

INTERACTION BETWEEN EXCITONS IN 3D AND 2D LASER-EXCITED SEMICONDUCTORS

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Analytical expressions of the exciton-exciton interaction in 3D and 2D laser-excited semiconductors are approximately derived in rather easy-to-handle forms which enable us to study in detail their dependences on the exciton momenta, the momentum transferred between two excitons and the electron-hole mass ratio.

I. Introduction

Highly excited materials have currently been investigated deeply and widely because of their promising potential in practice. In semiconductors excited by lasers in the below-gap spectral region, a quasi-equilibrium exciton gas could be established that would govern the optical and kinetical properties of the semiconductors, leading to the appearance of many novel physical phenomena capable to be exploited for various application purposes.

As a factor of nonlinearity the interaction between excitons in the semiconductors plays the most important role bringing about a variety of physical effects, among them Bose-Einstein condensation [1, 2], optical bistability [3–8], new zones and polarization feature of photo-luminescence [9, 10], nonlinear optical nutation [11], density-dependent level shift and broadening [7, 12, 13], ultrafast phase relaxation [14], optical Stark effect [15], the third order optical susceptibility [16, 17], polariton dispersion anomalies [18–20], etc. Recently, the interest in the exciton-exciton interaction has been further stimulated in the man-made low-dimensional semiconductor structures [7, 12, 16, 17, 21] such as (multiple-) quantum-wells, quantum-wires, quantum-well-wires or quantum-dots. Since the Coulomb force between charge-carriers behaves quite distinctly for different dimensionalities, the intra- as well as the inter-excitonic interactions must also be considered separately in every case.

The aim of this paper is to study in detail the effective exciton-exciton interaction in 3D and 2D laser-excited semiconductors with emphasis to its explicit dependences on the exciton momenta, the momentum transferred between two excitons and the electron-hole mass ratio. Such dependences in fact cannot be obtained analytically because their expressions are formulated in terms of very complicated multifold integrals [4, 22–26] that may only be evaluated numerically for general values

of the exciton momenta and mass. Therefore, in handling the exciton-exciton interaction for further theoretical calculations it often has to be taken as a constant equal to its value when the two interacting excitons possess the same momenta and the momentum transfer is zero [1, 2, 16, 17, 20, 22, 23]. In section III simple, but reasonable approximations are invoked to analytically evaluate the integrals of the exciton-exciton interaction which are derived rigorously in section II from the original electron-hole Hamiltonian of the semiconductors. In section IV the results are shown graphically and finally we end up with a conclusion.

The unit system we use in this paper is that with $\hbar = c = 1$ (\hbar the Planck constant, c the velocity of light).

II. Many-exciton system Hamiltonian

Being composite quasi-particles, each of which comprises an electron and a hole, excitons are neither bosons nor fermions and, consequently, the many-exciton problem is genuinely a very delicate one. Most often a bosonic representation is introduced to describe excitons as bosons which interact with each other in an effective manner so that both kinematical and dynamical interactions will be properly incorporated. The authors of [22, 23] perform a Usui transformation [27] on the semiconductor electron-hole Hamiltonian

$$\begin{aligned}
 H_{e-h} = & \sum_k \{ E_e(k) e_k^+ e_k + E_h(k) h_k^+ h_k + \\
 & + \frac{1}{2} \sum_{pq} V_q [e_{q+k}^+ e_{p-q}^+ e_p e_k + h_{q+k}^+ h_{p-q}^+ h_p h_k - 2e_{k+q}^+ h_{p-q}^+ h_p e_k] \}, \quad (1)
 \end{aligned}$$

where e_k^+ and e_k (h_k^+ and h_k) creates and destroys an electron (a hole) having momentum k and energy $E_e(k) = E_g + k^2/2m_e$ ($E_h(k) = k^2/2m_h$) with E_g and m_e (m_h) being the band gap and the electron (hole) effective mass, V_q is the Coulomb potential; they obtain a bosonic Hamiltonian with the exciton-exciton interaction part consisting of seven terms (see Eqs. (6.25b–d) in [23]), one of which is then excluded by the so-called ordering procedure or the procedure ensuring the one-to one correspondence between fermionic and bosonic subspaces. The six remaining terms can be written as eight terms in the form of Eqs. (6.2.8–10) in [23]. The ordering procedure is indeed quite confusing and not natural. To avoid this confusion, in [4] a transformation due to Marumori et al. [28] is applied to map the electron-hole-light Hamiltonian into a bosonic one which accounts for coupling between light and semiconductor. However, the Hamiltonian of [4] seems to be incorrect because it would lead to a strengthening of the dipole transition (see Eq. (2.16) in [4] where $r_2 \approx 1 - \sqrt{2} < 0$ means a strengthening of the dipole transition when the exciton density increases) but not to the bleaching as experimentally observed [29] and theoretically predicted (see e.g. Eq. (7) in the latter citation of [22] or Fig. 5 of [26]). Moreover, one always meets some uncertainties in having things to do with the special projection operator

\hat{P} defined by Eq. (A8) in [4] (such a remark is also made in [30]). Whereas in [24], an effective many-exciton Hamiltonian is ad hoc written down without any derivation. Hereinafter, following the clear and precise method outlined in [26], we will present with more detailed argumentations a low-density expansion of the fermionic Hamiltonian (1) in terms of exciton bosonic operators.

Assume a situation when almost all of the electron-hole pairs in a two-direct band semiconductor exist in bound states called excitons. Consider the most general bound electron-hole pair state, i.e. the exciton state, whose operators are determined by

$$b_{\nu k} = \frac{1}{\sqrt{V}} \sum_p f_{\nu}^*(p - \beta k) h_p e_{k-p}, \quad (2)$$

$$b_{\nu k}^{\dagger} = \frac{1}{\sqrt{V}} \sum_p f_{\nu}(p - \beta k) e_{k-p}^{\dagger} h_p^{\dagger}, \quad (3)$$

where ν specifies the exciton quantum state, V is the sample volume or area in the 3D and 2D case, resp., and $\beta = m_b/(m_e + m_h)$. Functions f_{ν} describe the relative motion of the electron-hole pair in an exciton and satisfy the ortho-normalization and completeness conditions:

$$\sum_p f_{\mu}^*(p) f_{\nu}(p) = V \delta_{\mu\nu}, \quad (4)$$

$$\sum_{\nu} f_{\nu}^*(p) f_{\nu}(q) = V \delta_{pq}. \quad (5)$$

Unlike the zero-excitation regime, in laser-excited semiconductors f_{ν} are not the well-known hydrogen-like functions. The choice of the actual form for f_{ν} is still open [30]. Instructively, f_{ν} can be calculated by variationally minimizing the somehow re-normalized single-exciton energy with respect to f_{ν}^* (see e.g. [26] p. 59). Yet, disregarding the concrete form of f_{ν} and taking into account only (4) and (5), we can verify the following useful relations:

$$h_p e_q = \frac{1}{\sqrt{V}} \sum_{\nu} f_{\nu}(\alpha p - \beta q) b_{\nu p+q}, \quad (6)$$

$$e_p^{\dagger} h_q^{\dagger} = \frac{1}{\sqrt{V}} \sum_{\nu} f_{\nu}^*(\alpha q - \beta p) b_{\nu p+q}^{\dagger} \quad (7)$$

with $\alpha = 1 - \beta$. The definitions (2) and (3) of b and b^{\dagger} yield a rather complicated, but closed set of nonbosonic bi- and trilinear commutators (see Eqs. (6), (7) and (11) to (13) in [31]) showing that for arbitrary density real excitons are never ideal and obey a modified statistics of quasi-paraferrions. At low excitation level we can carry out an appropriate pairing procedure and up to order n^2 ($n = N/V$ and N are the pair density and number) exciton operators may be treated as bosonic [26]. Indeed, in the vanishing pair density limit there is essentially one electron-hole pair, so that $\hat{N} = \hat{N}_e = \hat{N}_h = 1$, where $\hat{N}_e = \sum_p e_p^{\dagger} e_p$ and $\hat{N}_h = \sum_p h_p^{\dagger} h_p$ denote the elec-

tron and hole number operators. Making use of this fact, we can act on H_{e-h} in (1) by $\hat{f} = [\sum_{p'} e_{p'}^+ e_{p'}]^i \cdot [\sum_{p''} h_{p''}^+ h_{p''}]^j$ with $i, j = 0, 1$ or 2 and perform the pairing procedure with the aid of (6) and (7) (as done in [26] p. 57 in r -representation) to get, up to order n^2 , the following bosonic Hamiltonian in k -representation:

$$H_x = \sum_{p\mu\nu} E_{\mu\nu}(p) b_{\mu p}^+ b_{\nu p} + \frac{1}{2} \sum_{\substack{pp'q \\ \nu\nu'\mu'\mu}} U_{\nu\nu'\mu'\mu}(p, p', q) b_{\nu p+q}^+ b_{\nu' p'-q}^+ b_{\mu' p'} b_{\mu p}. \quad (8)$$

In (8) $E_{\mu\nu}(p)$ serves as the intra-excitonic interaction energy, whereas the inter-excitonic one is given by

$$U_{\nu\nu'\mu'\mu}(p, p', q) = \sum_{i=1}^7 U_{\nu\nu'\mu'\mu}^{(i)}(p, p', q), \quad (9)$$

$$U_{\nu\nu'\mu'\mu}^{(1)}(p, p', q) = \frac{V_q}{V^2} \sum_{kk'} f_\nu(k - \beta p - \beta q) f_{\nu'}(k' - \beta p' + \beta q) \times f_\mu^*(k' - \beta p') f_\mu^*(k - \beta p), \quad (10)$$

$$U_{\nu\nu'\mu'\mu}^{(2)}(p, p', q) = \frac{V_q}{V^2} \sum_{kk'} f_\nu(k - \beta p + \alpha q) f_{\nu'}(k' - \beta p' - \alpha q) \times f_\mu^*(k' - \beta p') f_\mu(k - \beta p), \quad (11)$$

$$U_{\nu\nu'\mu'\mu}^{(3)}(p, p', q) = -\frac{2V_q}{V^2} \sum_{kk'} f_\nu(k - \beta p - \beta q) f_{\nu'}(k' - \beta p' - \alpha q) \times f_\mu^*(k' - \beta p') f_\mu(k - \beta p), \quad (12)$$

$$U_{\nu\nu'\mu'\mu}^{(4)}(p, p', q) = -\frac{1}{V^2} \sum_{kk'} V_k f_\nu(k - k' - \beta p + \alpha q) \times f_{\nu'}(k - \beta p' + \beta q) f_\mu^*(k - k' + q - \beta p') f_\mu^*(k - \beta p), \quad (13)$$

$$U_{\nu\nu'\mu'\mu}^{(5)}(p, p', q) = -\frac{1}{V^2} \sum_{kk'} V_k f_\nu(k - k' - \beta p - \beta q) \times f_{\nu'}(k - p + \alpha p' - \alpha q) f_\mu^*(k - k' - p - q + \alpha p') f_\mu^*(k - \beta p), \quad (14)$$

$$U_{\nu\nu'\mu'\mu}^{(6)}(p, p', q) = \frac{2}{V^2} \sum_{kk'} V_k f_\nu(k - k' - \beta p - \beta q) \times f_{\nu'}(k - p + \alpha p' - \alpha q) f_\mu^*(j - p - q + \alpha p') f_\mu^*(k - \beta p), \quad (15)$$

$$U_{\nu\nu'\mu'\mu}^{(7)}(p, p', q) = \frac{2}{V^2} \sum_{kk'} V_k f_\nu(k - k' - \beta p - \beta q) \times f_{\nu'}(k - k' - p + \alpha p' - \alpha q) f_\mu^*(k - k' - p - q + \alpha p') f_\mu^*(k - \beta p); \quad (16)$$

$U_{\nu\nu'\mu'\mu}^{(1)}$ to $U_{\nu\nu'\mu'\mu}^{(3)}$ come from the Coulomb interaction only, while $U_{\nu\nu'\mu'\mu}^{(4)}$ to $U_{\nu\nu'\mu'\mu}^{(7)}$ are also due to the fermion exchange reflecting the kinematical character of the exciton-exciton interaction. At this moment, a very delicate feature arises. The matter is that to arrive at the effective many-exciton Hamiltonian as expressed by (5.7) to (5.9) of [26] (or by (4.2), (4.3) of [24] or by (6.2.8) to (6.2.10) of [23]), in (8) the term proportional to $U_{\nu\nu'\mu'\mu}^{(7)}$ must be skipped. This skip was silently made in [26] with no explanations. We are here attempting to suggest reasons for such a skip. Since we limit ourselves to the low-density case, the two incoming excitons are to be ideal and fermion exchanges do not occur before the “switch-on” of the Coulomb interaction. The electrons (holes) of the scattered excitons, however, thanks to the so-called final-state interaction, may be exchanged just after the Coulomb interaction “switch-off”. Therefore, the Feynman diagrams corresponding to the interaction mechanisms for which $U_{\nu\nu'\mu'\mu}^{(6)}$ and $U_{\nu\nu'\mu'\mu}^{(7)}$ are responsible will look as sketched respectively in Fig. 1a and b. Logically, the diagram in Fig. 1b should not

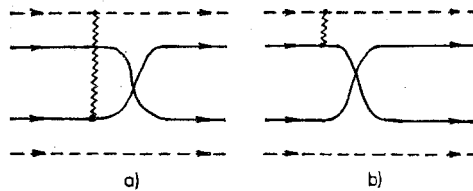


Fig. 1. Feynman diagrams due to a) $U_{\nu\nu'\mu'\mu}^{(6)}$ — and b) $U_{\nu\nu'\mu'\mu}^{(7)}$ — term. Solid, dashed and zigzag lines render electron, hole and the Coulomb force, resp. An exciton is represented by drawing an electron line by a hole one.

be taken into account because its interaction mechanism is of the intra-excitonic kind which had been conventionally accounted for by determining the exciton operators as in Eqs. (2) and (3). The correct many-exciton Hamiltonian thus remains as written in Eq. (8) but in the sum for $U_{\nu\nu'\mu'\mu}$ (see Eq. (9)) i runs from 1 to 6 only.

III. Approximative analytical expressions of the inter-excitonic interaction

Knowing the magnitudes and the momentum- as well as material-parameter dependence of the inter-excitonic interaction terms $U_{\nu\nu'\mu'\mu}^{(i)}$ defined in the previous Section is necessary to interpret different experimental findings. However, being formulated by means of multifold integrals of exciton wave functions $f_{\nu}(p)$ (see (10) to (15)) these terms cannot be evaluated analytically for arbitrary momenta and material parameters even in a particular case when $\nu \equiv \nu' \equiv \mu' \equiv \mu \equiv 1$ S [23].

The difficulty rests in the fact that the exciton wave functions $f_\nu(p)$ are some combinations of fractionary functions like $(1 + xp^2)^{-\nu}$, e.g. for $\nu = 1S$ we have

$$f_{1S}^{3D}(p) = \frac{8\sqrt{(\pi a_{3D}^2)}}{(1 + p^2 a_{3D}^2)^2}, \quad (17)$$

$$f_{1S}^{2D}(p) = \frac{2\sqrt{(2\pi) a_{2D}}}{(1 + p^2 a_{2D}^2)^{3/2}} \quad (18)$$

with $a_{3D}(a_{2D})$ being the 3D (2D) Bohr radius of the exciton. (In stating so we have recognized that the exact forms of $f_\nu(p)$ are unknown and for the evaluation of the inter-excitonic interaction we should, in principle, confine them to those of the isolated excitons [26, 30]). The presence of fractionary functions under integral symbols generates unavoidable difficulties in calculating the integrals $U_{\nu\nu',\mu,\mu'}^{(i)}$, especially their angular parts. In [1] a Silver's approximation [32] was applied for the Coulomb force $V_{k-k'}$. Such an approximation seems quite crude (see Eq. (4.5) in [1]) and we avoid it by changing the variables so that the Coulomb force enters our formulae only in forms of V_k , but not $V_{k-k'}$ (see (13) to (16)). Further, according to [33, 34] we approximate $(1 + xp^2)^{-\nu}$ by $\exp(-xyp^2)$, understanding that its criterion of validity reads: $k_B T \ll \alpha\beta E_b$ (see [33]) where k_B , T and E_b are the Boltzmann constant, temperature of the thermal quasi-equilibrium exciton gas and the exciton binding energy. Since for large p the exponential functions tend to zero faster than the fractionary ones, the results of integrations may be underestimated. We shall make it more accurate by multiplying the exponential functions by constants C_ν which are to be determined from the requirement that approximated wave functions should still obey the normalization condition: $\sum_p |f_\nu(p)|^2 = V$. Doing so for $\nu = 1S$ we shall have $C_{1S}^{3D} = \pi^{1/4}$ and $C_{1S}^{2D} = \sqrt{(1.5)}$, and thus the final approximative wave functions to be used for our further calculations are of the forms:

$$f_{1S}^{3D}(p) = 8\pi^{3/4} a_{3D}^{3/2} \exp(-2p^2 a_{3D}^2). \quad (19)$$

$$f_{1S}^{2D}(p) = 2\sqrt{(3\pi) a_{2D}} \exp(-3p^2 a_{2D}^2/2). \quad (20)$$

Now using the 3D and 2D formulae for V_q

$$V_q^{3D} = \frac{4\pi e^2}{V\epsilon_0 q^2}, \quad (21)$$

$$V_q^{2D} = \frac{2\pi e^2}{S\epsilon_0 q}, \quad (22)$$

where e and ϵ_0 are the electron charge and the static dielectric constant, and for clearness the 2D sample area is now labelled by S , we can easily integrate (10) to (15).

The results look quite convenient to handle:

$$U_{1S1S1S1S}^{(i) \ 3D}(p, p', q) = \frac{E_b^{3D} a_{3D}^3}{V} W_{3D,i}(p, p', q), \quad (23)$$

$$U_{1S1S1S1S}^{(i) \ 2D}(p, p', q) = \frac{E_b^{2D} a_{2D}^2}{S} W_{2D,i}(p, p', q), \quad (24)$$

where

$$W_{3D,1}(p, p', q) = \frac{8\pi}{(qa_{3D})^2} \exp(-2\beta^2 q^2 a_{3D}^2), \quad (25)$$

$$W_{3D,2}(p, p', q) = \frac{8\pi}{(qa_{3D})^2} \exp(-2\alpha^2 q^2 a_{3D}^2), \quad (26)$$

$$M_{3D,3}(p, p', q) = -\frac{16\pi}{(qa_{3D})^2} \exp[-(\alpha^2 + \beta^2) q^2 a_{3D}^2], \quad (27)$$

$$W_{3D,4}(p, p', q) = -32\pi \exp\{-2[\beta^2(p - p' + q)^2 + \alpha^2 q^2] a_{3D}^2\} \\ \times \int_0^1 \exp(2\alpha^2 q^2 a_{3D}^2 x^2) dx, \quad (28)$$

$$W_{3D,5}(p, p', q) = -32\pi \exp\{-2[\alpha^2(p - p' + q)^2 + \beta^2 q^2] a_{3D}^2\} \\ \times \int_0^1 \exp(2\beta^2 q^2 a_{3D}^2 x^2) dx, \quad (29)$$

$$M_{3D,6}(p, p', q) = \frac{128\pi}{\sqrt{3}} \exp\{-2[\alpha^2(p - p' + q)^2 + \beta^2 q^2] a_{3D}^2\} \\ \times \int_0^1 \exp\{2[\alpha(p - p' + q) - \beta q]^2 a_{3D}^2 x^2/3\} dx, \quad (30)$$

$$W_{2D,1}(p, p', q) = \frac{2\pi}{qa_{2D}} \exp(-3\beta^2 q^2 a_{2D}^2/2), \quad (31)$$

$$W_{2D,2}(p, p', q) = \frac{2\pi}{qa_{2D}} \exp(-3\alpha^2 q^2 a_{2D}^2/2), \quad (32)$$

$$W_{2D,3}(p, p', q) = -\frac{4\pi}{qa_{2D}} \exp[-3(\alpha^2 + \beta^2) q^2 a_{2D}^2/4], \quad (33)$$

$$W_{2D,4}(p, p', q) = -\sqrt{(1.5\pi)} \exp\{-3[\beta^2(p - p' + q)^2 + \alpha^2 q^2] a_{2D}^2/2\} \\ \times \int_0^{2\pi} \exp(3\alpha^2 q^2 a_{2D}^2 \cos^2 x/2) dx, \quad (34)$$

$$W_{2D,5}(p, p', q) = -\sqrt{(1.5\pi)} \exp\{-3[\alpha^2(p - p' + q)^2 + \beta^2 q^2] a_{2D}^2/2\} \\ \times \int_0^{2\pi} \exp(3\beta^2 q^2 a_{2D}^2 \cos^2 x/2) dx, \quad (35)$$

$$W_{2D,6}(p, p', q) = 2\sqrt{(2\pi)} \exp\{-3[\alpha^2(p - p' + q)^2 + \beta^2 q^2] a_{2D}^2/2\} \\ \times \int_0^{2\pi} \exp\{[\alpha(p - p' + q) - \beta q]^2 a_{2D}^2 \cos^2 x/2\} dx. \quad (36)$$

From (25) to (36) it is worth noticing that in both 3D and 2D cases the direct inter-excitonic interaction terms ($i = 1, 2, 3$) depend only on q – the momentum transferred between two colliding excitons, while those of the exchange terms ($i = 4, 5, 6$) are functions also of the exciton momentum difference ($p - p'$). So, for excitons with equal momenta $p = p'$ the exciton-exciton interaction depends just upon the momentum transfer. The dependences of $W_{3D,i}$ and $W_{2D,i}$ on the material parameters α and β are explicit, too. For $v, v', \mu', \mu \neq 1S$ the calculations will be more cumbersome but performable within the approximations mentioned above.

IV. Graphical illustrations

To make the analytical expressions obtained in Section III evident for the eyes, we shall now show them graphically. In Fig. 2a we draw the 3D q -dependences of different interaction mechanisms $W_{3D,i}(p, p, q) \equiv W_{3D,i}(q) \equiv W_{3D,i}$ of the direct (exchange) channels taken altogether $W_{3D,d} = W_{3D,1} + W_{3D,2} + W_{3D,3}$ ($W_{3D,ex} = W_{3D,4} + W_{3D,5} + W_{3D,6}$) and of their total contribution $W_{3D} = W_{3D,d} + W_{3D,ex}$. The material parameters used are those of CdS, namely $\alpha = 0.132$ and $\beta = 0.868$. The same for the 2D case are plotted in Fig. 2b. We see that $W_{3D(2D),i}$ taken individually are very large (for $i = 1, 2$ or 3 they even diverge at $q = 0$). Fortunately, they compensate with each other and their total contribution is finite. It equals to a positive value at $q = 0$ and then decreases with increasing q until a certain value of q is reached beyond which $W_{3D(2D)}$ may be negative. For q large enough, say $qa_{3D(2D)} \geq 5$, all the interaction mechanisms give no contributions. This fact permits us to choose an adequate cut-off to avoid the seeming divergence that sometimes might happen in integrating different physical functions over the whole momentum space. Since $W_{3D(2D),d}(q = 0) = 0$ it follows $W_{3D(2D),ex}(q = 0) = W_{3D(2D)}(q = 0)$ revealing that

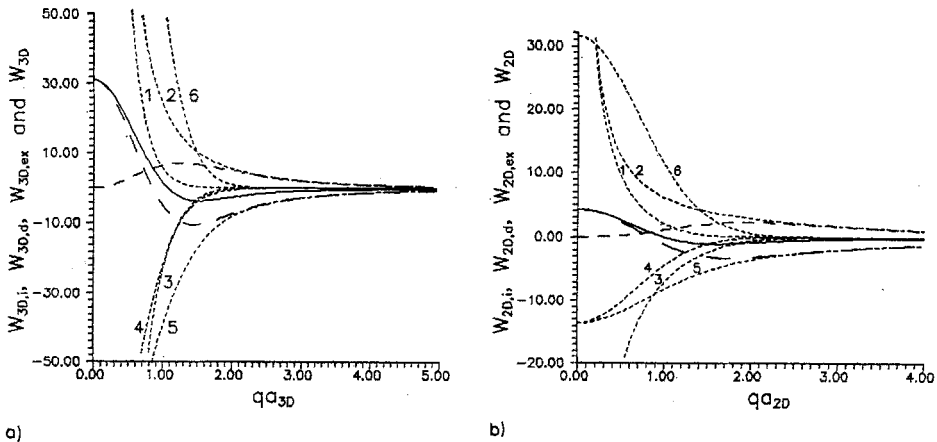


Fig. 2. a) $W_{3D,1}$ to $W_{3D,6}$ (1 to 6 short-dashed curves), $W_{3D,d}$ (the medium-dashed curve), $W_{3D,ex}$ (the long-dashed curve) and W_{3D} (the solid curve) as functions of qa_{3D} for CdS. b) The same for "2D".

when putting $W_{3D(2D)}(q) = W_{3D(2D)}(q = 0)$ (as done e.g. in [1, 2, 16, 17, 20, 22, 23]) one unintentionally ignores the direct inter-excitonic interaction mechanisms which in fact give non-negligible values for $q > 0$ (see Fig. 2). From the figure we also see that the mutual exciton-exciton interaction is remarkably reduced in the 2D case as compared with that in the 3D one. This is due to the narrower extension of 2D excitons in the real space ($a_{2D} = 0.5a_{3D}$) producing a stronger local charge neutralization and hence a weaker dipole-dipole kind inter-excitonic interaction. (The intra-excitonic interaction, of course, by the same reason, is enhanced: $E_b^{2D} = 4E_b^{3D}$).

How the material parameters affect the q -dependences is demonstrated in Fig. 3 for the 2D case (the 3D case is not shown because of its similarity), where W_{2D}

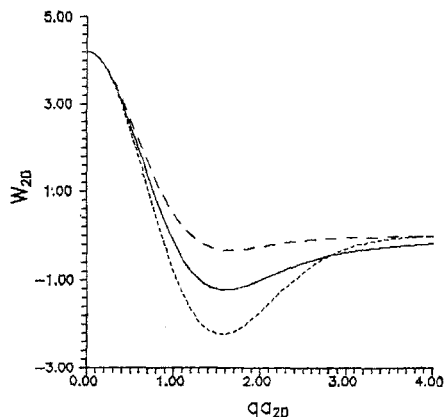


Fig. 3. W_{2D} vs qa_{2D} for $\sigma = 0, 0.2$ and 1 (the long-dashed, solid and short-dashed curves, resp.). The unphysical limiting values of $\sigma = 0$ and 1 are chosen just for a demonstrative purpose.

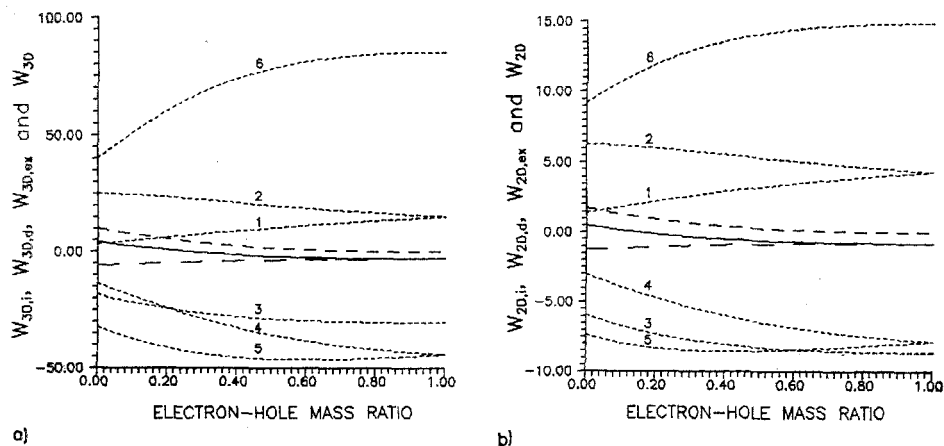


Fig. 4. a) Functions $W_{3D,1}$ to $W_{3D,6}$ (1 to 6 short-dashed curves), $W_{3D,d}$ (the medium-dashed curve), $W_{3D,ex}$ (the long-dashed curve) and W_{3D} (the solid curve) vs σ for $qa_{3D} = 1$. b) The same for "2D".

are drawn as functions of qa_{2D} for three values of the electron-hole mass ratio $\sigma = m_e/m_h = 0, 0.2$ and 1 . The larger the value of σ the deeper the minimum of the curves $W_{2D}(q)$.

Figure 4 represents $W_{3D(2D),i}$, $W_{3D(2D),d}$, $W_{3D(2D),ex}$ and $W_{3D(2D)}$ versus the electron-hole mass ratio for $qa_{3D(2D)} = 1$. Though the variations versus σ of $W_{3D(2D),i}$, especially for $i = 6$, are visible, $W_{3D(2D),d}$, $W_{3D(2D),ex}$ and $W_{3D(2D)}$ are slowly-varying functions of σ . For $qa_{3D(2D)} = 1$ the total inter-excitonic interaction slightly decreases when σ increases from 0 to 1.

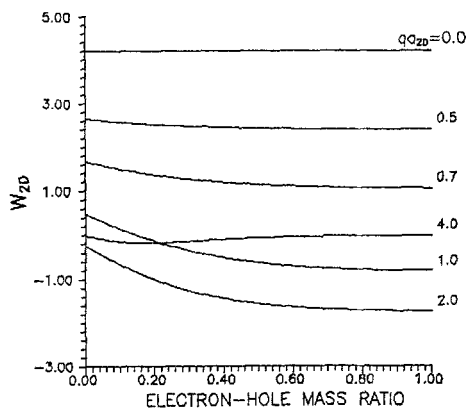


Fig. 5. The σ -dependences of W_{2D} for some values of qa_{2D} which are indicated directly on the figure.

Finally, we sketch in Fig. 5 the dependences of W_{2D} on σ for several values of qa_{2D} (the similar figure for the 3D case is not shown). For $q = 0$, W_{2D} does not depend on σ at all. For larger q it decreases slightly with increasing σ , but for $qa_{2D} \geq 4$ it becomes again almost independent of σ .

V. Conclusion

In conclusion, we have shown that it is possible to investigate analytically the dependences of the exciton-exciton interaction on the exciton momenta, the momentum transfer and the electron-hole mass ratio in 3D and 2D laser-excited semiconductors. Hopefully, such dependences would be useful toward getting better agreement with experimental data. Up to now many problems of the physics of highly excited media have been questioned and they often suffer from different ways of solution which not very seldom could result in contradictory outcomes. Take, for example, the problem of the density-dependent exciton lineshape determined by the function

$F(k, \omega)$. For the lowest energy level one has

$$F(k\omega) = \frac{1}{\pi} \frac{\Gamma(k\omega)}{[\omega - E(k) - \Delta(k\omega)]^2 + \Gamma^2(k\omega)}, \quad (37)$$

where the level shift Δ and damping Γ linearized in density are derived from the dressed exciton retarded Green function as

$$\Delta(k\omega) = U(0)N + \sum_q U(q)N_q + \sum_{pq} \frac{[U(k-p) + U(p-q)]U(k-p)N_q}{E(k) + E(q) - E(p) - E(k+q-p)}, \quad (38)$$

$$\Gamma(k\omega) = \pi \sum_{pq} [U(k-p) + U(p-q)]U(k-p)N_q \times \delta[E(k) + E(q) - E(p) - E(k+q-p)] \quad (39)$$

with N_q being the exciton distribution. If one restricts oneself to the Hartree-Fock approximation only (the first term of (38)), the level shift will be blue in both 3D and 2D cases [7, 13, 20]. On the other hand, if one pays attention just to the first correlation correction (the remaining part of (38)), the level will red shift (again in both cases of dimensionality). The total shifts are, however, opposite in the two cases: the 3D total shift is red, while that of the 2D case is blue [7]. The point to be stressed here is that the results just mentioned above can only serve as a first orientation for the level shifts [13] because in calculating (38) (and also (39)) too many simplified approximations have been made (see [13] and [35] to be aware of the discrepancies between results). If we use $U(q)$ in the exact forms of (9) to (15), the sums over p and q in (38) and (39) will not be performable even with the help of a computer. On the contrary, the approximative formulae (25) to (36) being expressed in terms of exponential functions could give advantage to the calculations of expressions like (38) and (39) or others. In the near future we plan to do such work not only for 3D and pure 2D structures, but also for real quasi-2D structures called (multiple-) quantum-wells which have already been fabricated and begun to give practical applications.

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