Modern Physics Letters B, Vol. 6, No. 7 (1992) 405–410

GENERATION OF SQUEEZED EXCITONS IN SEMICONDUCTORS BY COHERENT LIGHT

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Received 30 January 1992

In this letter, we show that semiconductor excitons generated by coherent light may become squeezed. This is due to their interactions both with the light and among themselves. Based on a polariton theory and using a secular approximation, analytical expressions for exciton quadrature variances have been derived, which enable us to study easily the dependence of exciton squeezing degree on the pumping light intensity, frequency detuning and strength of exciton-exciton, as well as exciton-photon, interactions.

Quadrature and number squeezed states of light[1] have strongly stimulated many physicists working in quantum optics during the last decade. Interests in squeezing phenomenon have even gone far beyond quantum optics and become a fresh topic for those working in quantum field theory[2] and condensed matter physics.[3,4] Nowadays, people begin to speak of soliton squeezing,[5] polariton squeezing,[6] phonon squeezing,[4] exciton squeezing,[7] biexciton squeezing,[8] etc.

In this letter we are concerned with squeezing of excitons that may be excited in a semiconductor by optical pumping. Under weak pumping there exist in the semiconductor a few excitons and the system Hamiltonian can be written in the form

$$H = Ea^+a + \omega b^+b + g(a^+b + b^+a)\,, \tag{1}$$

where a, a^+ and b, b^+ denote the operators of excitons with energy E and photons with frequency ω (in units such that $\hbar = c = 1$), g is the exciton-photon coupling constant. It has been shown[7] that while the system is described by (1), squeezed excitons can appear only and only if the photon were initially prepared in a squeezed state. Situation might change when one pumps the semiconductor by an intense light. Then there will be established a system of high density interacting excitons. The Hamiltonian in this case reads

$$H = Ea^+a + \omega b^+b + g(a^+b + b^+a) + fa^+a^+aa \tag{2}$$

PACS Nos.: 42.50.Dv & Md, 71.35z & 71.36+c.

with f being responsible for the interaction between excitons. As is already well-known, the presence of f may bring about various interesting nonlinear phenomena such as biexciton formation,[9] optical bistability (see e.g. Ref. 10), collapse and revival of oscillations,[11] giant polaritons with possible anomalous dispersion,[12] and others. Here our attempt is to point out one more effect which is also a consequence of the exciton-exciton interaction. Namely, we will show that when $f \neq 0$, the exciton may become squeezed even for input light prepared initially in a coherent state but not necessarily a squeezed one.

Applying a Bogolubov transformation technique with real function-coefficients u_1, u_2, v_1 and v_2 (for their explicit expressions see e.g. Ref. 7), we can diagonalize the quadratic part of (2) into the operators α_1, α_1^+ and α_2, α_2^+ of the two-branch polaritons which become interacting via the last term in (2). The transformed Hamiltonian will look like

$$H = \sum_\nu \Omega_\nu \alpha_\nu^+ \alpha_\nu + f \sum_{\mu\nu\zeta\xi} v_\mu v_\nu v_\zeta v_\xi \, \alpha_\mu^+ \alpha_\nu^+ \alpha_\zeta \alpha_\xi \,, \tag{3}$$

with μ, ν, ζ, ξ = 1 or 2 and Ω_ν being the polariton eigenenergy (for explicit expressions of Ω_ν see e.g. Ref. 7). Though, in a sense, (3) is simpler as compared with (2), it is still not exactly solvable. To proceed further, we at this moment resort to the so-called secular approximation.[11] Such an approximation allows us to be left with only the "resonant" terms in (3) which are

$$H = \sum_\nu \Omega_\nu \alpha_\nu^+ \alpha_\nu + \sum_{\nu\mu} f_{\nu\mu} \alpha_\nu^+ \alpha_\mu^+ \alpha_\mu \alpha_\nu \,, \tag{4}$$

where $f_{11} = f v_1^4$, $f_{22} = f v_2^4$ and $f_{12} = f_{21} = f v_1^2 v_2^2$. Note that (4) resembles a particular type of Hamiltonians describing two modes of light.[13] From (4) it is easy to set up the equations of motion for the polariton operators and then solve them to obtain

$$\alpha_\nu(t) = \exp\{-i[\Omega_\nu + 2f_{\nu\nu}\alpha_\nu^+(0)\alpha_\nu(0) + 2f_{\nu\mu}\alpha_\mu^+(0)]t\} \cdot \alpha_\nu(0)\,. \tag{5}$$

In (5), $\mu \neq \nu$. To investigate squeezing of excitons we must define their time-dependent quadratures $X_\nu(t)$ as below

$$X_\nu(t) = \frac{1}{2}\,(i)^{\nu-1}[a^+(t) - (-1)^\nu a(t)]\,. \tag{6}$$

Now, anticipate that the semiconductor is initially in the ground state with no excitons present and the light is in the coherent state characterized by a complex quantity z. Denoting by $|i\rangle$ the initial state of the semiconductor-light system we thus have

$$|i\rangle = D_b(z)|0, 0\rangle\,, \tag{7}$$

where $D_b(z)$ labels the displacement operator for the photon. To determine whether exciton squeezing occurs we have to follow the time-evolution of the normally ordered quadrature variances of the exciton denoted by $\langle i| : (\Delta X_\nu(t))^2 : |i\rangle \equiv \langle : (\Delta X_\nu(t))^2 : \rangle$. If in the course of time either of the variances ($\nu = 1$ or 2) becomes negative one says that the generation of squeezed excitons is possible. To analytically derive expressions for the variances, it is convenient to go to the polariton representation, i.e.,

$$a(t) \Rightarrow v_1\alpha_1(t) + v_2\alpha_2(t)\,, \tag{8}$$

$$a^+(t) \Rightarrow v_1\alpha_1^+(t) + v_2\alpha_2^+(t)\,, \tag{9}$$

$$D_b(z)|0,\,0\rangle \Rightarrow D_{\alpha_1}(u_1,\,z)D_{\alpha_2}(u_2 z)|0,\,0\rangle\,. \tag{10}$$

With the aid of (7) to (9) we obtain, after some lengthy operatoric and algebraic-trigonometric manipulations, the following formula which looks quite cumbersome but is rather easy to handle analytically (here z is assumed to be real for simplicity):

$$\begin{aligned}\langle:(\Delta X_\nu(t))^2:\rangle = {} & \frac{1}{2}\,\nu_1^2[u_1^2 z^2 - (-1)^\nu N_1(t) - 2S_1^2(t)] \\ & + \frac{1}{2}\,v_2^2[u_2^2 z^2 - (-1)^\nu N_2(t) - 2S_2^2(t)] \\ & + v_1 v_2[Q(t) - (-1)^\nu R(t) - 2S_1(t)S_2(t)]\,,\end{aligned} \tag{11}$$

with

$$\begin{aligned}N_{1,\,2}(t) = {} & z^2u_{1,\,2}^2 \exp\{z^2u_{1,\,2}^2[\cos(4fv_{1,\,2}^4 t) - 1] \\ & + z^2u_{1,\,2}^2[\cos(4fv_{1,\,2}^2 v_{2,\,1}^2 t) - 1]\}\cos[2(\Omega_{1,\,2} + fv_{1,\,2}^4)t \\ & + z^2u_{1,\,2}^2 \sin(4fv_{1,\,2}^4 t) + z^2u_{2,\,1}^2 \sin(4fv_{1,\,2}^2 v_{2,\,1}^2 t)]\,,\end{aligned} \tag{12}$$

$$\begin{aligned}S_{1,\,2}(t) = {} & z^2u_{1,\,2}^2 \exp\{z^2u_{1,\,2}^2[\cos(2fv_{1,\,2}^4 t) - 1] \\ & + z^2u_{2,\,1}^2[\cos(2fv_{1,\,2}^2 v_{2,\,1}^2 t) - 1]\}\cos[\Omega_{1,\,2} t \\ & + z^2u_{1,\,2}^2 \sin(2fv_{1,\,2}^4 t) + z^2u_{2,\,1}^2 \sin(2fv_{1,\,2}^2 v_{2,\,1}^2 t)]\,,\end{aligned} \tag{13}$$

$$\begin{aligned}Q(t) = {} & z^2u_1u_2 \exp\{z^2u_1^2[\cos(2fv_1^2(v_2^2 - v_1^2)t) - 1] \\ & + z^2u_2^2[\cos(2fv_2^2(v_2^2 - v_1^2)t) - 1]\}\cos[(\Omega_2 - \Omega_1)t \\ & + z^2u_1^2 \sin(2fv_1^2(v_2^2 - v_1^2)t) + z^2u_2^2 \sin(2fv_2^2(v_2^2 - v_1^2)t]\,,\end{aligned} \tag{14}$$

$$\begin{aligned}R(t) = {} & z^2u_1u_2 \exp\{z^2u_1^2[\cos(2fv_1^2(v_2^2 + v_1^2)t) - 1] \\ & + z^2u_2^2[\cos(2fv_2^2(v_2^2 + v_1^2)t) - 1]\}\cos[(\Omega_2 + \Omega_1 + 2fv_1^2v_2^2)t \\ & + z^2u_1^2 \sin(2fv_1^2(v_2^2 + v_1^2)t) + z^2u_2^2 \sin(2fv_2^2(v_2^2 + v_1^2)t)]\,.\end{aligned} \tag{15}$$

The remainder of this letter is reserved for graphically illustrating our above-obtained formula. Since we are dealing with excitons in semiconductors, we must in numerical calculations take parameters suited to semiconductors. Typically, g/E spans a range of about from 0.001 to 0.05, while f/E from 0.0005 to 0.005. In

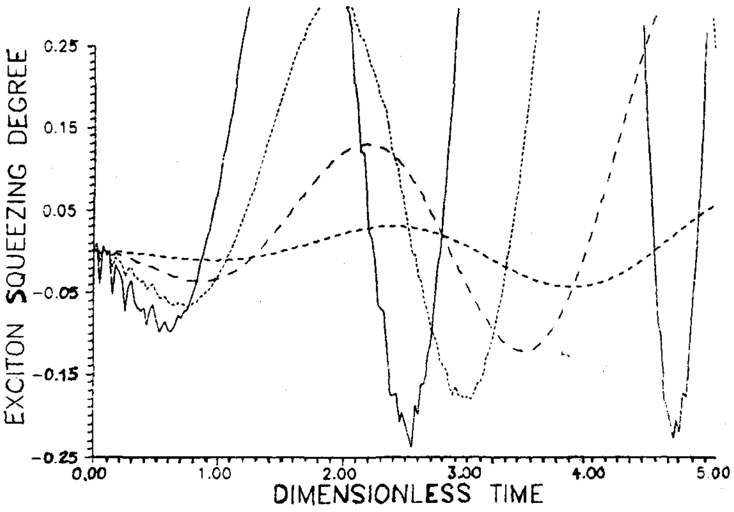

Fig. 1. Exciton squeezing degree $\langle :(\Delta X_1(t))^2: \rangle$ versus dimensionless time Et for $\omega/E = 1$, $g/E = 0.02$ and $f/E = 0.002$. The medium-dashed, long-dashed, short-dashed, and solid curves correspond respectively to $z = 5$, 10, 15, and 20.

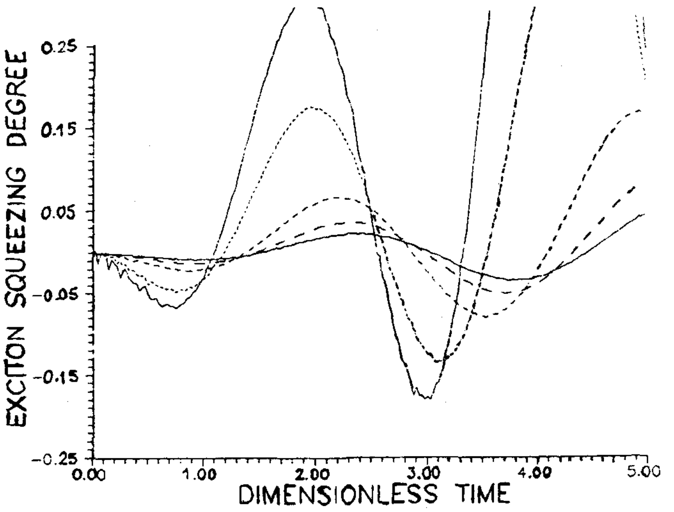

Fig. 2. Same as Fig. 1 but with z fixed at 15 while ω/E varies near the perfect resonance value: $\omega/E = 1$ (solid curve with the deepest antipeaks), 0.97 (short-dashed curve), 0.93 (medium-dashed curve), 0.90 (long-dashed curve) and 0.87 (solid curve with the most shallow antipeaks).

Fig. 1, $\langle :(\Delta X_1(t))^2: \rangle$ (hereinafter referred to as exciton squeezing degree) is plotted against the dimensionless time Et for $\omega/E = 1$, $g/E = 0.02$, $f/E = 0.002$ and $z = 5$, 10, 15, and 20. As can be seen, increasing the initial light intensity z causes a better degree of exciton squeezing (the negative antipeaks become deeper). Figure 2

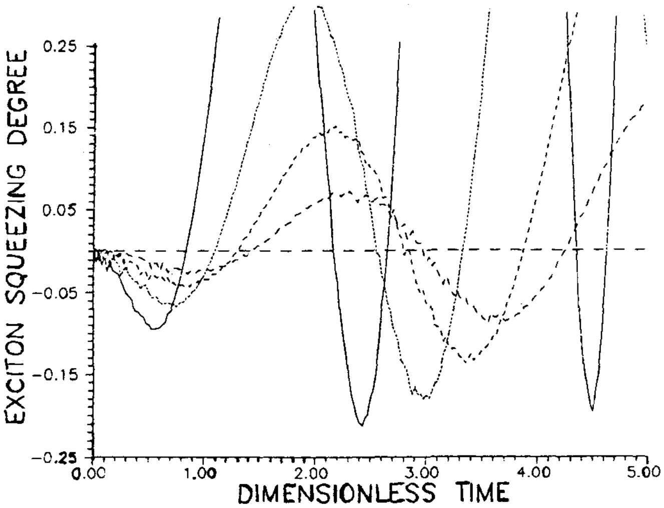

Fig. 3. Same as Fig. 1 but with $z = 15$, and $f/E = 0$ (long-dashed straight horizontal line) and $f/E = 0.0005, 0.001, 0.002, 0.004$ corresponding to decreasing antipeaks depth.

plots the same as in Fig. 1 but the detuning is changing with z fixed at 15 (see the figure caption). Obviously the perfect resonance detuning ($\omega = E$ or $\omega/E = 1$) gives the best squeezing. By going farther off the perfect resonance the squeezing becomes worse and already at $\omega/E = 0.87$ the exciton can possess just quite a small degree of squeezing. For $\omega/E < 0.87$ the squeezing proves to be negligible. That is why in the last two figures we will fix ω to be equal to E. The dependence of the exciton squeezing degree on the strength of the exciton-exciton interaction, f, is represented in Fig. 3, from which one observes that larger values of f favour more the squeezing process of excitons. For $f = 0$, as it should be, no squeezing can occur for coherent input light. This clearly stresses the significant role of f. Finally, we demonstrate the role exciton-photon interaction plays in our system by Fig. 4. When $g = 0$ the exciton and the photon are disconnected and as a consequence the stated problem loses its meaning. The greater g the higher degree of squeezing may be obtained. It is interesting however to note that in Fig. 4 we have used $\omega/E = 0.95$ instead of 1. For $\omega/E = 1$ the coefficients u_ν, v_ν turn out to be g-independent (they all then equal to $\sqrt{0.5}$), and therefore the influence of g is reflected only through Ω_ν. Since at perfect resonance $\Omega_\nu = E - (-1)^\nu g$ and in semiconductors $g \ll E$, the influence of g is very small (not shown). However, by moving away from perfect resonance the coefficients u_ν, v_ν will strongly depend on g and the role of g then becomes visible, as seen from Fig. 4 for $\omega/E = 0.95$.

Summing up the results we arrive at the conclusion that coherent light may generate squeezed excitons in semiconductors provided that the light intensity is high enough in order to make the excitons interact ($f \neq 0$). By adjusting light frequency and by choosing semiconductors with the appropriate characteristics one can reach a desired squeezing degree of excitons.

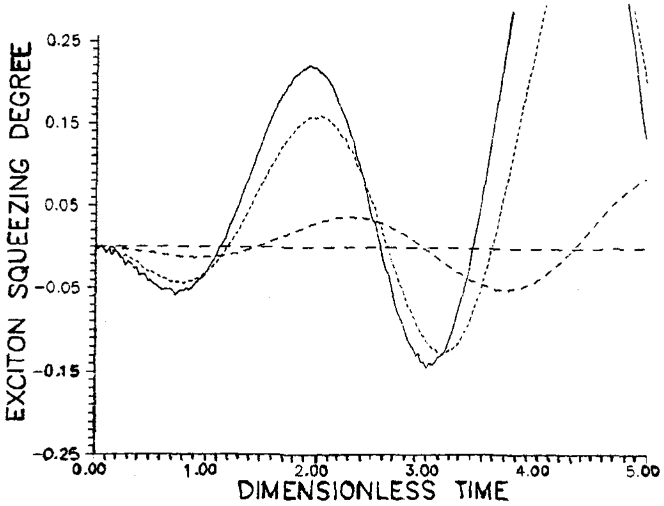

Fig. 4. Same as Fig. 1 but with $\omega/E = 0.95$, and $g/E = 0.001$ (long-dashed curve, squeezing is almost impossible for small g), 0.01 (medium-dashed curve), 0.03 (short-dashed curve) and 0.05 (solid curve).

Acknowledgements

The authors thank Profs. Nguyen Van Hieu and Dao Vong Duc for their constant support, and the Institute of Theoretical Physics of Vietnam for providing them with computer facilities. This work is financially supported by the Hanoi Pedagogic College.

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