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# SEMICONDUCTOR EXCITON OPTICAL BISTABILITY AS INFLUENCED BY FREE CARRIERS

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Exciton density in an optically excited semiconductor is calculated as a function of pumping light intensity. In a certain domain in the detuning-intensity plane there may appear bistability of exciton density which is shown considerably influenced by the presence of free carriers. The carrier-induced effect manifests itself in the modification of phase diagrams leading to changes of the hysteresis loop size in both intensity-controlled and frequency-controlled bistabilities.

## 1. Introduction

Optical bistability (OB) as a typical example of a first-order nonequilibrium phase transition in physical systems far away from thermal equilibrium was first theoretically predicted by Szoke *et al.*[1] In the last ten years the phenomenon of OB has gained great attention, both theoretically and experimentally, mostly because of its practical potential for all-optical data processing devices. An optical bistable system is one which can exhibit two steady output states for the same input parameters. Optical nonlinearity and feedback form the basis of OB. In a highly excited semiconductor there may coexist different kinds of quasi-particles such as excitons, biexcitons, trions, free carriers, etc. The interactions among the quasiparticles play important roles in generating various nonlinear collective processes in the semiconductor. Several mechanisms have been proposed for optical nonlinearities. As for the feedback, it can also be provided by different origins. In Refs. 2–5 the feedback was due to the configuration of a crystal platelet resonator of width of order of $1\mu m$. The reflectivity of the crystal surfaces determines the possibility of OB which

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should disappear at all if the crystal width tends to infinity as the situation in a bulky material. The authors of Refs. 6 and 7 have shown that the local field effect can give rise to a feedback in bulky samples. In Ref. 8 we have suggested a new mechanism of OB in bulky semiconductors by taking into account the intracrystal photon energy renormalization caused by the interaction of the photon with the valence electron, and in Ref. 9 the mechanism we suggested has been applied to overcome some difficulties connected with the so-called "double" and "strange" poles appearing in the expression for the complex dielectric function. In all those works cited above however, the authors have only considered a semiconductor as a system consisting only of excitons/biexcitons and intramedium photons which are driven by an external monochromatic light field. In fact, such models are not complete. In reality, it is known that a semiconductor may also contain electrons, holes (free or bound) and/or other kinds of elementary excitations. A number of works have pointed out the role of the free carriers in many physical processes. References 10 and 11 considered the influence of free electrons on the absorption and the luminescence of excitons. In Ref. 12 the exciton–electron and exciton–hole interactions were taken into account to study the phase transition for the excitonic excitations in semiconductors. Especially in Refs. 13 and 14 attention has been paid to the influence of free carriers on the turbulence and chaotic behaviour. Further, a linear response theory for a highly excited exciton–electron–hole system in the athermal stage of evolution has just recently been developed.[15]

The present paper performs a theoretical treatment of possible bistable behaviour of the exciton density in a direct-gap semiconductor subjected to a driving laser field with emphasis paid to the influence of free carriers present simultaneously in the semiconductor. Our results show that such an influence might be significant and making use of it could enable one to operate a bistable device more effectively by preparing the sample with a necessary concentration of free carriers in advance.

We shall utilize the system of units with $\hbar = c = V = 1$, where $\hbar$, $c$, and $V$ are the Planck constant, velocity of light in the vacuum and volume of the sample respectively.

## 2. Light-Driven Equation for Excitons in the Presence of Free Carriers

Let us consider an excited semiconductor modelled as a system of interacting excitons and free carriers which is placed under the action of an externally driving laser field with given wave-vector $\mathbf{k}$, complex amplitudes $E_{\mathbf{k}}^{(\pm)}$ and frequency $\Omega_k$. The laser frequency is assumed to be in resonance with the exciton energy so that the excitons are generated directly by the laser. The carriers are anticipated to be already present in the semiconductor by e.g., doping or other means which we are not particularly interested in. They occupy some portion of the energy bands and interact only among themselves and with the excitons but not with the resonant laser. Thus in our theory the carrier concentration as well as the carrier–exciton interaction appear as influencing parameters. Since we shall deal with the behaviour of the exciton density versus the light field intensity, we may lay the Coulomb force among the carriers aside. The part of the total Hamiltonian of the system under consideration that we are interested in then has the form

$$H = \sum_{\mathbf{p}} \omega_x(p) a^+_{\mathbf{p}} a_{\mathbf{p}} + g_k(E^{(-)}_{\mathbf{k}} e^{-i\Omega_k t} a^+_{\mathbf{k}} + \text{h.c.}) + \sum_{\mathbf{p},\mathbf{q},\mathbf{l}} \left\{ \frac{1}{2}\nu_{x-x}(\mathbf{p},\mathbf{q},\mathbf{l}) a^+_{\mathbf{p}+\mathbf{l}} a^+_{\mathbf{q}-\mathbf{l}} a_{\mathbf{p}} a_{\mathbf{q}} + \nu_{x-e}(\mathbf{p},\mathbf{q},\mathbf{l}) a^+_{\mathbf{p}+\mathbf{l}} e^+_{\mathbf{q}-\mathbf{l}} e_{\mathbf{p}} a_{\mathbf{q}} + \nu_{x-h}(\mathbf{p},\mathbf{q},\mathbf{l}) a^+_{\mathbf{p}+\mathbf{l}} h^+_{\mathbf{q}-\mathbf{l}} h_{\mathbf{p}} a_{\mathbf{q}} \right\} . \tag{1}$$

In (1) $a^+_{\mathbf{p}}(a_{\mathbf{p}})$, $e^+_{\mathbf{p}}(e_{\mathbf{p}})$ and $h^+_{\mathbf{p}}(h_{\mathbf{p}})$ are boson operators for excitons and fermion operators for electrons and holes. $\omega_x(p)$ denotes the exciton energy. $g_k$ is the exciton-light coupling factor. $\nu_{x-x}(\mathbf{p},\mathbf{q},\mathbf{l})$, $\nu_{x-e}(\mathbf{p},\mathbf{q},\mathbf{l})$ and $\nu_{x-h}(\mathbf{p},\mathbf{q},\mathbf{l})$ are the exciton–exciton, exciton–electron and exciton–hole interaction potentials respectively. While the exciton–exciton interaction has a dipole–dipole-like character, an electron (a hole) interacts with an exciton like a charge with a dipole. The explicit expressions of $\nu_{x-x}$, $\nu_{x-e}$ and $\nu_{x-h}$ can be found e.g., in Refs. 12, 16 and 17.

The Heisenberg equation of motion for the $\mathbf{k}$-mode exciton operator can be derived from the Hamiltonian (1) as

$$i\frac{\partial}{dt} a_{\mathbf{k}} = (\omega_x(k) - i\gamma_k) a_{\mathbf{k}} + g_k E^{(-)}_{\mathbf{k}} e^{-i\Omega_k t} + \sum_{\mathbf{p},\mathbf{q}} \left\{ \frac{1}{2}\nu_{x-x}(\mathbf{p},\mathbf{q},\mathbf{k}-\mathbf{p}) a^+_{\mathbf{p}+\mathbf{q}-\mathbf{k}} a_{\mathbf{q}} a_{\mathbf{p}} + \nu_{x-e}(\mathbf{p},\mathbf{q},\mathbf{k}-\mathbf{p}) e^+_{\mathbf{p}+\mathbf{q}-\mathbf{k}} e_{\mathbf{q}} a_{\mathbf{p}} + \nu_{x-h}(\mathbf{p},\mathbf{q},\mathbf{k}-\mathbf{p}) h^+_{\mathbf{p}+\mathbf{q}-\mathbf{k}} h_{\mathbf{q}} a_{\mathbf{p}} \right\} \tag{2}$$

where $\gamma_k$ is the phenomenologically introduced damping of the $\mathbf{k}$-mode exciton. In order to consider the dependence of the exciton density on the external laser intensity, we have to take the average of Eq. (2) over the state of the system. In the Hatree–Fock approximation, we have :

$$\begin{aligned} \sum_{\mathbf{p},\mathbf{q}} \nu_{x-x}(\mathbf{p},\mathbf{q},\mathbf{k}-\mathbf{p}) \langle a^+_{\mathbf{p}+\mathbf{q}-\mathbf{k}} a_{\mathbf{q}} a_{\mathbf{p}} \rangle &\approx \sum_{\mathbf{p},\mathbf{q}} \nu_{x-x}(\mathbf{p},\mathbf{q},\mathbf{k}-\mathbf{p}) \langle a^+_{\mathbf{p}+\mathbf{q}-\mathbf{k}} a_{\mathbf{q}} \rangle \langle a_{\mathbf{p}} \rangle + \sum_{\mathbf{p},\mathbf{q}} \nu_{x-x}(\mathbf{p},\mathbf{q},\mathbf{k}-\mathbf{p}) \langle a^+_{\mathbf{p}+\mathbf{q}-\mathbf{k}} a_{\mathbf{p}} \rangle \langle a_{\mathbf{q}} \rangle \\ &= \sum_{\mathbf{p}} \nu_{x-x}(\mathbf{p},\mathbf{q},0) N^{ex}_{\mathbf{p}} \langle a_{\mathbf{k}} \rangle + \sum_{\mathbf{q}} \nu_{x-x}(\mathbf{k},\mathbf{q},0) N^{ex}_{\mathbf{q}} \langle a_{\mathbf{k}} \rangle . \end{aligned} \tag{3}$$

Note that $\langle a_{\mathbf{k}}\rangle$ is finite because our system is in nonequilibrium: the excitons generated by the external laser constitute the coherent state $N_{\mathbf{q}}^{ex} = a_{\mathbf{q}}^{+}a_{\mathbf{q}}$. To simplify the calculation, let us approximately take $\nu_{x-x}(\mathbf{p},\,\mathbf{q},\,\mathbf{l}) \approx \nu_{x-x}(0,0,0) = \nu = 52\pi I_x a_x^3/3$ (see e.g., Ref. 12), here $I_x$ and $a_x$ are the exciton binding energy and Bohr radius respectively. Then, from (3) we get

$$\sum_{\mathbf{p},\,\mathbf{q}} \nu_{x-x}(\mathbf{p},\,\mathbf{q},\,\mathbf{k}-\mathbf{p})\langle a_{\mathbf{p}+\mathbf{q}-\mathbf{k}}^{+}a_{\mathbf{q}}a_{\mathbf{p}}\rangle \approx 2\nu\sum_{\mathbf{q}} N_{\mathbf{q}}^{ex}\langle a_{\mathbf{k}}\rangle = 2\nu n^{ex}\langle a_{\mathbf{k}}\rangle \tag{4}$$

with $n^{ex} = \sum_{\mathbf{q}} N_{\mathbf{q}}^{ex}$ being the exciton density. Similarly we also have:

$$\sum_{\mathbf{p},\,\mathbf{q}} \nu_{x-e}(\mathbf{p},\,\mathbf{q},\,\mathbf{k}-\mathbf{p})\langle e_{\mathbf{p}+\mathbf{q}-\mathbf{k}}^{+}e_{\mathbf{q}}a_{\mathbf{p}}\rangle \approx \nu_{x-e}\sum_{\mathbf{q}} N_{\mathbf{q}}^{e}\langle a_{\mathbf{k}}\rangle = \mu n^{e}\langle a_{\mathbf{k}}\rangle/2 \tag{5}$$

and

$$\sum_{\mathbf{p},\,\mathbf{q}} \nu_{x-h}(\mathbf{p},\,\mathbf{q},\,\mathbf{k}-\mathbf{p})\langle h_{\mathbf{p}+\mathbf{q}-\mathbf{k}}^{+}h_{\mathbf{q}}a_{\mathbf{p}}\rangle \approx \nu_{x-h}\sum_{\mathbf{q}} N_{\mathbf{q}}^{h}\langle a_{\mathbf{k}}\rangle = \mu n^{h}\langle a_{\mathbf{k}}\rangle/2 \tag{6}$$

where $N_{\mathbf{q}}^{e} = e_{\mathbf{q}}^{+}e_{\mathbf{q}}$, $N_{\mathbf{q}}^{h} = h_{\mathbf{q}}^{+}h_{\mathbf{q}}$ and $n^{e} = \sum_{\mathbf{q}} N_{\mathbf{q}}^{e}$, $n_{\mathbf{q}}^{h} = \sum_{\mathbf{q}} N_{\mathbf{q}}^{h}$ are the density of electrons and holes respectively. In (5) and (6) we again used the approximative formulae given in Ref. 12 for the exciton–carrier coupling constants, namely $\nu_{x-e}(\mathbf{p},\,\mathbf{q},\,\mathbf{l}) \approx \nu_{x-h}(\mathbf{p},\,\mathbf{q},\,\mathbf{l}) \approx \nu_{x-e}(0,0,0) = \nu_{x-h}(0,0,0) = \mu/2 = 24\pi I_x a_x^3$ . No essential features will be lost if we put $n^{e} = n^{h} = n$. Then Eq. (2) becomes

$$i\frac{\partial}{dt}\langle a_{\mathbf{k}}\rangle = \{\omega_x(k) + \nu n^{ex} + \mu n - i\gamma_k\}\langle a_{\mathbf{k}}\rangle + g_k E_{\mathbf{k}}^{(-)}e^{-i\Omega_k t}\ . \tag{7}$$

In the stationary regime a particular solution of Eq. (7) can be sought in the form

$$\langle a_{\mathbf{k}}\rangle = A_{\mathbf{k}}\,e^{-i\Omega_k t} \tag{8}$$

where $A_k$ needs to be determined later. Substituting Eq. (8) into Eq. (7) we obtain

$$g_k E_{\mathbf{k}}^{(-)} = \{(\nu n^{ex} + \mu n - \Delta_k) - i\gamma_k\}A_{\mathbf{k}} \tag{9}$$

in which $\Delta_k = \Omega_k - \omega_x(k)$ denotes the frequency detuning. From Eq. (9) we can easily arrive at the following equation

$$g_k^2|E_{\mathbf{k}}^{(-)}|^2 = \{(\nu n^{ex} + \mu n - \Delta_k)^2 + \gamma_k^2\}\,n^{ex}\ . \tag{10}$$

In obtaining Eq. (10) we assumed that the number of excitons with momenta different from $\mathbf{k}$ is negligibly small, i.e., we assumed $|A_{\mathbf{k}}|^2 = \langle a_{\mathbf{k}}\rangle^2 = n^{ex}$. For brevity, in what follows we will drop the index $\mathbf{k}$. In Eq. (10) the exciton damping $\gamma$ is a constant and does not depend on the exciton density as well as on the carrier concentration. In realistic conditions, $\gamma$ may be density-dependent. Experimentally, in some situations the density-dependent shift of the exciton energy level is hardly observable because of compensations of several many-body effects being simultaneously taken into account. In such cases the occurrence of OB is interpreted as caused solely by the dependence of the exciton damping on the density.[18] For the sake of simplicity we can follow Ref. 19 to treat the density-dependent exciton damping in a phenomenological way by considering the interaction couplings as complex constants, i.e., $\nu \to \nu - i\chi_1$ and $\mu \to \mu - i\chi_2$, where $\chi_1$ and $\chi_2$ are some real constants describing the said dependence of the exciton damping. With this phenomenology Eq. (10) will get the form

$$\left\{(\nu n^{ex} + \mu n - \Delta)^2 + (\gamma + \chi_1 n^{ex} + \chi_2 n)^2\right\} n^{ex} = g^2 I \tag{11}$$

where $\Gamma = \gamma + \chi_1 n^{ex} + \chi_2 n$ serves as the density-dependent damping of the exciton and $I = |E^{(-)}|^2$ labels the light intensity. Since Eq. (11) is a cubic (with respect to $n^{ex}$) equation, it might exhibit bistability. Obviously, if $\nu = 0$ and $\chi_1 = 0$, i.e., there are no exciton-exciton interactions, Eq. (11) becomes a linear one and no bistability can appear. Therefore, it is the exciton-exciton interaction that plays the decisive role in bringing about the phenomenon of exciton bistability. Physically speaking, this interaction induces an energy shift of the exciton ($\propto \nu$) and/or a broadening of the exciton level width ($\propto \chi_1$) that causes discontinuous density jumps when the light intensity is sweeping back and forth. For convenience we introduce the following normalized dimensionless quantities: light intensity $\tilde{I}$, exciton density $\tilde{n}$, carrier density $\rho$ and frequency detuning $d$ which are scaled:

$$\tilde{I} = |E^{(-)}|^2/E_s^2\ ; \quad \tilde{n} = \nu n^{ex}/\gamma\ ; \quad \rho = \mu n/\gamma\ ;$$
$$d = \Delta/\gamma\ ; \quad E_S^2 = \gamma^3/(\nu g^2)\ . \tag{12}$$

Equation (11 ) now becomes quite simple:

$$\left\{(\tilde{n} + \rho - d)^2 + (1 + \xi_1 \tilde{n} + \xi_2 \rho)^2\right\} \tilde{n} = \tilde{I} \tag{13}$$

where $\xi_1 = \chi_1/\nu$ and $\xi_2 = \chi_2/\mu$. Note that the role of the carriers is played though $\rho$ that enters our basic equation (13) as a controllable parameter. As seen from (13), such carriers result in an additional shift of the exciton energy ($\propto \rho$ in the first parentheses of Eq. (13)) and also in an additional broadening of the exciton level width ($\propto \xi_2\rho$ in the second parentheses of Eq. (13)). In the next section, we shall see in more detail how the presence of the carriers modifies the characteristics of the exciton bistability.

## 3. Bistable Exciton Density

In this section we shall concern ourselves with the carrier-influenced bistable behaviour of the exciton density as a function of light intensity and frequency detuning. Though Eq. (13) is a cubic equation with respect to $\tilde{n}$, bistable behaviour could occur only under certain conditions. It is easy to find from Eq. (13) that the conditions for $\tilde{n}$ to be a tri-valued function of $\tilde{I}$ are

$$\xi_1 < 1/\sqrt{3} \quad \text{and} \quad d > d_c = \rho + (1+\xi_2\rho)(\xi_1+\sqrt{3})/(1-\xi_1\sqrt{3})\ . \tag{14}$$

Clearly, if the carrier density is vanished, that is $\rho = 0$, the conditions (14) are completely coincided with those in Ref. 19 where no carriers are present. When the conditions (14) are met, the function $\tilde{I} = \tilde{I}(\tilde{n})$ has two extrema at $\tilde{n} = \tilde{n}_\pm$ ($\tilde{n}_- < \tilde{n}_+$) determined by

$$\tilde{n}_\pm = \frac{1}{3}\frac{1}{1+\xi_1^2}\Big\{2((d-\rho)-\xi_1(1+\xi_2\rho)) \pm \Big(((d-\rho)-\xi_1(1+\xi_2\rho))^2 - 3((1+\xi_2\rho)+(d-\rho)\xi_1)^2\Big)^{\frac{1}{2}}\Big\}\ . \tag{15}$$

The values of the light intensity corresponding to $\tilde{n}_\pm$ are labelled by $\tilde{I}_\mp$ ($\tilde{I}_+ < \tilde{I}_-$), i.e.,

$$\tilde{I}_\pm = \tilde{I}(\tilde{n}_\mp)\ . \tag{16}$$

In the interval of light intensity $\tilde{I} \in [\tilde{I}_+, \tilde{I}_-]$, one value of $\tilde{I}$ answers to three values of exciton density $\tilde{n}$ (see Figs. 2 and 3) among them one is unstable.[20] The phase diagrams sketched in Fig. 1 indicate the regions of frequency detuning and intensity in which bistable behaviour of $\tilde{n}$ takes place. For the sake of simplicity, from now on, we choose $\xi_1 = \xi_2 = \xi$, and carry out preliminary consideration of the free carrier-influenced bistability of exciton density. We can see from Fig. 1 that, with fixed values of d and $\xi$, when the carrier density ($\sim \rho$) increases the critical intensity

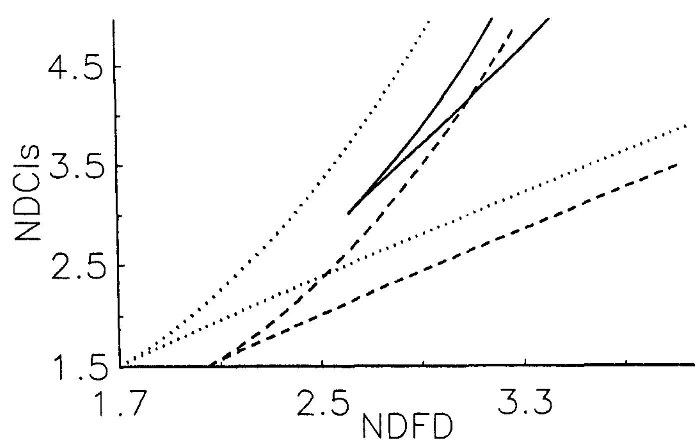


Fig. 1. The phase transition diagrams. The normalized dimensionless critical intensities (NDCIs), $\tilde{I}_\pm$, versus the normalized dimensionless detuning (NDFD), d, for $\rho = 0$ and $\xi_1 = \xi_2 = 0$ (dotted curves), $\rho = 0.35$ and $\xi_1 = \xi_2 = 0$ (dashed curves) and $\rho = 0.35$ and $\xi_1 = \xi_2 = 0.1$ (solid curves). The regions in between a pair of curves correspond to bistability phases.

value $\tilde{I}_+$ from which the density bistable behaviour begins to occur will be decreased. This fact provides a possibility to observe bistability at lower values of light intensity at a given frequency detuning by increasing the carrier density in semiconductors. On the other hand, with the same values of $\rho$ and d, the critical intensity $\tilde{I}_+$ will increase when $\xi$ increases.

Figures 2 and 3 represent the dependence of exciton density ($\sim \tilde{n}$) on light intensity ($\sim \tilde{I}$) with $d = 3$. When $\xi = 0$, and $\rho$ changes, the dependence is shown in Fig. 2, and when $\xi$ changes with $\rho$ fixed to be 0.35, it is shown in Fig. 3. For fixed d and $\xi$, Fig. 2 shows that both critical intensities $\tilde{I}_\pm$ and hysteresis loop size ($\propto (\tilde{I}_- - \tilde{I}_+)$) of the density-intensity characteristics decrease when the carrier density increases. It is known that when the hysteresis loop size decreases one can use driving pulses with lower height to change states of the memory element.[21] However, if the carrier density becomes large (for example when $d = 3$ and $\rho = 0.7$, see the dashed curve in Fig. 2) the hysteresis loop size will be very small. If $\rho$ increases further bistable behaviour may disappear at all. So, one should properly control the carrier density to reach necessary behaviour of the characteristic curves. On the other side, Fig. 3 shows that with fixed values of $\rho$ and d (for example $\rho = 0.35$ and $d = 3$) and varying $\xi$, bistable behaviour also changes noticeably but the manner of change differs from the previous case. Namely, if the nonlinearity of exciton damping increases (i.e., $\xi$ increases), the critical intensities $\tilde{I}_\pm$ will also increase, although the hysteresis loop size decreases. In view of the above analysis, one can optimize the operation of a bistable device by suitably controlling the carrier density as well as by choosing materials that have adequate nonlinearity of exciton damping.

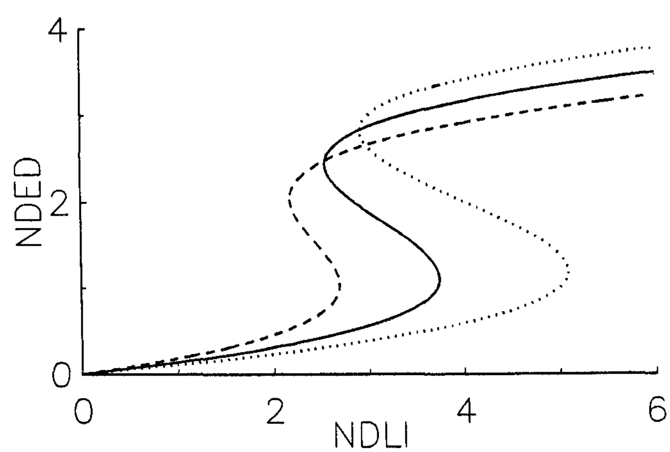


Fig. 2. The normalized dimensionless exciton density (NDED), $\tilde{n}$, versus the normalized dimensionless light intensity (NDLI), $\tilde{I}$, with $\xi_1 = \xi_2 = 0$ and $d = 3$ for $\rho = 0$ (dotted curve), $\rho = 0.35$ (solid curve) and $\rho = 0.7$ (dashed curve).

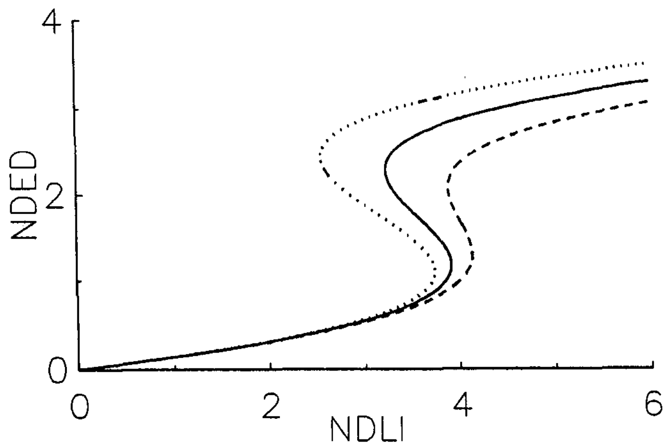


Fig. 3. The same as in Fig. 2 but with $\rho = 0.35$ and $d = 3$ for $\xi_1 = \xi_2 = 0$ (dotted curve), $\xi_1 = \xi_2 = 0.05$ (solid curve) and $\xi_1 = \xi_2 = 0.1$ (dashed curve).

To see how bistability of exciton density may arise by varying frequency but keeping the intensity fixed, let us return to the phase diagram in Fig. 1. Imagine that we are formally going along a line $d = const$ ($d > d_c$), we then shall meet the curves of the phase diagram at two points $\tilde{I}_+$ and $\tilde{I}_-$ (see Fig. 4). It is well known that with one and the same value of $\tilde{I} \in [\tilde{I}_+, \tilde{I}_-]$, the exciton density can have three values among which one is unstable and the two others are stable, i.e., the density–intensity bistability takes place. Now, instead, if we go along a line $\tilde{I} = const$ ($\tilde{I} > \tilde{I}_c$, $\tilde{I}_c$ is a threshold intensity which is given e.g., for the case of $\rho = 0$ by $\tilde{I}_c = 2(d - \xi_1)\left\{(3\xi_1^2 + 2\xi_1 + d)^2 + (\xi_1^2 + 2d\xi_1 + 3)^2\right\}/(27(1 + \xi_1^2)))$, we shall also meet the curves at two points $d_+$ and $d_-$ (see again Fig. 4). It is possible to foresee that, inside the interval of $d \in [d_+, d_-]$ with a fixed value of $\tilde{I} > \tilde{I}_c$, there will be two stable values of $\tilde{n}$. The phenomenon of OB occurring like this can conventionally be called the density-frequency bistability or the frequency-controlled bistability as compared to the above-described kind which is now referred to as the intensity-controlled one. The latter kind of bistability was already investigated for the Bose-condensation state,[22] for the nonlinear notation[16] and for a system consisting of many excitons and biexcitons.[23] Mathematically, we can consider $\tilde{n}$ as a function of d which contains $\tilde{I}$ as a parameter. From (13) it is easier to express d as a function of $\tilde{n}$. The (d-$\tilde{n}$) dependence reads

$$\mathrm{d} = \tilde{n} + \rho - \left\{\tilde{I}/\tilde{n} - (1 + \xi_2\rho + \xi_1\tilde{n})^2\right\}^{1/2} \qquad \text{when} \qquad \mathrm{d} < \tilde{n} + \rho \tag{17}$$

and

$$\mathrm{d} = \tilde{n} + \rho + \left\{\tilde{I}/\tilde{n} - (1 + \xi_2\rho + \xi_1\tilde{n})^2\right\}^{1/2} \qquad \text{when} \qquad \mathrm{d} > \tilde{n} + \rho\,. \tag{18}$$

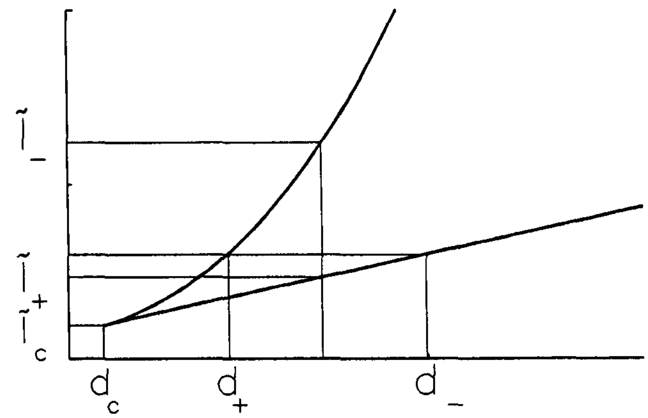


**Fig. 4.** Schematic phase diagram in the ($\tilde{I}$-d) plane. In between the two bold curves is the bistable phase. The outside is monostable.

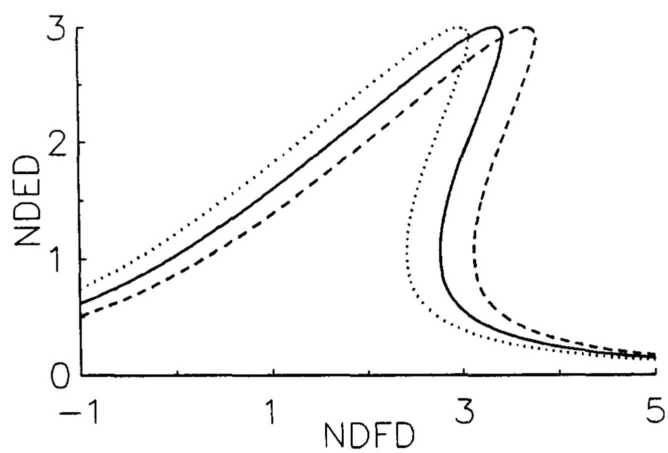


**Fig. 5.** The normalized dimensionless exciton density (NDED), $\tilde{n}$, versus the normalized dimensionless frequency detuning (NDFD), d, with $\xi_1 = \xi_2 = 0$ and $\tilde{I} = 3$ for $\rho = 0$ (dotted curve), $\rho = 0.35$ (solid curve) and $\rho = 0.7$ (dashed curve).

Using Eqs. (17) and (18) we can plot $\tilde{n}$ versus d in Figs. 5 and 6. In Fig. 5, $\tilde{I} = 3$, and $\rho$ changes with fixed $\xi = 0$. In Fig. 6, $\tilde{I} = 3$, while $\xi$ varies with fixed $\rho = 0.35$. Figure 5 shows that when $\xi$ is fixed both the critical detunings $d_+$ and $d_-$ increases with growing $\rho$. The hysteresis loop size, i.e., the difference $(d_- - d_+)$, is, however, almost unchanged. Finally, from Fig. 6 we have the following picture: when $\rho$ and $\tilde{I}$ are constant, the increase in $\xi$ causes a worse bistable behaviour. For $\xi = 0.1$, for instance, bistability begins to disappear (see the dashed curve in Fig. 6).

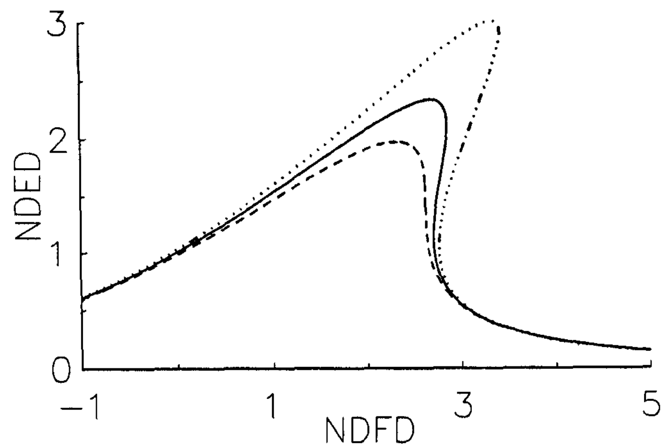


Fig. 6. The same as in Fig. 5 but with $\rho = 0.35$ and $\tilde{I} = 3$ for $\xi_1 = \xi_2 = 0$ (dotted curve), $\xi_1 = \xi_2 = 0.05$ (solid curve) and $\xi_1 = \xi_2 = 0.1$ (dashed curve).

Summing up all the discussion we have observed a quite complicated manner of the changing of the exciton density bistable behaviour in the presence of free carriers. Therefore, special attention has to be paid to such effects in operating realistic semiconductor bistable devices which should possess some unavoidable content of free carriers mostly due to impurities. From another point of view, by measuring bistability characteristics one could experimentally determine both the sign and the magnitude of the exciton–carrier coupling constant if the carrier concentration is known (or, vice versa, determine the carrier concentration if the exciton–carrier interaction is known). A method for such experimental determinations was outlined in Ref. 24.

## 4. Conclusion

In conclusion, we have investigated two kinds (intensity-controllable and frequency-controllable) of the optically bistable behaviour of the exciton density in a semiconductor containing a finite concentration of free carriers. The calculations show that the phenomenon of exciton bistability is heavily influenced by the presence of the carriers. The carriers may appear in an excited semiconductor e.g., by doping or by irradiating with an additional light beam of frequency exceeding the band gap. So, by adjusting the degree of doping or the additional light intensity one can control the amount of the carriers toward a desirable operation of a bistable device.

To conclude we note that similar possibilities of controlling the hysteresis loop size in the bistability phenomenon were considered in Ref. 25 where use of squeezed light but not of free carriers was made.

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