a_{\mathbf{p}'}^+ a_{\mathbf{p}'-l} a_{\mathbf{p}+l} + i(gV)^{1/2} \times \{\mathcal{E}_k^{(-)} e^{-i\Omega_k} b_{\mathbf{k}}^+ - \mathcal{E}_k^{(+)} e^{+i\Omega_k} b_{\mathbf{k}}\} \quad (3)$$

$$H_3 = \frac{G_2}{V} \sum_{\mathbf{p}, \mathbf{p}', l} [a_{\mathbf{p}}^+ a_{\mathbf{p}'}^+ a_{\mathbf{p}'-l} b_{\mathbf{p}+l} + b_{\mathbf{p}+l}^+ a_{\mathbf{p}'-l}^+ a_{\mathbf{p}}] \quad (4)$$

$$H_4 = \frac{G_3}{V} \sum_{\mathbf{p}, \mathbf{p}', l} [a_{\mathbf{p}}^+ e_{\mathbf{p}'}^+ e_{\mathbf{p}'-l} b_{\mathbf{p}+l} + b_{\mathbf{p}+l}^+ e_{\mathbf{p}'-l}^+ e_{\mathbf{p}}] + \frac{G_4}{V} \sum_{\mathbf{p}, \mathbf{p}', l} [a_{\mathbf{p}}^+ h_{\mathbf{p}'}^+ h_{\mathbf{p}'-l} b_{\mathbf{p}+l} + b_{\mathbf{p}+l}^+ h_{\mathbf{p}'-l}^+ h_{\mathbf{p}}] \quad (5)$$

$$H_5 = \frac{F_2}{V} \sum_{\mathbf{p}, \mathbf{p}', l} a_{\mathbf{p}}^+ e_{\mathbf{p}'}^+ e_{\mathbf{p}'-l} a_{\mathbf{p}+l} + \frac{F_3}{V} \sum_{\mathbf{p}, \mathbf{p}', l} a_{\mathbf{p}}^+ h_{\mathbf{p}'}^+ h_{\mathbf{p}'-l} a_{\mathbf{p}+l}. \quad (6)$$

In writing formulae (2) to (6) we have used some notations with their meanings as follows: $(e_{\mathbf{p}}, e_{\mathbf{p}}^+)$, $(h_{\mathbf{p}}, h_{\mathbf{p}}^+)$, $(a_{\mathbf{p}}, a_{\mathbf{p}}^+)$ and $(b_{\mathbf{p}}, b_{\mathbf{p}}^+)$, denote respectively the operators of electrons, holes, excitons and photons with energies $E_{c\mathbf{p}}$, $E_{h\mathbf{p}}$, $E_{a\mathbf{p}}$ and

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frequency $E_{\mathbf{b}\mathbf{p}}$. The field $\mathcal{E}_{\mathbf{k}}$ is treated as classical and monochromatic at the wave-vector \mathbf{k} . The sample volume is labelled V and $G_{1,2,3,4}$, $F_{1,2,3}$ and g , mean coupling constants which are for simplicity anticipated momentum-independent.

2. Excitonic stabilities, instabilities and chaos

Now, within the Hartree-Fock approximation we set up from the Hamiltonians (1) to (6) the equations of motion for the \mathbf{k} -mode of excitons and photons. These modes are directly governed by the monochromatic field $\mathcal{E}_{\mathbf{k}}$ and therefore they can be treated as coherent modes [14, 15]. Performing the necessary averaging procedure we get the following system of nonlinear differential equations:

$$\frac{d\langle a \rangle}{dt} = -i\{E_a + 4F_1 \cdot n_a + n[F_2 + F_3] - i\gamma_a\}\langle a \rangle + i\{G_1 - 2G_2 \cdot n_a - n[G_3 + G_4]\}\langle b \rangle \quad (7)$$

$$\frac{d\langle b \rangle}{dt} = (gV)^{1/2} \mathcal{E}^{(-)} \cdot e^{-i\Omega t} - i\{E_b - i\gamma_b\}\langle b \rangle + i\{G_1 - 2G_2 \cdot n_a - n[G_3 + G_4]\}\langle a \rangle \quad (8)$$

where $\langle a \rangle \equiv \langle a_{\mathbf{k}} \rangle$, $\langle b \rangle \equiv \langle b_{\mathbf{k}} \rangle$, $E_a \equiv E_{a\mathbf{k}}$, $E_b \equiv E_{b\mathbf{k}}$, $n_a = V^{-1} \langle a^\dagger a \rangle \equiv V^{-1} | \langle a \rangle |^2$, $\Omega_{\mathbf{k}} \equiv \Omega$, $n = \sum_{\mathbf{p}} (\langle e_{\mathbf{p}}^+ e_{\mathbf{p}} \rangle + \langle h_{\mathbf{p}}^+ h_{\mathbf{p}} \rangle) / 2V$ and we assumed $\mathcal{E}_{\mathbf{k}}^{(+)} = \mathcal{E}_{\mathbf{k}}^{(-)} = \mathcal{E}$ for simplicity. We have also introduced in a phenomenological way the transverse exciton (γ_a^{-1}) and photon (γ_b^{-1}) lifetime.

Solutions of the equation system in a transient regime are sought in the forms:

$$\langle a \rangle = Q\{X_a(t) + iY_a(t)\} e^{-i\Omega t} \quad (9)$$

$$\langle b \rangle = Q\{X_b(t) + iY_b(t)\} e^{-i\Omega t} \quad (10)$$

with $Q = (\gamma_a V / F_1)^{1/2} / 2$ introduced for convenience. Then we arrived at the following nonlinear differential coupled equations:

$$\frac{dX_a}{d\tau} = -X_a - (\varepsilon - \eta - X_a^2 - Y_a^2)Y_a - [\varphi - \chi - \rho(X_a^2 + Y_a^2)]Y_b \quad (11)$$

$$\frac{dY_a}{d\tau} = -Y_a + (\varepsilon - \eta - X_a^2 - Y_a^2)X_a + [\varphi - \chi - \rho(X_a^2 + Y_a^2)]X_b \quad (12)$$

$$\frac{dX_b}{d\tau} = -\sigma X_b - \Delta Y_b - [\varphi - \chi - \rho(X_a^2 + Y_a^2)]Y_a + P \quad (13)$$

$$\frac{dY_b}{d\tau} = -\sigma Y_b + \Delta X_b + [\varphi - \chi - \rho(X_a^2 + Y_a^2)]X_a \quad (14)$$

with the following dimensionless scaled notations:

$$\begin{aligned} \tau &= \gamma_e t; \quad \varepsilon = \frac{\Omega - E_a}{\gamma_a}; \quad \Delta = \frac{\Omega - E_b}{\gamma_a}; \\ \varphi &= \frac{G_1}{\gamma_a}; \quad \sigma = \frac{\gamma_b}{\gamma_a}; \quad \rho = \frac{G_2}{2F_1}; \\ \eta &= \frac{2nF}{\gamma_a}; \quad \chi = \frac{2nG}{\gamma_a}; \quad P = 2 \frac{\mathcal{E}(gF_1)^{1/2}}{\gamma_a^{3/2}} \end{aligned} \quad (15)$$

where $G = G_3 = G_4$, $F = F_2 = F_3$ are taken as in [16].

The exciton (photon) dimensionless density can be expressed as $N_a = X_a^2 + Y_a^2$, ($N_b = X_b^2 + Y_b^2$). $\varepsilon(\Delta)$, and φ describe the exciton (photon) detuning and the bare exciton-photon interaction, respectively. η and χ arise from the exciton-carrier interaction and the screening of the exciton-photon interaction by free carriers (they both depend on the carrier density). The so-called exciton-assisted exciton-photon transition matrix element is denoted by ρ . $P^2 \equiv I$ labels the laser (dimensionless) intensity which can serve as a control parameter in our problem. It is worth noticing at this moment that in the case of $\eta = \chi = \rho = 0$ our eqs (11–14) coincide with the equations established in [8].

The steady state solutions of eqs (11–14) can easily be found. Using these solutions we can obtain the following relations:

$$I \equiv P^2 = \frac{\{N_a(\alpha N_a^4 + \beta N_a^3 + \delta N_a^2 + \vartheta N_a + \lambda)\}}{(\varphi - \chi - \rho N_a)^2} \quad (16)$$

and

$$N_b = \frac{N_a\{\varepsilon - \eta - N_a\} + 1}{(\varphi - \chi - \rho N_a)^2} \quad (17)$$

where

$$\alpha = \rho^4; \quad \beta = 2\rho^2[\Delta + 2\rho(\chi - \varphi)]; \quad \mu = \chi - \varphi \quad (18)$$

$$\delta = \sigma^2 + [\Delta + 2\rho(\chi - \varphi)]^2 + 2\rho^2 \times [\sigma + \Delta(\eta - \varepsilon) + (\varphi - \chi)^2] \quad (19)$$

$$\vartheta = 2\{[2\rho(\chi - \varphi) - \Delta][\Delta(\varepsilon - \eta) + \sigma(\varphi - \chi)^2] + \sigma[\Delta + \sigma(\eta - \varepsilon)]\} \quad (20)$$

$$\lambda = [\sigma + (\eta - \varepsilon)\Delta + (\chi - \varphi)^2]^2 + [\Delta - \sigma(\eta - \varepsilon)]^2. \quad (21)$$

We now depict in Fig. 1 and Fig. 2 the exciton and photon density N_a and N_b vs. the laser intensity taking the same set of material parameters as used in [8]. A straightforward linear stability analysis together with the Hurwitz criteria

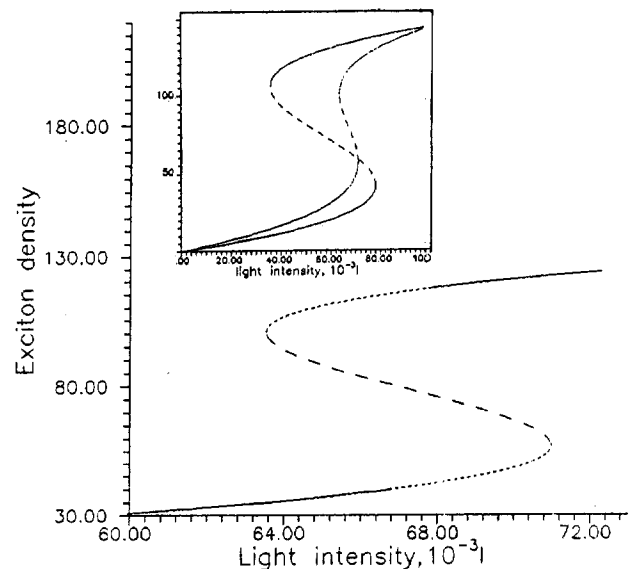


Fig. 1. Dimensionless exciton density N_a as a function of light intensity $I = P^2$ for $\varepsilon = 118$, $\varphi = 23.6$, $\sigma = 10$, $\Delta = 0$, $\rho = 0$, $\chi = 0$ and $\eta = 0$. The inset represents in an extended range of I the cases with $\chi = 6$, $\eta = 7.5$ (curve with large hysteresis loop) and $\chi = \eta = 0$ (curve with smaller hysteresis loop).

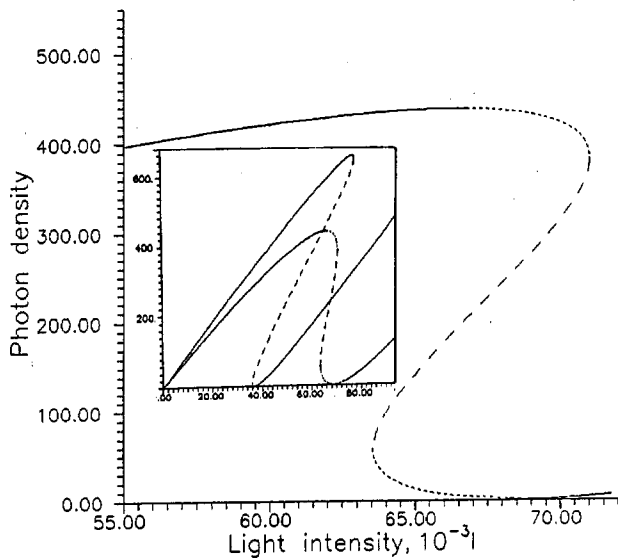


Fig. 2. Dimensionless photon density N_b as a function of light intensity $I = P^2$ for the same parameters as in Fig. 1.

yield the following classifications. The bistability curves possess considerable stable regions in the lower and upper branches (see the solid lines in Figs 1, 2). The whole middle branches of negative slope which are long-dashed and small short-dashed parts in the leftmost (rightmost) side of the upper (lower) branches violate the Hurwitz conditions and therefore should be unstable. However, further to the studies in [8, 9], here we are able to distinguish the difference in the nature of instability in the above-mentioned dashed parts of the bistability curves. But before doing this we would like to stress that unlike the model of a collection of two-level atoms placed in a cavity [10, 11], where the entire upper branch of the S curve is unstable, in our case, i.e. in exciton systems, just a small segment of the upper branch becomes unstable leaving the remainder stable (see Fig. 1). Such a behaviour also conflicts with that of the model suggested in [12] where the upper branch of the S-shaped curve of the carrier density *vs.* the driving amplitude has a small leftmost stable segment which is entirely followed by a large domain of instability.

Now coming back to the nature of instability in the unstable domains. As for the long-dashed parts in Figs 1 and 2 the unstable behaviour is understood in the sense of the usual linear stability theory. That is, even an infinitely small departure from a point in the long-dashed parts can make the system deviate from it. However, the direction of deviation is sensitive to the initial conditions. If the system is initially very near but above (below) a point in the long-dashed parts, then in the course of time it will eventually relax towards the point in the upper (lower) branch which corresponds to the same input laser intensity. (The plots are not shown because of lack of space.) Concerning the short-dashed parts in Figs 1 and 2 they display very rich information on instability nature and on replacement of one kind of stability by another one. Upon decreasing the laser intensity from the stable domain of the lower branch of the photon density bistability curve we first meet at about $P = 259.2196$ ($10^{-3}I = 67.1948$) a Hopf bifurcation point below which periodic solutions emerge. We have carried out a computer experiment on the solutions of the equation system (11–14) in the short-dashed domain of the lower branch of the

photon density curve in Fig. 2 with $\chi = \eta = \rho = 0$. The solutions which correspond to the short-dashed domain of the upper branch of the exciton density bistability curve are of the same nature and not shown because of lack of space. In Figs 3 and 4 the time structures of the photon density are plotted, respectively for $P = 258.9044$ and 258.4822 ($10^{-3}I = 67.0315$ and 66.8131). These are periodic self-oscillations corresponding to period-2 and period-8, respectively. The plots of period-1 and period-4 self-oscillations computed for $P = 259.2193$ and 258.5747 ($10^{-3}I = 67.1946$ and 66.8609), respectively, are not shown because of lack of space. For a further decrease in the control parameter P , we have observed an intensity window inside which chaotic self-pulsations appear. For example, Fig. 5 represents chaotic time structure of the photon density when $P = 258.4591$ ($10^{-3}I = 66.8011$).

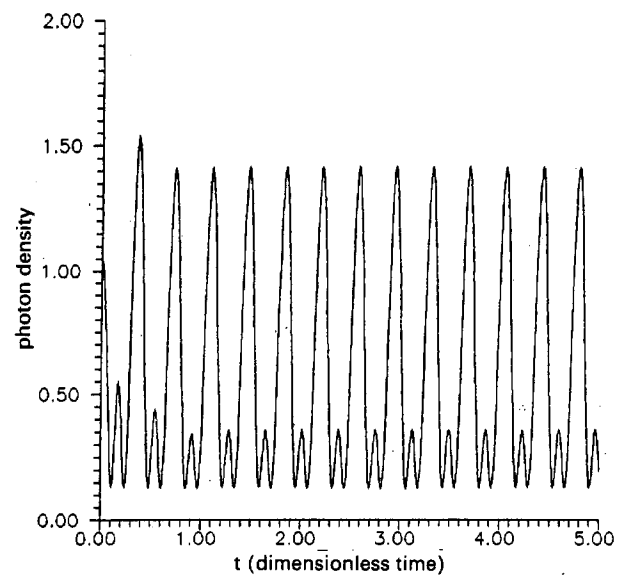


Fig. 3. Time-evolution of the photon density when the system initially suffers a slight displacement from the steady state with $P = 258.9044$ ($10^{-3}I = 67.0315$) in the lower branch of the bistability curve in Fig. 2. Period-2 oscillation.

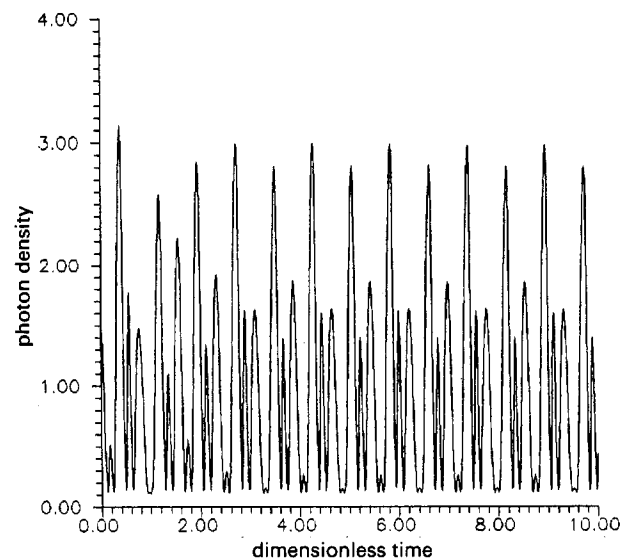


Fig. 4. The same as in Fig. 3 but $P = 258.4822$ ($10^{-3}I = 66.8131$). Period-8 oscillation.

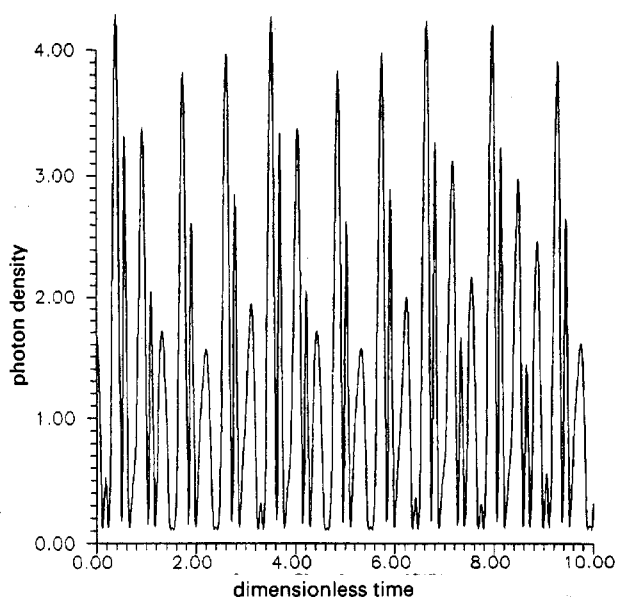


Fig. 5. The same as in Fig. 3 but $P = 258.4591$ ($10^{-3}I = 66.8011$). Chaotic self-pulsation.

In order to see the chaotic nature more clearly, we have trace in Fig. 5(a) the phase portraits which correspond to the self-pulsation drawn in Fig. 5. Transparently, the phase portraits in Fig. 5(a) is indeed chaotic attractors but not limit cycles (as for the periodic self-oscillations in Figs 3, 4) or fixed focus (as for the relaxation to the steady state). The plotted time in Fig. 5(a) is up to 60. For longer times the dark region becomes larger and in principle for time tending to infinity the trajectory 100% darkens the attractor space. The lower branch leftmost short-dashed part next to the chaos window as already studied in [8, 9] manifest a chain of multi-peaked pulses (see Fig. 6 for the case when $P = 255.6630$ ($10^{-3}I = 65.3636$)). All these results show that in laser-pumped photon-exciton systems one of the possible routes to chaos is the period-doubling route. It seems that such a route to chaos in photon-exciton systems is pointed out by us for the first time. Although a number of papers

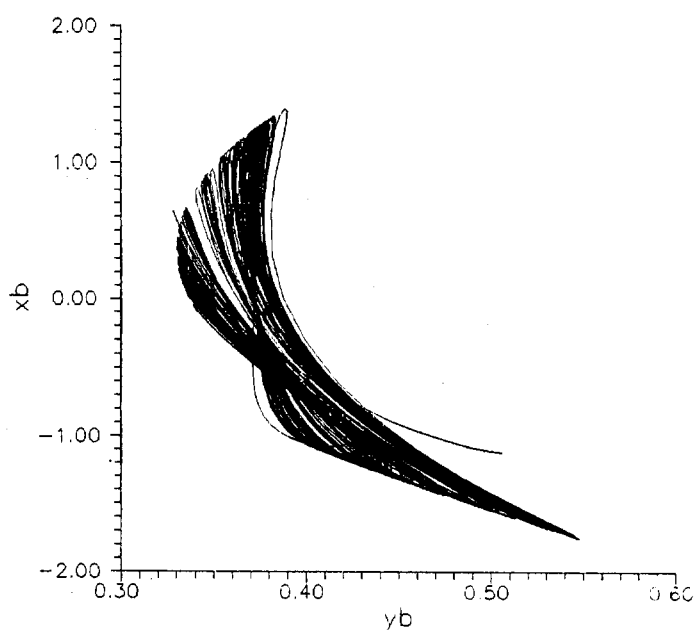


Fig. 5(a). Phase portrait corresponding to Fig. 4. The plotted (scaled) time is up to 60.

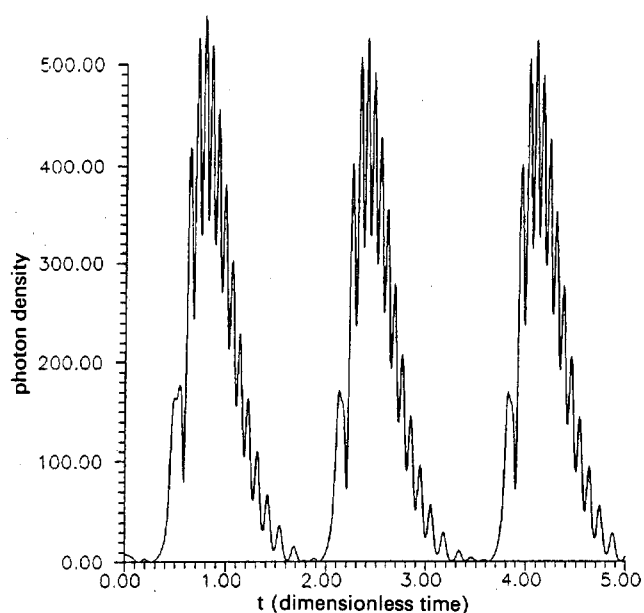


Fig. 6. The same as in Fig. 3 but $P = 255.6630$ ($10^{-3}I = 65.3636$). A chain of 16-peaked pulses.

were published by the Kishinhev group [8, 9], their authors just numerically calculated the solutions at some particular points in the unstable domains, therefore they have not found the instability nature leading to the chaos of the system as in this work.

Before making conclusions we wish to discuss the role of the free carriers co-existent with the exciton in the system. From the inset of Figs 1 and 2 we see that within the same interval of values of the control parameter P corresponding to the unstable dashed part of the curves with $\chi = \eta = 0$ (free carriers absent) the curves with $\chi, \eta \neq 0$ (free carriers present) have stable domains. This means that the free carrier density can serve as another control parameter for the dynamics of the system. This also suggests a possible way to control the dynamical system by using simultaneously two control parameters, the laser intensity and the free carrier density, for practical purposes. Say, if we want to exploit the system as a converter of c.w. light into pulsed light we may use the curves with $\chi, \eta = 0$. Alternatively, if we want to have a memory element it is more favourable to use the curves with $\chi, \eta \neq 0$, i.e. we should create free carriers, say, by an additional pump into the bands.

3. Conclusion

In conclusion, we have performed a computer experiment on the stability analysis of the lower (upper) branch of the photonic (excitonic) optical bistability curve for a chosen set of parameters as in [8]. For other parameter sets the physical picture may be drastically changed. Namely, taking into account the exciton-assisted photon-exciton interaction ($\rho \neq 0$) the steady state curves might develop five branches bringing about the so-called tri-stability [6, 7]. Under certain circumstances the state curves may however be of single-stability type [8, 9]. Efficiently detailed stability analysis for such single- and tri-stability cases are still missing up to now. Not speaking already about chaotic regimes, even the domain of periodic solutions proves to

contain many different new phenomena like optical multi-stability for periodic solutions [13]. Thus, the topic is very wide and this work must be supplemented and expanded by further investigations.

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