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SQUEEZED EXCITONS IN SEMICONDUCTORS

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The concept is introduced of squeezed excitons that appear during the process of their interaction with squeezed photons. Situations are considered where the excitons initially are in the Fock or coherent states. For the excitons to become squeezed it is more favourable to prepare them beforehand in a coherent state than in a Fock one. The squeezing degree of excitons depends on that of the initial photons and on detuning.

It is perhaps the interaction between bodies that makes the universe so beautifully simple and at the same time so awfully complicated from both macro- and micro-points of view. Here we are concerned with the interaction between photons and excitons in semiconductors that not only, as is well-known, reconstructs the eigenenergy spectrum of the whole system[1,2] or causes nutation process[3] but also, as will be shown, leads to possible exchange of squeezing property between photons and excitons.[4]

Nonclassical squeezed states of light were proposed for the first time in quantum optics in 1970[5] and since then have stimulated a lot of attention (see Refs. 6 and 7 and the references cited therein) because they promise prospective applications in reducing light natural noise[8–10] and in detecting gravity waves.[9,10] Their existence also predicts similar conceptions in other-than-optics fields.[11] In this letter we try to introduce the concept of squeezed excitons in semiconductors which are expected to bring benefits by raising physical measurement accuracies in condensed matter. It is not excluded that there may exist different mechanisms of generating squeezed excitons. But here we consider only the simplest one which is a consequence of light-matter interaction.

We start from a two-mode Hamiltonian[12] (Planck's constant and velocity of light are put equal to unity everywhere)

$$H = Ea^+a + \omega c^+c + g(a^+c + c^+a) , \tag{1}$$

where operators a, a^+ (c, c^+) are specified as those of excitons (photons) with energy E (frequency ω) and g is the coupling constant which is assumed real for simplicity. Using Bogolubov transformations we can exactly diagonalize (1)

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into[1,2,13]

$$H = \sum_{\nu} \Omega_\nu \alpha_\nu^+ \alpha_\nu ; \quad \nu = 1, 2 , \tag{2}$$

with

$$\Omega_\nu = \frac{1}{2}\{E + \omega - (-1)^\nu[(E-\omega)^2 + 4g^2]^{1/2}\} , \tag{3}$$

$$\alpha_\nu = u_\nu c + v_\nu a ; \quad \alpha_\nu^+ = u_\nu c^+ + v_\nu a^+ , \tag{4}$$

$$u_\nu = [1 + g^2/(\Omega_\nu - E)^2]^{-1/2} ; \quad v_\nu = g u_\nu / |\Omega_\nu - E| , \tag{5}$$

and

$$a = \sum_\nu v_\nu \alpha_\nu ; \quad a^+ = \sum_\nu v_\nu \alpha_\nu^+ , \tag{6}$$

$$c = \sum_\nu u_\nu \alpha_\nu ; \quad c^+ = \sum_\nu u_\nu \alpha_\nu^+ . \tag{7}$$

From (2) immediately follow the time-dependences of operators α_ν and α_ν^+:

$$\alpha_\nu(t) = \alpha_\nu(0)\exp(-i\Omega_\nu t) ; \quad \alpha_\nu^+(t) = \alpha_\nu^+(0)\exp(+i\Omega_\nu t) . \tag{8}$$

Putting first (4) into the right-hand side of (8) and then (8) into the right-hand side of (6, 7) yields

$$a(t) = \sum_\nu v_\nu[u_\nu c(0) + v_\nu a(0)]\exp(-i\Omega_\nu t) , \tag{9}$$

$$c(t) = \sum_\nu u_\nu[u_\nu c(0) + v_\nu a(0)]\exp(-i\Omega_\nu t) , \tag{10}$$

and similar expressions for $a^+(t)$ and $c^+(t)$.

Now anticipating that the photon is initially squeezed, i.e., either $\langle(\Delta X_c(0))^2\rangle \equiv \langle(\Delta(c^+(0) + c(0)))^2\rangle/4$ or $\langle(\Delta P_c(0))^2\rangle \equiv \langle(\Delta(c^+(0) - c(0)))^2\rangle/4$ is less than[5–7] 0.25 ($\langle \ldots \rangle$ denotes the mean value of $(\ldots)$ over the eigenstate of H), we can, with the help of (9, 10) and of the similar ones for a^+, c^+, calculate the exciton quadratures $\langle(\Delta X_a(t))^2\rangle$ and $\langle(\Delta P_a(t))^2\rangle$. If the exciton at $t = 0$ is in a coherent state,[14] we get

$$\left.\begin{matrix}\langle(\Delta X_a(t))^2\rangle \\ \langle(\Delta P_a(t))^2\rangle\end{matrix}\right\} = \frac{1}{4} + \sin^2\left(\frac{1}{2}\delta t\right)\cdot\left\{\begin{matrix}X(t) \\ P(t)\end{matrix}\right. , \tag{11}$$

where

$$\left.\begin{array}{c} X(t) \\ P(t) \end{array}\right\} = \frac{\beta}{2} m\left[1 \pm \left(1 + \frac{1}{m}\right)^{1/2} \cos(St - y)\right], \tag{12}$$

$\delta = \Omega_1 - \Omega_2$, $\beta = 4g^2/(d^2 + 4g^2)$, $d = E - \omega$, $S = \Omega_1 + \Omega_2$, $m = sh^2 r$, r and y are two real numbers which together with two others (see Refs. 5–9) characterize a given state of the initial squeezed photon. For the case where the exciton at $t = 0$ is in a Fock state with a certain number n of excitons, we have

$$\left.\begin{array}{c} \langle(\Delta X_a(t))^2\rangle \\ \langle(\Delta P_a(t))^2\rangle \end{array}\right\} = \frac{1}{4} + \frac{n}{2}\left[\cos^2\left(\frac{1}{2}\delta t\right) + \gamma \cdot \sin^2\left(\frac{1}{2}\delta t\right)\right] + \sin^2\left(\frac{1}{2}\delta t\right) \cdot \left\{\begin{array}{c} X(t) \\ P(t) \end{array}\right. \tag{13}$$

with $\gamma = d^2/(d^2 + 4g^2)$. From (11–13) we arrive at the following conclusions.

1) If the photons are non-squeezed, i.e. $m = 0$, the functions $X(t)$ and $P(t)$ vanish (see (12)), and hence the exciton quadratures can never be less than 0.25 (see (11) and (13)), indicating the impossibility of generation of squeezed excitons. On the other hand, for $m \neq 0$ there always exist time intervals inside which either $X(t)/\beta$ or $P(t)/\beta$ (suppose $\beta \neq 0$) becomes negative (it is so because the amplitude of the cosine function in (12), $(1 + 1/m)^{1/2}$, is always greater than unity for $m \neq 0$ (see (12)).

2) If there are no photon-exciton interaction, i.e., $g = 0$, then $\beta = 0$ (and $\gamma = 1$) yielding both $X(t)$ and $P(t)$ equal to zero irrespective of whether m differs from zero or not. Thus, combining 1) and 2) we emphasize that the simultaneous requirements $m \neq 0$ and $g \neq 0$ are the necessary and sufficient conditions for the exciton to be squeezed.

3) Because of the presence of the additional term proportional to n (n positive integer) in (13), the inequality of $\langle(\Delta X_a(t))^2\rangle$ (or $\langle(\Delta P_a(t))^2\rangle < 0.25$ is easier to fulfill in (11) than in (13). That means that preparing the initial exciton in a coherent state favours the formation of squeezed excitons.

4) The larger m the stronger is the obtainable exciton squeezing, but, instead, the shorter is the time duration during which the exciton spends in the squeezed state (see Fig. 1). For the limit of large m, e.g. $m \gg 1$, we can write $(1 + 1/m)^{1/2} \approx 1 + 1/(2m)$ and thus analysing (12) leads to the fact that exciton squeezing can reach the maximally possible degree, i.e. $\langle(\Delta X_a(t))^2\rangle$ (or $\langle(\Delta P_a(t))^2\rangle) \approx 0$, in both the coherent and Fock initial exciton cases at times $t = \pi(1 + 4k)/\delta$; $k = 1, 2, 3, \ldots$, if we arrange $y = \pi(S(1 + 4k)/\delta - 1 - 2k')$ (or $y = \pi(S(1 + 4k)/\delta - 2k')$), $k, k' = 1, 2, 3, \ldots$, and tune the photon frequency so that $d = 0$.

5) For a fixed value of m, the larger the detuning $D = |d|$ the weaker is the obtainable exciton squeezing and the time duration during which the exciton spends in the squeezed state is independent of D (see Fig. 2). Mathematically,

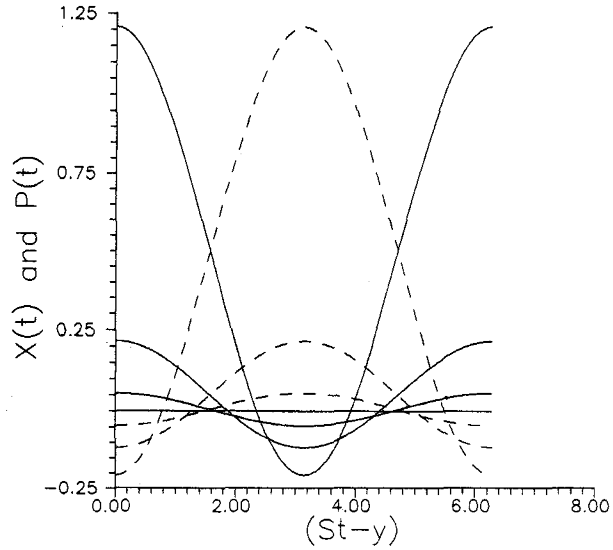

Fig. 1. Functions $X(t)$ (solid lines) and $P(t)$ (dashed lines) versus $(St - y)$ for zero detuning $D = 0$ and different m ($m = 0.01$, 0.1 and 1 corresponding to increasing (decreasing) central peaks' (antipeaks') height (depth) of the curves $P(t)$ ($X(t)$)). The straightforward line $X(t)$, $P(t) = 0$ represents the case of $m = 0$. Negative parts of these functions lying under the line $X(t)$, $P(t) = 0$ become deeper and deeper, i.e., exciton squeezing becomes stronger and stronger, for increasing m.

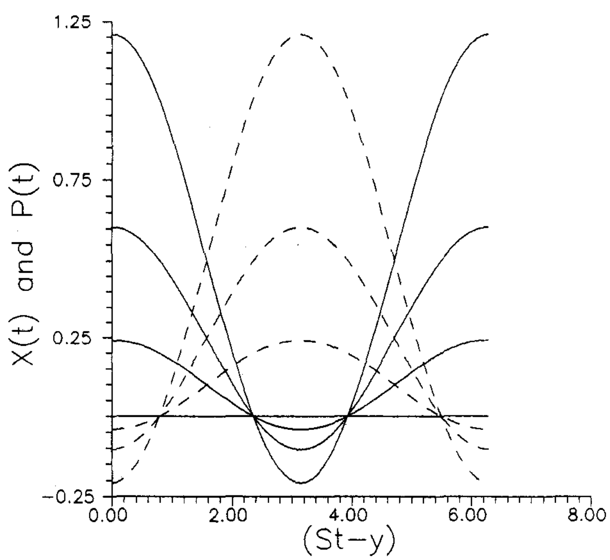

Fig. 2. The same as in Fig. 1 but for a fixed value of $m = 1$ and different detunings D ($D = 200$, 100 and 0 meV corresponding to increasing (decreasing) central peaks' (antipeaks') height (depth) of the curves $P(t)$ ($X(t)$)). The straightforward line $X(t)$, $P(t) = 0$ represents the case of $D = \infty$. Negative parts of these functions lying under the line $X(t)$, $P(t) = 0$ become deeper and deeper, i.e., exciton squeezing becomes stronger and stronger, for decreasing D. Constant g is taken equal to 50 meV which is suitable for CdS or CuCl.

when $D \to \infty$ ($\beta \to 0$), i.e., the detuning is very large, no squeezing transfer can occur between excitons and photons because both $X(t)$ and $P(t)$ then tend to zero.

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