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SQUEEZED STATE OF BIEXCITONS IN EXCITED SEMICONDUCTORS

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Exact time-evolution is analytically derived from a coherent photon-exciton system subjected to a monochromatic radiation field resonant with the exciton-biexciton transition. It is shown that biexcitons, which appear under the resonant pumping, might periodically behave as squeezed quasiparticles, if photons or/and excitons are initially in squeezed states. The squeezing degree of the initial photons or/and excitons, the pumping intensity and the off-resonance detuning are demonstrated to govern the transfer of squeezing from the photon-exciton system to the biexciton.

1. Introduction

Novel nonclassical states of the radiation field called squeezed states manifest many interesting quantum fluctuation properties.[1,2] The paramount significance of the squeezed state is their ability to raise the precision of optical measurements beyond the standard quantum noise limit. More concretely speaking, such states can reduce quantum noise as much as wanted for one observable and, at the same time, do no violate the well-recognized Heisenberg uncertainty relation by correspondingly increasing quantum noise for the other conjugate observable. For practical use, information of course should be carried by that observable whose quantum noise was reduced. The first experimental confirmation[3] of the existence of the squeezed state of light in 1985 and its first principal application realization[4] in 1987 promised a very good future of applying squeezed light, e.g., to detect gravitons or to almost losslessly transmit optical communication.

There are several kinds of work dealing with the squeezing phenomenon. One kind suggests mechanisms of producing squeezed light in different physical systems.[5–10] Another kind investigates the alteration of the squeezing of light due to the light-matter interaction.[11–13] The third kind considers how various linear and nonlinear optical systems respond to the squeezed light.[14] The fourth kind studies

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possibility of periodic exchange of statistical properties between interacting light fields[15] and so forth.

Besides, as noted in Ref. 2, new concepts in optics such as squeezed states, antibunching or subpoissonian statistics,... possess a certain community: their analogy must exist in other areas, say, in condensed matter physics and in quantum field theory. In Ref. 16 we have outlined a possible transfer of squeezing from quantum light to condensed matter quasiparticles, whereas in Ref. 17 the concept of squeezed excitons in semiconductors has been introduced. Our attempt in this paper is to propose a way to produce squeezed biexcitons in an excited semiconductor which is modelled as a photon-exciton system. As noted in Ref. 16, biexcitons are optically passive in the sense that they cannot appear directly by one-photon absorptions. Therefore, to generate biexcitons one should pump the system by a classical electromagnetic field which is spectroscopically resonant with the exciton-biexciton transition. We shall show that the necessary conditions for the biexciton to become squeezed are simultaneous nonzero pump intensity and nonzero initial squeezing degree of the photon or/and the exciton.

For convenience we shall use the $\hbar = c = 1$ unit system.

2. Working Hamiltonian

Consider a coherent photon-exciton system which is placed in the presence of a monochromatic classical laser field characterized by an angular frequency Ω and complex amplitudes $A^{(\pm)}$ associated with the time oscillating parts $\exp(\pm i\Omega t)$. Because of the coherence nature of the system and the monochromaticity of the field we can for brevity suppress the momentum (wave vector) indices in all of the symbols and formulae afterwards. Suppose the laser frequency is nearly equal to the energy gap between the exciton and biexciton levels so that one can realize the exciton-assisted optical generation of biexcitons. Both excitons and biexcitons in fact behave as composite nonbosonic quasiparticles.[18] However, we confine ourselves here only to the low density limit and thus we may assume that these quasiparticles are ideal bosons. Denoting by a, a^+ (b, b^+ or c, c^+) the bosonic operators of an exciton (a biexciton or a photon) with energy E (energy ε or frequency ω), we can write down the Hamiltonian of the system under consideration in the form:

$$\begin{aligned} H = {} & Ea^+a + \varepsilon b^+b + \omega c^+c + g(a^+c + c^+a) \\ & + f\left[A^{(-)}e^{-i\Omega t}b^+a + A^{(+)}e^{+i\Omega t}a^+b\right] \end{aligned} \tag{1}$$

where g stands for the photon-exciton coupling constant[19] and f for the exciton-biexciton transition dipole.[22,23] In principle, we can set up from Eq. (1) the equations of motion for the quasiparticle operators to find their time development. But these differential equations containing terms proportional to $\exp(\pm\Omega t)$ are difficult to solve. To get rid of such time dependences we could go into an interaction picture. To do so and to explicitly reflect the laser resonance at the energy $(\varepsilon - E)$, we should consider the overall Hamiltonian (1) as composed of two parts, a "free" Hamiltonian H_0

$$H_0 = E(a^+a + c^+c) + (E + \Omega)b^+b \tag{2}$$

and an "interaction" Hamiltonian H_{int}

$$H_{\text{int}} = H - H_0 \ . \tag{3}$$

It is easy to verify that in such an interaction representation the fast time oscillatons $\exp(\pm i\Omega t)$ will be automatically eliminated and then we shall have the following form of the working Hamiltonian $\mathcal{H}$:

$$\begin{aligned}\mathcal{H} = e^{+iH_0t}H_{\text{int}}e^{-iH_0t} &= (\varepsilon - E - \Omega)b^+b + (\omega - E)c^+c \\ &+ g(a^+c + c^+a) + f\left[A^{(-)}b^+a + A^{(+)}a^+b\right] \ .\end{aligned} \tag{4}$$

Note that the choice of H_0, like in Eq. (2), differs from that in, e.g., Ref. 22 and will lead to an adequate pole at frequency $\Omega = \varepsilon - E$ in the complex dielectric function. The pole, however, is strange[24] for the problems involving the giant two-photon-biexciton transition and such a strangeness can be removed by using an appropriate effective Hamiltonian.[25–27] In what follows we shall for simplicity handle the working Hamiltonain (4) assuming the laser amplitude to be real, i.e. $A^{(+)} = A^{(-)} = A$.

3. Diagonalization of $\mathcal{H}$

Many problems in physics need a diagonalization of an appropriate Hamiltonian. In general, one can analytically diagonalize bilinear Hermitian Hamiltonians.[28] Yet, in particular, some kinds of non-Hermitian Hamiltonians might also be diagonalized.[29] In the case of two-mode systems the diagonalization yields the well-known polariton.[30,19] Our working Hamiltonian here is that of a three-mode kind whose diagonalization can be performed by canonically transformating a, b, c-operators to new d_ν-operators ($\nu = 1, 2$ and 3) as below:

$$d_\nu = u_\nu c + v_\nu a + w_\nu b \tag{5}$$

$$d_\nu^+ = u_\nu c^+ + v_\nu a^+ + w_\nu b^+ \tag{6}$$

where real functions u_ν, u_ν and w_ν are required to be determined. If we demand the d_ν-operators to also be bosonic, then the functions u_ν, u_ν and w_ν must obey the following orthogonalization and normalization conditions:

$$u_\nu u_\mu + v_\nu v_\mu + w_\nu w_\mu = \delta_{\nu\mu} \ . \tag{7}$$

If we further request the function u_ν, v_ν and w_μ to be such that they will diagonalize the Hamiltonian (4) into the form:

$$\mathcal{H} = \sum_\nu U_\nu d_\nu^+ d_\nu \tag{8}$$

with U_ν being the eigenenergies of the new "free" excitations described by the operators d_ν and d_ν^+, these transformation functions and quantities U_ν should satisfy the equation system as below:

$$(U_\nu - \lambda)u_\nu - gv_\nu = 0 \tag{9}$$

$$-gu_\nu + U_\nu v_\nu - fAw_\nu = 0 \tag{10}$$

$$-fAv_\nu + (U_\nu - \delta)w_\nu = 0 \tag{11}$$

where λ and δ stand for $\omega - E$ and $\varepsilon - E - \Omega$. The eigenenergies U_ν are of course the nontrivial solutions of the equation determined by the determinant D:

$$D = \begin{vmatrix} U_\nu - \lambda & -g & 0 \\ -g & U_\nu & -fA \\ 0 & -fA & U_\nu - \delta \end{vmatrix} \tag{12}$$

while the expressions for the transformation functions read

$$u_\nu = \left\{ 1 + \frac{g^2\left[(U_\nu - \delta)^2 + f^2 I\right]}{\left[U_\nu(U_\nu - \delta) - f^2 I\right]^2} \right\}^{-1/2} \tag{13}$$

$$v_\nu = \frac{g(U_\nu - \delta)u_\nu}{U_\nu(U_\nu - \delta) - f^2 I} \tag{14}$$

$$w_\nu = \frac{fAv_\nu}{U_\nu - \delta} \tag{15}$$

with $I = A^2$ being the external pump laser intensity. From Eqs. (9–11) it is easy to prove that

$$\sum_\nu u_\nu^2 = \sum_\nu v_\nu^2 = \sum_\nu w_\nu^2 = 1 \tag{16}$$

and

$$\sum_\nu u_\nu v_\nu = \sum_\nu u_\nu w_\nu = \sum_\nu v_\nu w_\nu = 0\ . \tag{17}$$

Applying Eqs. (16, 17) to Eqs. (5, 6) we can obtain the following useful transformations which are inverse to Eqs. (5, 6)

$$c = \sum_\nu u_\nu d_\nu\ , \quad c^+ = \sum_\nu u_\nu d_\nu^+ \tag{18}$$

$$a = \sum_\nu v_\nu d_\nu\ , \quad a^+ = \sum_\nu v_\nu d_\nu^+ \tag{19}$$

$$b = \sum_\nu w_\nu d_\nu\ , \quad b^+ = \sum_\nu w_\nu d_\nu^+\ . \tag{20}$$

4. Exact Time-Evolution

Going now to the Heisenberg representation for Hamiltonian $\mathcal{H}$ in (8) and using it to set up the equations of motion for the operators d and d^+. These equations, thanks to the diagonalized form of $\mathcal{H}$, are exactly solvable giving their solution with the following explicit time-dependence:

$$d_\nu(t) = d_\nu(0)e^{-iU_\nu t} , \tag{21}$$

$$d_\nu^+(t) = d_\nu^+(0)e^{+iU_\nu t} . \tag{22}$$

Substituting Eqs. (21, 22) into Eqs. (20) and taking into account Eqs. (5, 6) for the operators $d_\nu(0)$ and $d_\nu^+(0)$, we get expressions for the biexciton operators which depend on time and also on the initial conditions of the system under consideration. Namely, they are

$$b(t) = \alpha(t)c(0) + \beta(t)a(0) + \gamma(t)b(0) , \tag{23}$$

$$b^+(t) = \alpha^*(t)c^+(0) + \beta^*(t)a^+(0) + \gamma^*(t)b^+(0) . \tag{24}$$

In Eqs. (23, 24) α, β and γ are complex functions of time:

$$\alpha(t) = \sum_\nu u_\nu w_\nu e^{-iU_\nu t} , \tag{25}$$

$$\beta(t) = \sum_\nu v_\nu w_\nu e^{-iU_\nu t} , \tag{26}$$

$$\gamma(t) = \sum_\nu w_\nu^2 e^{-iU_\nu t} . \tag{27}$$

The time-dependence described by Eqs. (23, 24) is exact and reveals that the behaviour of the biexciton at an arbitrary moment t is governed not only by its own behaviour at $t = 0$ but also by the initial behaviour of the photon and the exciton. This allows us to expect that at some moment or during some interval of time biexcitons will possess some specific feature which they do not possess at $t = 0$ but photons or/and excitons do. As will be shown, the squeezing proves to be one among such specific features. To study the squeezing property of biexcitons we need to define their quadrature operators as

$$P = -\frac{i}{2}(b - b^+) \text{ and } X = \frac{1}{2}(b + b^+) . \tag{28}$$

It is well-known that the squeezed state arises when the normally-ordered variance $\langle N(\dots)\rangle$ (the average $\langle\dots\rangle$ is understood usually as in the quantum sense) of fluctuation of either quadrature operator becomes negative. We therefore have to follow the dependence on time of $\langle N\left\{[\Delta P(t)]^2\right\}\rangle$ and $\langle N\left\{[\Delta X(t)]^2\right\}\rangle$ in order to check whether they might be negative during their evolution. With the aid of Eqs. (28, 23–24) we can arrive at

$$\left.\begin{array}{l}\langle N\{[\Delta P(t)]^2\}\rangle\\ \langle N\{[\Delta X(t)]^2\}\rangle\end{array}\right\} = \mp\frac{1}{2}\left\{\mathrm{Re}\left[\alpha^2(t)\langle[\Delta c(0)]^2\rangle\right.\right.$$
$$+\beta^2(t)\langle[\Delta a(0)]^2\rangle + \gamma^2(t)\langle[\Delta b(0)]^2\rangle]$$
$$\mp|\alpha(t)|^2\left[\langle c^+(0)c(0)\rangle - \langle c^+(0)\rangle\langle c(0)\rangle\right]$$
$$\mp|\beta(t)|^2\left[\langle a^+(0)a(0)\rangle - \langle a^+(0)\rangle\langle a(0)\rangle\right]$$
$$\mp|\gamma(t)|^2\left[\langle b^+(0)b(0)\rangle - \langle b^+(0)\rangle\langle b(0)\rangle\right]\} \tag{29}$$

where $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are determined by Eqs. (25–27).

5. Possible Squeezing of Biexcitons

The relation (29) derived in the previous section are very helpful to analyze the change in time of the biexciton behaviour for a wide choice of initial conditions. As for photons and excitons we anticipate that they may be at $t = 0$ in a Fock, a coherent or a squeezed state. The possibility of the appearance of squeezed excitons has been suggested in Ref. 17. Initial biexcitons, however, are assumed to be in either Fock or coherent states only. Hence, there will be in total $3 \times 3 \times 2 = 18$ possible combinatons of initial states of the three kinds of quasiparticles. Respectively labelled by fff, cff ,scf,... the situations when at $t = 0$ all the three kinds of quasiparticles are in Fock states, photons are coherent while excitons and biexcitons are Fock, photons are squeezed whereas excitons are coherent and biexcitons are in Fock states,.... Carrying out then the averages in the rhs of the relations (29) over the corresponding states at $t = 0$, we come to the final results (LHS stand for the lhs of Eq. (29)):

For the fff-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\alpha(t)^2 n_c + |\beta(t)|^2 n_a + |\gamma(t)|^2 n_b\right\}\ . \tag{30}$$

For the fcf-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\alpha(t)|^2 n_c + |\gamma(t)|^2 n_b\right\}\ . \tag{31}$$

For the cff-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\beta(t)|^2 n_a + |\gamma(t)|^2 n_b\right\}\ . \tag{32}$$

For the ccf-case:

$$\mathrm{LHS} = \frac{1}{2}|\gamma(t)|^2 n_b\ . \tag{33}$$

For the ffc-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\alpha(t)|^2 n_c + |\beta(t)|^2 n_a\right\}\ . \tag{34}$$

For the fcc-case:

$$\mathrm{LHS} = \frac{1}{2}|\alpha(t)|^2 n_c \ . \tag{35}$$

For the cfc-case:

$$\mathrm{LHS} = \frac{1}{2}|\beta(t)|^2 n_a \ . \tag{36}$$

For the ccc-case:

$$\mathrm{LHS} = 0 \ . \tag{37}$$

For the fsf-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\alpha(t)|^2 n_c + |\gamma(t)|^2 n_b + S_a^{(\pm)}(t)\right\} \ . \tag{38}$$

For the sff-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\beta(t)|^2 n_a + |\gamma(t)|^2 n_b + S_c^{(\pm)}(t)\right\} \ . \tag{39}$$

For the csf-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\gamma(t)|^2 n_b + S_a^{(\pm)}(t)\right\} \ . \tag{40}$$

For the scf-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\gamma(t)|^2 n_b + S_c^{(\pm)}(t)\right\} \ . \tag{41}$$

For the ssf-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\gamma(t)|^2 n_b + S_a^{(\pm)}(t) + S_c^{(\pm)}(t)\right\} \ . \tag{42}$$

For the fsc-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\alpha(t)|^2 n_c + S_a^{(\pm)}(t)\right\} \ . \tag{43}$$

For the sfc-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{|\beta(t)|^2 n_a + S_c^{(\pm)}(t)\right\} \ . \tag{44}$$

For the csc-case:

$$\mathrm{LHS} = \frac{1}{2}S_a^{(\pm)}(t) \ . \tag{45}$$

For the scc-case:

$$\mathrm{LHS} = \frac{1}{2}S_c^{(\pm)}(t) \ . \tag{46}$$

For the ssc-case:

$$\mathrm{LHS} = \frac{1}{2}\left\{S_a^{(\pm)}(t) + S_c^{(\pm)}(t)\right\} \ . \tag{47}$$

In Eqs. (30–47) n_a, n_b and n_c denote respectively the number of excitons, biexcitons and photons in Fock states while $S_a^{(\pm)}(t)$ and $S_c^{(\pm)}(t)$ are of the forms:

$$s_a^{(\pm)}(t) = |\beta(t)|^2 m_a \pm \text{Re}\left[\beta^2(t)e^{i\theta_a}\sqrt{m_a(m_a+1)}\right] \quad (48)$$

$$s_c^{(\pm)}(t) = |\alpha(t)|^2 m_c \pm \text{Re}\left[\alpha^2(t)e^{i\theta_c}\sqrt{m_c(m_c+1)}\right] \quad (49)$$

where $m_a = \sinh^2 r_a$ ($m_c = \sinh^2 r_c$). r_a (r_c) together with $\theta_a(\theta_c)$ are the two real quantities that characterize the squeezing degree (see, e.g., Ref. 2) of the initial exciton (photon). As can be seen from Eqs. (30–37), LHS can never be negative, i.e. no squeezing of biexcitons arises, if neither photons nor excitons are initially squeezed. On the other hand, if the initial photon or/and exciton are in squeezed states, then one can expect for the biexciton to be squeezed also during their time evolution disregarding the initial state of biexcitons is Fock or coherent (see Eqs. (38–47)). It is so because the expressions $S_a^{(\pm)}(t)$, $S_c^{(\pm)}(t)$ given by Eqs. (48–49) can during their time variation become much negative and in cases absolute value of the negative magnitude of $S_{a,c}^{(\pm)}(t)$ predominates the positive part proportional to n_a or/and n_b or/and n_c (see Eqs. (38–44)), squeezing of biexcitons comes into being. Such adjustments naturally also mean that the most favourable for the squeezing of biexcitons is the situation under which at $t = 0$ both the photon and the exciton are squeezed, while the biexciton must be in a coherent state, i.e. the situation corresponding to the ssc-case (see Eq. (47)).

To investigate in more details the roles the pump laser intensity and detuning play in the squeezing process of biexcitons, it is necessary to express the functions $S_{a,c}^{(\pm)}(t)$ in more evidence of their variables. Using Eqs. (48, 49) for the case of $\theta_a = \theta_c = 0$ and $m_a = m_c = m$ we can get the following algebraico-trigonometrical formulae:

$$\begin{aligned}\frac{1}{2}\left[S_a^{(\pm)}(t) + S_c^{(\pm)}(t)\right] &= 2w_1^2 w_2^2 \sin^2\left[\frac{1}{2}(U_2 - U_1)t\right] \\ &\times \left\{m \mp \sqrt{m(m+1)}\cos\left[(U_2+U_1)t\right]\right\} \\ &+2w_1^2 w_3^2 \sin^2\left[\frac{1}{2}(U_3 - U_1)t\right]\left\{m \mp \sqrt{m(m+1)}\cos[(U_3+U_1)t]\right\} \\ &+2w_2^2 w_3^2 \sin^2\left[\frac{1}{2}(U_3 - U_2)t\right]\left\{m \mp \sqrt{m(m+1)}\cos[(U_3+U_2)t]\right\} . \quad (50)\end{aligned}$$

Quite interestingly to remark that among the transformation functions u_ν, v_ν and w_ν defined by Eqs. (13–15) only w_ν enter Eqs. (50). Returning to Eqs. (13–15) we observe that w_ν are proportional to the laser amplitude A (see Eq. (15)) whereas both u_ν and v_ν remain infinte even when the laser field vanishes, i.e, $I = A^2 \to 0$ (see Eqs. (13, 14)). That means that a laser "switch-off" causes the vanishing of w_ν and, hence, also of $S_{a,c}^{(\pm)}(t)$ resulting in the impossibility for biexcitons to be squeezed. Physically, such a situation corresponds to the coexistence of two

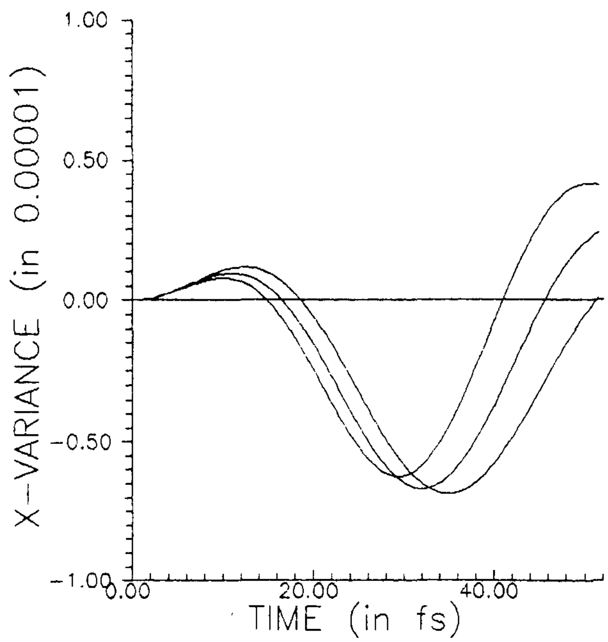

Fig. 1. The time-dependent normally-ordered variance of fluctuation of the biexciton quadrature operator X in the ssc-case which is graphed for $m = 0.01$, $\theta = 0$, $I = 10\ \mathrm{MW/cm^2}$ and $\delta = \lambda = 10$, 5.0 and 0.0 meV corresponding to decreasing antipeaks.

independent subsystem: one is the photon-exciton subsystem and the other is the biexciton subsystem (see Eq. (1) for $A = 0$). Once subsystems are independent, nothing can be transferred between them, not speaking already about squeezing. It is the pump field that connects the two subsystems. Thus the role of the pump field here is very important. We see further that if we put in Eqs. (50) $m = 0$ they will immediately vanish. Combining the argumentations so far we arrive at a conclusion that the necessary conditions for the biexciton to become squeezed are $A \neq 0$ and $m \neq 0$, i.e. the presence of a pump field and the nonzero squeezing degree of the initial photon (or/and exciton) are required.

In the rest of this section we wish to graphically illustrate the time-dependence of the normally-ordered variance of fluctuation of the biexciton quadrature operator X, $\langle N\left\{[\Delta X(t)^2\right\}\rangle$, in the most favourable ssc-case (hereinafter called "X-variance" for brevity). For numerical evaluations we take laser detuning δ, intensity I and initial squeezing degree m as controllable parameters, while for the material sample we choose CuCl which has ε (ground state dielectric constant) = 5, $E = 3204$ meV, a_x (exciton Bohr radius) = 3×10^{-7} cm, m_c/m_0 (m_c and m_0 the effective mass of an electron in the crystal and the mass of a free electron) = 0.4, m_h/m_0 (m_h the effecive mass of a hole in the crystal) = 4.2, E_x^b (exciton binding energy) = 200 meV and ε = 6372 meV. With such material values we are able to estimate g

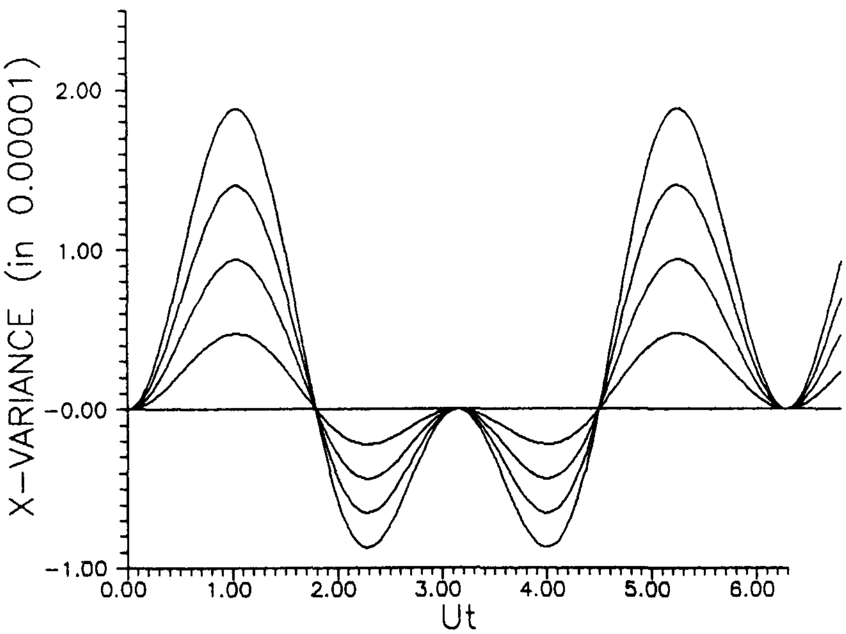

Fig. 2. The normally-ordered variance of fluctuation of the biexciton quadrature operator X in the ssc-case as functions of Ut with $U = (g^2 + f^2 I)^{1/2}$. The fixed parameters are $m = 0.05$ and $\theta = \lambda = \delta = 0$. The pump laser intensity has the values $I = 5.0$, 10, 15 and 20 MW/cm^2 corresponding to decreasing (increasing) antipeaks (peaks).

and f in Eq. (1) following their formulae given respectively in Refs. 19 and 21. Namely, we have got $g = 60$ meV and $f = 9 \times 10^{-5}$ eV^{-1}. In Fig. 1 X-variance is graphed against time for $m = 0.01$, $\theta = \theta_a = \theta_c = 0$, $I = 10$ MW/cm^2 and several detunings $\delta(= \lambda) = 0.0$, 5.0 and 10 meV. Visually, the biexciton becomes squeezed after some time (when X-variance is negative) and this behaviour is kept for some period of time. The achievable squeezing degree of the biexciton increases when the laser frequency approaching the perfect resonance of the exciton-to-biexciton transition, i.e. when $\delta \to 0$. Figure 2 represents X-variance as functions of Ut, where $U = (g^2 + f^2 I)$, for $m = 0.05$, $\theta = \lambda = \delta = 0$ and various pump laser intensities $I = 5.0$, 10, 15 and 20 MW/cm^2. As seen from Fig. 2, the appearance of squeezed biexcitons is clearly possible and the stronger the pump the larger the achievable degree of squeezing, i.e. the deeper the antipeaks. On the other hand, however, if the pump field is too strong, it may damage the sample or bring about a variety of high excitation effects that cannot be treated properly up to now. Thus, an optimal pump field must be applied. Finally, we demonstrate in Fig. 3 how the squeezing degree of the biexciton depends on that of the initial photons and excitons under the same physical situation as in Figs. 1 and 2, i.e. in the ssc-case. Due to the algebraico-trigonometrical nature of Eqs. (50) this dependence displays itself in

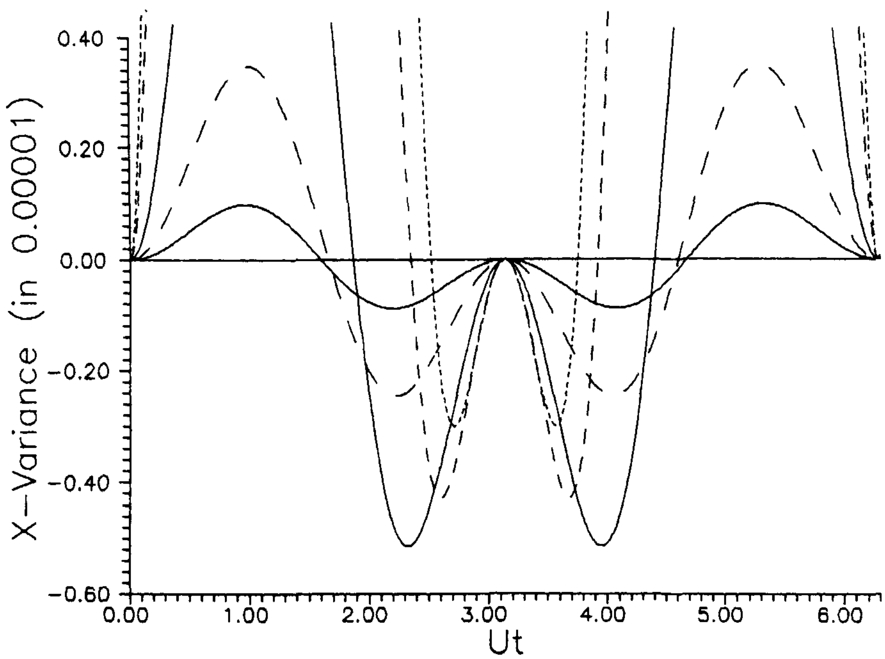

Fig. 3. The same as in Fig. 2, but I is fixed to be 10 MW/cm^2 while m equals to 0.001 (solid curve with the most shallow antipeaks), 0.01 (long-dashed curve, 0.1 (solid curve with the deepest antipeaks), 1 (medium-dashed curve) and 2 (short-dashed curve).

a quite delicate manner. To follow the alteration of the X-variance on m, we fix the detuning and the intensity to be $\delta = \lambda = 0$ and $I = 10$ MW/cm^2. For simplicity we again put as before $\theta = 0$. For some values of m which we had chosen for plotting, we have noticed the following behaviour. For m growing, say, from 0.001 to 0.1 the biexciton squeezing degree seems to increase (see the solid curves and the long-dashed curve in Fig. 3: the antipeaks become deeper and deeper). Nevertheless, if m increase further, say, from 0.1 to 2, the biexciton squeezing turns out to decrease (see in Fig. 3 the solid curve with the deepest antipeak and the medium-dashed and short-dashed curves: the antipeaks become more and more shallow).

6. Conclusions

In conclusion, we have given a try, as pointed out in Ref. 2, to expand the concept of squeezing to quasiparticles in condensed matter physics. Following the concept of squeezed excitons in semiconductors, which has been introduced in Ref. 17, we are here concerned with a possible appearance of squeezed biexcitons in excited semiconductors modelled as a photon-exciton system. In our treatment there is needed an external optical pump in the resonance region of the exciton-to-biexciton transition (the spectroscopical domain of the M-zone). We have shown that biexcitons might be squeezed when and only when m_a (or/and m_c) and A simultaneously differ from zero. In other words, it is necessary to resonantly pump the system as well as to prepare it beforehand such that it should possess some initial degree of squeezing. Summing up the results (see Sec. 5), it should be stressed that for obtaining best squeezing of biexcitons one has to suitably adjust all the controllable parameters. Since the squeezed light has had wide perspectives in practice, it is hoped that the squeezed quasiparticles in condensed matter could also find their applications in different branches of both science and technology. Nevertheless, before that, the validity of the introduced concepts must be seriously criticized and the existence of squeezed quasiparticles must be experimentally confirmed.

Finally, it is worth noticing that while practical applications of squeezed states in optics are already well recognized[31–33] possible utilizations of those states have just begun in condensed matter[34,35] as well as in quantum field theory.[36]

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