OPTICAL BISTABILITY DUE TO VIRTUAL FORMATION OF BIEXCITONS IN SEMICONDUCTORS

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Intrinsic optical bistability is theoretically investigated in a coherent photon-biexciton system. It is shown that bistability due to virtual two photon-biexciton transitions could occur if the system were pumped by a single external field. The conditions for the appearance of the bistability are determined.

I. INTRODUCTION

Optical bistability (OB) as a typical example of a first-order nonequilibrium phase transition in physical systems far away from thermal equilibrium was first theoretically predicted by Szoke et al. [1]. In the last few years the phenomenon of OB has gained great attention, both theoretically and experimentally, partly because of its practical potential for all-optical data processing devices (see e.g. [2]). Many physical systems should exhibit bistability. In semiconductors OB was first observed experimentally by Miller et al. [3] and Gibbs et al. [4]. In [5, 6] quantum theories were developed for excitonic OB. Recently, it has been pointed out that the two-photon resonance of the biexciton is also a source of large optical nonlinearities [7]. The fusion of two photons into a biexciton gives rise to an intensity dependence of the dielectric function [8] and this nonlinearity makes possible OB in wide-gap semiconductors like CuCl, CuBr, ... where biexcitons may be generated. Unlike refs. [8 – 10], where a Fabry-Perot cavity used as a feedback element, was described in ref. [11] the effect of OB in a coherent photon-biexciton system without any resonators was studied. This variant of a new type of OB is called resonatorless or intrinsic OB (see, for example [12]). The authors of [11] assumed that there were two external resonant pumping fields with intensities P_1 and P_2 acting on a semiconductor and directly creating coherent photons and biexcitons. According to their results OB could occur only if both pump fields were present. If either P_1 or P_2 were absent, there were either no photons or a monotonic growth of the photon number with increasing intensity P_1.

In this paper we would like to show that OB in a coherent photon-biexciton system can also take place even when only one pump field, say P_1, is present. In such a single-beam variant OB would be exhibited if certain conditions are met, which we should determine in the next section.

Throughout this paper we shall use such units that $\hbar = c = V = 1$ where $\hbar$ is the Planck constant, c is the velocity of light and V is the volume of the sample.

II. OPTICAL BISTABILITY

Let us consider the nonlinear nutation phenomenon in a coherent photon-biexciton system [13] under the action of an external periodic classical field:

$$E_k(t) = E_k^{(-)}\,e^{-i\omega_0 t} + E_k^{(+)}\,e^{i\omega_0 t} \tag{1}$$

with amplitudes $E_k^{(-)}, E_k^{(+)}$, frequency ω_0 and wave vector k. Suppose that ω_0 is such smaller than the energy of the biexcitons Ω_{2k}, but falls in the spectral region of that of the photons ω_k in the system (from now on referred to as inside photons). Then the external field can drive the inside photons, which in turn are coupled to the biexcitons of the system. Therefore, the effective Hamiltonian responsible for the problem of interest can be written in the form:

$$\begin{aligned} H_k(t) = {} & \Omega_{2k} b_{2k}^+ b_{2k} + \omega_k c_k^+ c_k + i m_k (c_k^+ c_k^+ b_{2k} - b_{2k}^+ c_k c_k) + \\ & + i(E_k^{(-)} c_k^+ \, e^{-i\omega_0 t} - E_k^{(+)} c_k \, e^{i\omega_0 t}) \end{aligned} \tag{2}$$

where $b_{2k}^+(b_{2k})$ and $c_k^+(c_k)$ are boson operators for biexcitons and inside photons with energies Ω_{2k} and ω_k, respectively. m_k is the matrix element of the two photon-biexciton transition. In general, into Hamiltonian (2) operators of dissipative systems must be added. However, the departure of photons and biexcitons from the coherent mode to incoherent ones can be described phenomenologically by introducing the damping constants γ_k and Γ_{2k} corresponding to the photons and biexcitons. Then the Heisenberg equations of motion with damping are as follows:

$$i\dot{b}_{2k} = \Omega_{2k} b_{2k} - i m_k c_k c_k - i\Gamma_{2k} b_{2k} \tag{3}$$

$$i\dot{c}_k = \omega_k c_k + 2\, i m_k c_k^+ b_{2k} - i\gamma_k c_k + iE_k^{(-)}\, e^{-i\omega_0 t}\,. \tag{4}$$

We now seek the particular stationary solutions of the system of nonlinear eqs. (3) and (4) in the form:

$$b_{2k} = \tilde{b}_{2k}\, e^{-2i\omega_0 t} \tag{5}$$

$$c_k = \tilde{c}_k\, e^{-i\omega_0 t}\,. \tag{6}$$

Inserting (5) and (6) into (3) and (4) and then multiplying (4) by $\tilde{c}_k$ we obtain

$$(2\omega_0 - \Omega_{2k} + i\Gamma_{2k})\,\tilde{b}_{2k} + i m_k \tilde{c}_k^2 = 0 \tag{7}$$

$$-\,2\, i m_k n \tilde{b}_{2k} + (\omega_0 - \omega_k + i\gamma_k)\,\tilde{c}_k^2 = iE_k^{(-)}\tilde{c}_k \tag{8}$$

In (8) $n = |c_k|^2 = \tilde{c}_k^* \tilde{c}_k$ denotes the inside photon number in the stationary states.

From (7) and (8) we obtain an equation allowing us to receive the driving field intensity dependence of the photon number n:

$$n = \frac{[(2\omega_0 - \Omega_{2k})^2 + \Gamma_{2k}^2]\, I}{[(2\omega_0 - \Omega_{2k})(\omega_0 - \omega_k) - \gamma_k \Gamma_{2k} - 2m_k^2 n]^2 + [\gamma_k(2\omega_0 - \Omega_{2k}) + \Gamma_{2k}(\omega_0 - \omega_k)]^2} \tag{9}$$

where the pump field intensity is labelled by $I = |E^{(-)}|^2 = |E^{(+)}|^2$. Equation (9) can also be represented in a different form, that is:

$$n = \frac{[(2\omega_0 - \Omega_{2k})^2 + \Gamma_{2k}^2]\, I}{[(\omega_0 - \omega_1)^2 + \gamma_1^2] \cdot [(\omega_0 - \omega_2)^2 + \gamma_2^2]} . \tag{10}$$

In formula (10) $\omega_1(\omega_2)$ and $\gamma_1(\gamma_2)$ are the eigenfrequencies and dampings of the polariton-like two-branch eigenmodes [13] of the system where there is a mixing between two photons and a biexciton. In other words, the photons here are self-inducedly renormalized due to virtual formation of biexcitons at high densities [14]. $\omega_{1,2}$ and $\gamma_{1,2}$ might be calculated from the requirement that the system of two equations (7) and (8) in absence of the external field should have nontrivial solutions.

$$\omega_{1,2}(k) = \tfrac{1}{2}(\omega_k + \tfrac{1}{2}\Omega_{2k}) \pm \frac{1}{2\sqrt{2}} \{[(\omega_k - \tfrac{1}{2}\Omega_{2k})^2 - (\gamma_k - \tfrac{1}{2}\Gamma_{2k})^2 + 4m_k^2 n)^2 + (\omega_k - \tfrac{1}{2}\Omega_{2k})^2 (\gamma_k - \tfrac{1}{2}\Gamma_{2k})^2]^{1/2} + (\omega_k - \tfrac{1}{2}\Omega_{2k})^2 + 4m_k^2 n - (\gamma_k - \tfrac{1}{2}\Gamma_{2k})^2\}^{1/2} , \tag{11}$$

$$\gamma_{1,2}(k) = \tfrac{1}{2}(\gamma_k + \tfrac{1}{2}\Omega_{2k}) \pm \frac{1}{2\sqrt{2}} \{[(\omega_k - \tfrac{1}{2}\Omega_{2k})^2 - (\gamma_k - \tfrac{1}{2}\Gamma_{2k})^2 + 4m_k n)^2 + (\omega_k - \tfrac{1}{2}\Omega_{2k})^2 (\gamma_k - \tfrac{1}{2}\Gamma_{2k})^2]^{1/2} - (\omega_k - \tfrac{1}{2}\Omega_{2k})^2 - 4m_k^2 n + (\gamma_k - \tfrac{1}{2}\Gamma_{2k})^2\}^{1/2} . \tag{12}$$

The dependence of n upon the pump field frequency ω_0 and its intensity I, following from (10), are rather complicated because the eigenfrequencies $\omega_{1,2}$ and dampings $\gamma_{1,2}$ themselves depend on n (see (11) and (12)). By analogy with the phenomenon of nonlinear nutation in a photon-exciton system [15] we can also here expect that there exist some intervals of frequency for each of the two polariton-like branches where the so-called frequency hysteresis loops can be formed when ω_0 is first increasing and then decreasing. In the following we shall study the intensity dependence of n with ω_0 being treated as a parameter which would obey some conditions for the appearance of OB. For this purpose, it is convenient to utilize eq. (9) which is now rewritten in new notations:

$$n = \frac{[(2\delta_k + \lambda_k)^2 + \Gamma_{2k}^2]\, I}{[(2\delta_k + \lambda_k)\,\delta_k - \gamma_k \Gamma_{2k} - 2m_k^2 n]^2 + [\gamma_k(2\delta_k + \lambda_k) + \Gamma_{2k}\delta_k]^2} \tag{13}$$

with new symbols determined as

$$\delta_k = \omega_0 - \omega_k , \quad \lambda_k = 2\omega_k - \Omega_{2k} . \tag{14}$$

Note that if the external pump field is in perfect resonance with the photons ($\omega_0 = \omega_k$) then for the wave vector $k = k_0$ satisfying the equation

$$2\omega_{k_0} = \Omega_{2k_0} \tag{15}$$

we have from (13)

$$n = \frac{\Gamma_{2k_0}^2 I}{(\gamma_{k_0}\Gamma_{2k_0} + 2m_{k_0}^2 n)^2} . \tag{16}$$

From this equation follows that $n = 0$ when $I = 0$ and $\partial n/\partial I$ is always positive. This means that n grows monotonically with increasing pump field intensity I and thus no OB could be observed. This result coincides with that of [11] when P_2 is set equal to zero.

Now suggest that the system has been pumped by a single beam in the off-resonance frequency region $\delta_k \neq 0$ where it is almost transparent for low incident light intensities. Only a small number of the inside photons is created by the weak coupling between the external field and the inside photons caused by the damping constant γ_k of the latter. Nevertheless, with increasing pump intensity the photon number in the system increased and the photon-biexciton interaction comes into play which shows itself in the virtual formation of biexcitons. By such a way the inside photons must be self-renormalized and their energy spectrum exhibit a gap the width of which enlarges when n increases [14]. When the light intensity becomes sufficiently high, it can result in a situation in which the renormalized spectrum (upper and lower polariton-like branches ω_1 and ω_2 in (11)) passes the incident frequency and hence, the number of inside photons increases abruptly. We have to find the light intensity and the necessary conditions at which the above-mentioned sudden increase of n should take place. For this aim we rewrite (13) in the form:

$$n[(n + \alpha_k)^2 + \beta_k^2] = \gamma_k^2 \dot{J} \tag{17}$$

where

$$\alpha_k = \frac{\gamma_k \Gamma_{2k} - (2\delta_k + \lambda_k)\,\delta_k}{2m_k^2}\,, \quad \beta_k = \frac{\gamma_k(2\delta_k + \lambda_k) + \Gamma_{2k}\delta_k}{2m_k^2}\,,$$
$$\gamma_k = \frac{(2\delta_k + \lambda_k)^2 + \Gamma_{2k}^2}{2m_k^2}\,, \quad \dot{J} = \frac{I}{2m_k^2}\,. \tag{18}$$

The conditions for OB of n are determined by requiring the equation $\partial \dot{J}/\partial n = 0$ to have two real positive solutions. This implies either

$$\beta_k > 0 \quad \text{and} \quad \alpha_k + \beta_k \sqrt{(3)} < 0 \tag{19}$$

or

$$\beta_k < 0 \quad \text{and} \quad \alpha_k - \beta_k \sqrt{(3)} < 0 \tag{20}$$

for a bistability to occur.

Substituting α_k and β_k from (18) into (19) it is easy to check that (19) will be satisfied if the following inequalities hold:

$$\delta_k > -\frac{\gamma_k \lambda_k}{2\gamma_k + \Gamma_{2k}}\,, \tag{21}$$

$$\delta_k < \frac{\sqrt{3}}{4}\left\{2\gamma_k + \Gamma_{2k} - \frac{\lambda_k}{\sqrt{3}} - \left[\left(2\gamma_k - \Gamma_{2k} + \frac{\lambda_k}{\sqrt{3}}\right)^2 + \frac{32}{3}\gamma_k \Gamma_{2k}\right]^{1/2}\right\}, \tag{22}$$

$$(23)\qquad \delta_k > \frac{\sqrt{3}}{4}\left\{2\gamma_k + \Gamma_{2k} - \frac{\lambda_k}{\sqrt{3}} + \left[\left(2\gamma_k - \Gamma_{2k} + \frac{\lambda_k}{\sqrt{3}}\right)^2 + \frac{32}{3}\gamma_k\Gamma_{2k}\right]^{1/2}\right\},$$

while (20) requires

$$(24)\qquad \delta_k < -\frac{\gamma_k\lambda_k}{2\gamma_k + \Gamma_{2k}},$$

$$(25)\qquad \delta_k < -\frac{\sqrt{3}}{4}\left\{2\gamma_k + \Gamma_{2k} - \frac{\lambda_k}{\sqrt{3}} + \left[\left(2\gamma_k - \Gamma_{2k} + \frac{\lambda_k}{\sqrt{3}}\right)^2 + \frac{32}{3}\gamma_k\Gamma_{2k}\right]^{1/2}\right\},$$

$$(26)\qquad \delta_k > -\frac{\sqrt{3}}{4}\left\{2\gamma_k + \Gamma_{2k} - \frac{\lambda_k}{\sqrt{3}} - \left[\left(2\gamma_k - \Gamma_{2k} + \frac{\lambda_k}{\sqrt{3}}\right)^2 + \frac{32}{3}\gamma_k\Gamma_{2k}\right]^{1/2}\right\}$$

If we are interested in the value of $k = k_0$ (see (15)) then $\lambda_{k_0} = 0$ and bistability would take place when the pump field frequency ω_0, the photon damping γ_{k_0} and that of the biexcitons Γ_{2k_0} obeyed the following relation:

$$(27)\qquad |\delta_{k_0}| > \frac{\sqrt{3}}{4}\left\{2\gamma_{k_0} + \Gamma_{2k_0} + \left[(2\gamma_{k_0} - \Gamma_{2k_0})^2 + \frac{32}{3}\gamma_{k_0}\Gamma_{2k_0}\right]^{1/2}\right\}.$$

To illustrate the result obtained above we restrict ourselves to a particular case where Γ_{2k_0} is supposed to be equal to $2\gamma_{k_0}$. Furthermore, we introduce two new normalized dimensionless parameters:

$$(28)\qquad \varkappa = \frac{\delta_{k_0}}{2\gamma_{k_0}} \quad \text{and} \quad \theta = \frac{m_{k_0}^2}{4\gamma_{k_0}^2}.$$

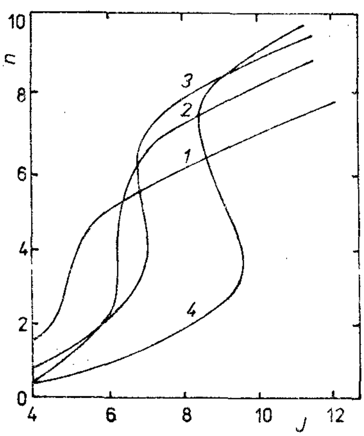

Fig. 1.

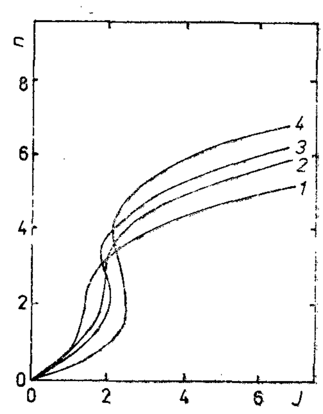

Fig. 2

Fig. 1. Photon number n as function of the pump field intensity J for $\theta = 0{\cdot}5$; $|\varkappa| = 1{\cdot}730$ (curve 1), $1{\cdot}866$ (curve 2), $2{\cdot}000$ (curve 3) and $2{\cdot}200$ (curve 4).

Fig. 2. The same as in fig. 1 for $\theta = 1{\cdot}0$.

Equation (17) now takes the form

$$j = \frac{2\theta}{1 + 4\varkappa^2} \left[\left(n + \frac{1 - 4\varkappa^2}{4\theta} \right)^2 + \frac{\varkappa^2}{\theta^2} \right] n \tag{29}$$

and the condition (27) becomes:

$$|\varkappa| > 1{\cdot}866 . \tag{30}$$

In figs. 1 and 2 we plot function n versus j for $\theta = 0{\cdot}5$ and 1 and $|\varkappa| = 1{\cdot}730$, $1{\cdot}866$, $2{\cdot}000$ and $2{\cdot}200$. It is clear that for $|\varkappa| = 1{\cdot}730$ n is a monotonic function of j, while OB displays itself for $|\varkappa| = 2{\cdot}000$ and $2{\cdot}200$. The curves with $|\varkappa| = 1{\cdot}866$ are the critical ones. For another case with the biexciton damping ignored the critical curves would show themselves at $|\varkappa| = 0{\cdot}866$.

III. DISCUSSION

We have shown that OB can be exhibited in a coherent photon-biexciton system by single-beam excitation method if the pump field frequency is tuned not to the exact two photon resonance of biexciton but just somewhat below or above this resonance point. In such off-resonance region at high densities there is a mixing between two photons and a biexciton which leads to the energy band splitting in the resonance domain to form two new renormalized bands with a gap between them. As we have seen, it is the dependence of the gap width upon the photon number n that is necessary for OB to occur in the system. The farther the excitation frequency from the exact resonant point (the greater $|\varkappa|$) the higher the beam intensity at which a discontinuous switching from a state of small (or large) n to a state of large (or small) n takes place, i.e. the bigger is the intensity hysteresis loops. (See figs. 1 and 2). This statement can be expressed quantitatively by the expression derived from (29) for the dependences of the switch-on j_{on} and switch-off j_{off} intensities upon $\varkappa^2$ (fig. 3)

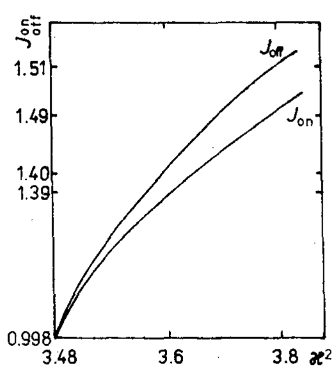

Fig. 3. Switch-on j_{on} and switch-off j_{off} as functions of $\varkappa^2$ ($\theta = 1$).

$$(31)\quad J^{\text{on}}_{\text{off}} = \\ = \frac{4\varkappa^2 - 1 \mp \frac{1}{2}\sqrt{[(1-4\varkappa^2)^2 - 48\varkappa^2]\{[1 - 4\varkappa^2 \mp \sqrt{((1-4\varkappa^2)^2 - 48\varkappa^2)}]^2 + 144\varkappa^2\}}}{432\theta^2(1+4\varkappa^2)}.$$

Also (31) shows that both switch-on and switch-off intensities are inversely proportional to θ^2, i.e. the stronger the photon-biexciton interaction or the smaller the photon damping, the lower the holding intensities. Conversely, by experimental measurements on OB one might estimate the magnitude of the photon-biexciton interaction (at least relative magnitudes from a material to another) as well as the dampings. As concerns the optical feedback, it is obvious that here no cavity geometry is needed for OB and one can expect this intrinsic OB could display a very fast response.

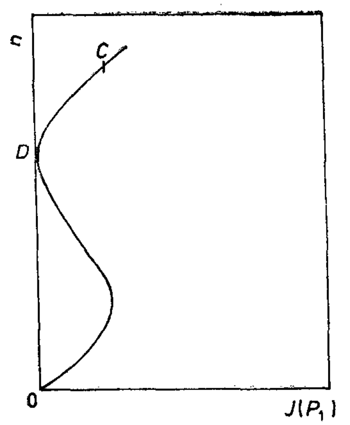

Fig. 4. Pump field intensity dependence of the photon number n (according to ref. [11]).

In comparison with ref. [11] we note that there is a difference in behaviour between OB treated there and here. In [11] moving from point C (see fig. 4) along the upper branch of the S-shaped curve $n = n(J)$ in the direction towards low intensities we arrive at point D where the input intensity (J here and P_1 there) vanishes but the number of the photons in the system is finite which is due to the presence of the second resonant pump field with intensity P_2. As in our variant there is solely one external field driving the inside photons n which should vanish when the intensity P_1 tends to zero (figs. 1 and 2).

Finally, it is worth noticing that according to [11] biexcitons can be generated either by the resonant field P_2 or by the real conversion of two photons into a biexciton. The steady states of the system then contain a large number of not only photons but also real biexcitons and thereby the interaction between biexcitons can play an important role, which may change the OB picture. Such an interaction was not taken into account yet in [11]. In our consideration the biexciton-biexciton interaction plays a negligible role, because only virtual biexcitons are formed.

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