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CHAOTIC DYNAMICS OF HIGH DENSITY PHOTON AND EXCITON SYSTEM

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We consider nonlinear dynamics of a system of interacting photons and excitons driven by a resonantly exciting light field. Using the linear stability analysis for nonlinear coupled equations of photon and exciton operators we investigated that, besides the usual unstable domain which is a negative slope branch, there are other unstable ones in the leftmost and rightmost sides of the lower and upper branches of the frequency-density bistable curves. Numerical study shows that the system has very rich information on the instability nature and a self-oscillation process leading to chaos can occur in these unstable domains.

1. Introduction

Not quite long ago, it was thought that chaotic self-oscillations in dissipative systems (and indeed, in conservation systems) were related exclusively to the excitation in a given system of a very large number of degrees of freedom. However, it is now clear that there are dissipative systems with a small number of degrees of freedom which have chaotic behaviour. The foundation lies in the instability of solutions of partial differential equations. This is one of the major reasons which have caused extensive study of chaotic dynamics in optical systems, both theoretically and experimentally in recent years, after the pioneering works of Bonifacio and Lugiato,[1] and Ikeda.[2] For reviews on this rapidly developing field of laser instabilities see Haken,[3] Arecchi and Harrison,[4] and Lugiato and Narducci.[5] Optical stabilities, instabilities and chaos in semiconductors have also received a great deal of attention.[6,7] However, chaotic

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dynamics of the interaction photons and excitons system have been studied quite recently.[8–10] In Refs. 8, 9, the Kishinhev group has considered the instability and chaos at some particular points in unstable domains of the steady stationary solution curve. In Ref. 10 we have considered the nonlinear dynamics of free carrier-exciton-photon system, and for the first time we have found in Ref. 10 that the period-doubling self-oscillation process is a possible route to chaos in the system. In these works,[8–10] however, the authors have only considered the nonlinear dynamics of the exciton-photon system in which dynamical control parameter is pumping light intensity.

In this paper we first present a linear stability analysis for dynamics equations of photonic and excitonic bistability, then we study instability chaotic behaviour of photon and exciton densities treating as the functions of pumping light field. Moreover, we want to focus to the case when chaotic dynamics of the system is controlled by the frequency of light.

We shall utilize the system of units with $\hbar = c = 1$ where $\hbar$, c are Planck constant and the velocity of light respectively.

2. Basic Equations

Supposing now that the semiconductor is irradiated by an external laser field with given wave vector k, complex amplitudes $\varepsilon_k^{(\pm)}$ and frequency Ω_k. The laser frequency is assumed to be resonant with the exciton level. The exciton system is thus directly driven by the laser field via the exciton-photon transition. Under the excitation of the laser field, there is a high density system of interacting photons and excitons in the semiconductor. The Hamiltonian which describes this system has the form[10]:

$$H = \sum_{\mathrm{p}} \{\omega_{ap} a_{\mathrm{p}}^{+} a_{\mathrm{p}} + \omega_{bp} b_{\mathrm{p}}^{+} b_{\mathrm{p}} - G[a_{\mathrm{p}}^{+} b_{\mathrm{p}} + b_{\mathrm{p}}^{+} a_{\mathrm{p}}]\} + \frac{F}{V} \sum_{\mathrm{p},\mathrm{p}',1} a_{\mathrm{p}}^{+} a_{\mathrm{p}'}^{+} a_{\mathrm{p}'-1} a_{\mathrm{p}+1} + i(gV)^{1/2} \{\varepsilon_{\mathrm{k}}^{(-)} e^{-i\Omega t} b_{\mathrm{k}}^{+} - \varepsilon_{\mathrm{k}}^{(+)} e^{+i\Omega t} b_{\mathrm{k}}\} \,. \tag{1}$$

In Eq. (1) we have used some notations with their meanings as follow: $(a_{\mathrm{p}}, a_{\mathrm{p}}^{+})$, and $(b_{\mathrm{p}}, b_{\mathrm{p}}^{+})$, denote respectively the operators of excitons and photons with energies ω_{ap} and frequency ω_{bp}. The field ε_{k} is treated as classical and monochromatic at the wave vector k. The sample volume is labelled by V and G, F and g, mean coupling constants which are, for simplicity, anticipated momentum-independent.

Now within the Hartree-Fock approximation, we set up from the Hamiltonian (1) the equations of motion for the k-mode of excitons and photons. These modes are directly governed by the monochromatic field ε_{k} and therefore they can be treated as coherent modes.[11] Performing the necessary averaging procedure we get the following system of nonlinear differential equations:

$$\frac{d\langle a\rangle}{dt} = -i\{\omega_a + 4Fn_a - i\gamma_a\}\langle a\rangle + iG\langle b\rangle \tag{2}$$

$$\frac{d\langle b\rangle}{dt} = -i\{\omega_b - i\gamma_b\}\langle b\rangle + iG\langle a\rangle + (gV)^{1/2}\varepsilon^{(-)}.e^{-i\Omega t} \tag{3}$$

where $\langle a\rangle \equiv \langle a_{\rm k}\rangle$, $\langle b\rangle \equiv \langle b_{\rm k}\rangle$, $\omega_a \equiv \omega_{ak}$, $\omega_b \equiv \omega_{bk}$, $\Omega = \Omega_k$ and $n_a = V^{-1}\langle a^+\rangle\langle a\rangle \equiv V^{-1}|\langle a\rangle|^2$. We assumed $\varepsilon_{\rm k}^{(+)} = \varepsilon_{\rm k}^{(-)} = \varepsilon$ for simplicity. We have also introduced in a phenomenological way the transverse exciton (γ_a^{-1}), photon (γ_b^{-1}) life-time.

Solutions of the equation system in a transient regime are sought in the forms:

$$\langle a\rangle = Q\{X_a(t) + iY_a(t)\}e^{-i\Omega t} \tag{4}$$

$$\langle b\rangle = Q\{X_b(t) + iY_b(t)\}e^{-i\Omega t} \tag{5}$$

with $Q = (\gamma_a V/F)^{1/2}/2$ introduced for convenience. Then we arrived at the following nonlinear differential coupled equations:

$$\frac{dX_a}{d\tau} = -X_a - (\delta - X_a^2 - Y_a^2)Y_a - \varphi Y_b \tag{6}$$

$$\frac{dY_a}{d\tau} = -Y_a + (\delta - X_a^2 - Y_a^2)X_a + \varphi X_b \tag{7}$$

$$\frac{dX_b}{d\tau} = -\sigma X_b - \Delta Y_b - \varphi Y_a + P \tag{8}$$

$$\frac{dY_b}{d\tau} = -\sigma Y_b + \Delta X_b + \varphi X_a \tag{9}$$

with the following dimensionless scaled notations:

$$\tau = \gamma_a t\ ; \qquad \delta = \frac{\Omega - E_a}{\gamma_a}\ ; \qquad \Delta = \frac{\Omega - E_b}{\gamma_a}$$

$$\varphi = \frac{G}{\gamma_a}\ ; \qquad \sigma = \frac{\gamma_b}{\gamma_a}\ ; \qquad P = 2\frac{\varepsilon(gF)^{1/2}}{\gamma_a^{3/2}}\ . \tag{10}$$

The dimensionless exciton and photon densities can be expressed as $N_a = X_a^2 + Y_a^2$, and $N_b = X_b^2 + Y_b^2$. In (10) Δ and φ have the meaning of photon detuning and the bare exciton-photon interaction, respectively. $P^2 \equiv I$ labels the dimensionless laser intensity which was treated as a dynamical control parameter in our previous work,[10] δ is exciton detuning which can serve a control parameter in this work.

The steady state solutions of Eqs. (6–9) can easily be found. Using these solutions we can obtain the following relations:

$$I \equiv P^2 = \frac{\sigma^2}{\varphi^2}N_a\left\{(\delta - N_a)^2 + \left(1 + \frac{\varphi}{\sigma}\right)^2\right\} \tag{11}$$

and

$$N_b = \frac{1}{\varphi^2} N_a \{(\delta - N_a)^2 + 1\} \ . \tag{12}$$

To arrive at Eqs. (11) and (12) we have assumed in this paper that $\Delta = 0$.

Equations (11) and (12) are basic equations of the problem of excitonic optical bistability. From the system of nonlinear coupled equations (6–9) we can consider properties of bistability, instability and chaos of the system in which the initial conditions are steady stationary solutions (11–12) with detuning δ fixed and control parameter is light intensity as in our previous work.[10] On the contrary in this work we consider the chaotic dynamics of the system when we fixed the value of the light intensity and changed the frequency detuning. Though Eqs. (11) and (12) are cubic equations with respect to N_a, bistable behaviour could occur only under certain conditions. It is easy to find from Eq. (11) that the condition for N_a (and then for N_b) to be a tri-valued function of I is:

$$\delta > \sqrt{3}\left(1 + \frac{\varphi}{\sigma}\right) \ . \tag{13}$$

When (13) is met, the function $I = I(N_a)$ has two extrema at $N_a^{(\pm)}(\delta)$ determined by

$$N_a^{(\pm)}(\delta) = \frac{2\delta \pm \sqrt{\delta^2 - 3\left(1 + \frac{\varphi}{\sigma}\right)^2}}{3} \tag{14}$$

the values of the light intensity (called the critical intensities) corresponding to $N_a^{(\pm)}$ are labelled by

$$I^{(\pm)} = I[N_a^{(\pm)}(\delta)] \ . \tag{15}$$

The phase diagram plotted in Fig. 1 indicates the region of frequency and intensity in which bistable behaviour of N_a and N_b takes place. Mathematically, from Eqs. (11), (12) we can consider δ as the variable of the functions N_a, N_b and I as a parameter, then the $\delta - N_a$ dependence will look like:

$$\delta = N_a - \frac{\varphi}{\sigma}\sqrt{\frac{I}{N_a} - \left(1 + \frac{\sigma}{\varphi}^2\right)} \quad \text{when} \quad \delta < N_a \tag{16}$$

and

$$\delta = N_a + \frac{\varphi}{\sigma}\sqrt{\frac{I}{N_a} - \left(I + \frac{\sigma}{\varphi}\right)^2} \quad \text{when} \quad \delta > N_a \tag{17}$$

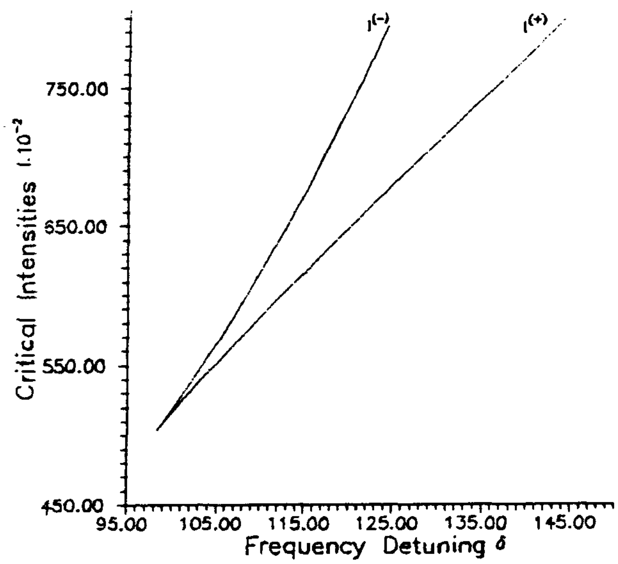

Fig. 1. The phase transition diagram. The dimensionless critical intensities $I^{\pm}$ versus dimensionless detuning δ. For $\varphi = 23.6$, $\sigma = 10$.

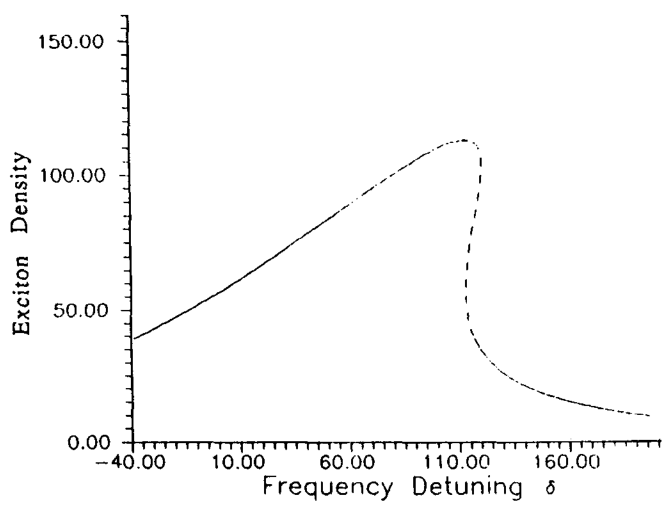

Fig. 2. Dimensionless exciton density N_a as a function of frequency detuning δ for $I = 65 \times 10^2$ $\varphi = 23.6$, $\sigma = 10$.

and the dependence $\delta - N_b$ can be determined through Eq. (12). Using Eqs. (13), (14) and (12) we can present N_a and N_b functions of δ by the graphs in Figs. 2 and 3. We can see in Figs. 2 and 3 that when δ changes and with a fixed value

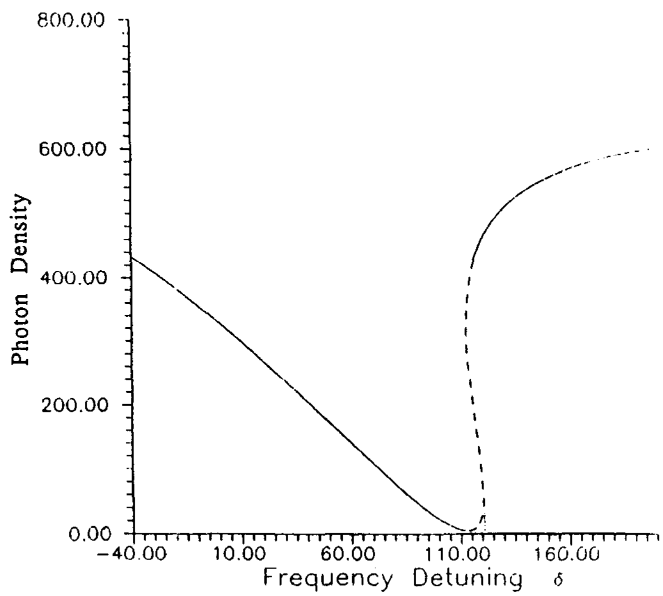

Fig. 3. Dimensionless photon density N_b as a function of frequency detuning δ for the same parameters as in Fig. 2.

of $I = [I^{(+)}, I(-)]$, there is a domain of δ that the $\delta - N_a(N_b)$ dependence is tri-valued. This character is called frequency-density bistability. The system of nonlinear coupled equations (6–9) and its steady stationary solutions (16–17) which present the dependence of δ on exciton and photon densities N_a and N_b are the basic equations of our problem.

3. Chaotic Dynamics

It is well known from the optical bistability that the whole middle branches of negative slope of the hysteresis loops are unstable. This means that even an infinitely small departure from a point in the negative slope domain can make the system remove from it. However, the direction of removal is very sensitive to the initial condition. If the system is initially very near but above (below) a point in this domain, then in the course of time it will eventually relax towards the point in the upper (lower) branch which corresponds to the same input laser frequency detuning (the plots are not shown because of lack of space).

We now study further the instabilities of the whole branch of steady stationary curves in Figs. 2, 3 within the framework of the linear stability analysis. Let us consider neighborhoods of the steady state solution, and these neighborhoods can be presented in the following forms:

$$X_{a,b} = \bar{X}_{a,b} + \xi_{a,b} e^{\lambda t}$$
$$Y_{a,b} = \bar{Y}_{a,b} + \eta_{a,b} e^{\lambda t} \tag{18}$$

where $\xi_{a,b}$, $\eta_{a,b}$ are small perturbations, $\bar{X}_{a,b}$, $\bar{Y}_{a,b}$ are steady-state solutions of Eqs. (6–9). Inserting (18) in Eqs. (6–9) and linearizing these equations we arrive at the system of linear algebraic equations for $\xi_{a,b}$, $\eta_{a,b}$, in which its characteristic equation is the form:

$$\det \begin{pmatrix} \lambda + 1 - 2\bar{X}_a\bar{Y}_a & \delta + \bar{X}_a^2 - \bar{Y}_a^2 & 0 & \varphi \\ -\delta - \bar{X}_a^2 + \bar{Y}_a^2 & \lambda + 1 + 2\bar{X}_a\bar{Y}_a & -\varphi & 0 \\ 0 & \varphi & \lambda + \sigma & 0 \\ -\varphi & 0 & 0 & \lambda + \sigma \end{pmatrix} = 0 \tag{19}$$

or

$$\lambda^4 + C_1\lambda^3 + C_2\lambda^2 + C_3\lambda + C_4 = 0 \, . \tag{20}$$

It is clear that if all real parts $\text{Re}\lambda_{1,2,3,4} < 0$ with λ_i are solutions of Eq. (20), then solutions (18) of the system of Eqs. (6–9) are unstable. Using the Hurwitz criteria for all real parts of λ_i negative, we have:

$$C_1 > 0 \, ; \quad C_4 > 0 \, ; \quad C_1C_2 - C_3 > 0 \, ; \quad (C_1C_2 - C_3)C_3 - C_4C_1^2 > 0 \tag{21}$$

where C_1, C_2, C_3 and C_4 are combinations of the parameters of the problem and the steady state solutions of Eqs. (6–9). Numerically studying the conditions (21) with the same set of material parameters as used in Ref. 9 we can plot in Figs. 2 and 3 the exciton and photon density N_a and N_b versus the frequency detuning with the instability domains presented by dashed lines. We want to stress that while the whole middle branches of negative slope is unstable, the instability domains include the leftmost and rightmost sides of the upper and lower branches. In this paper we limit ourselves to consider the instability and chaotic behavior of the rightmost side of the lower and upper branches in Figs. 2 and 3 respectively.

Let us now examine the dynamics of the system in which the initial conditions are the instability rightmost side of the lower and upper branches of the steady state solution curves in Figs. 2, 3. Taking the values of the parameters as in Ref. 8 $\varphi = 23.6$, $\sigma = 10$ while the intensity is chosen $I = 65 \times 10^2$, we have carried out numerical studies of equations (6–9). Upon increasing the frequency detuning from the stable domain (solid lines) of the lower (upper) branch of the frequency-density bistable curves of dimensionless exciton (photon) density versus frequency detuning, we first meet at about $\delta = 114.6158$ a Hopf bifurcation point below which periodic solutions emerge. In Figs. 4, 5 the time structure of the exciton

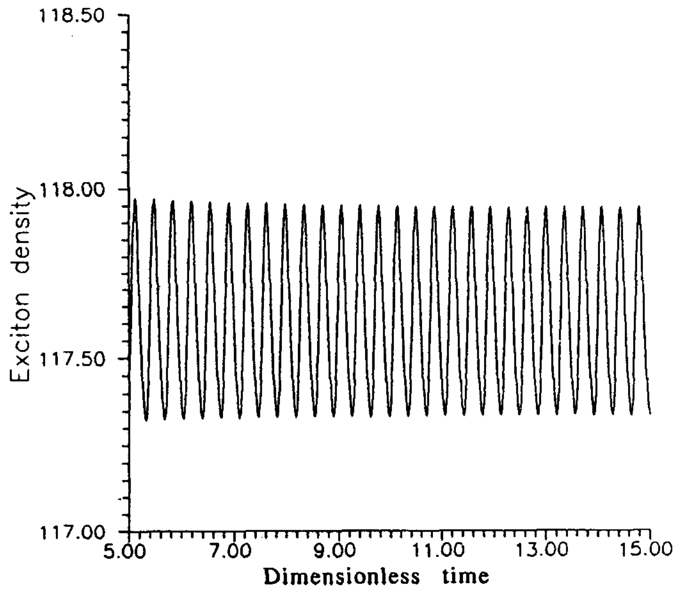

Fig. 4. Time-evolution of the exciton density when the system initially suffers a slight displacement from the steady state with $\delta = 116.1473$ in the lower branch of the bistability curve in Fig. 2. Periodic self-oscillation.

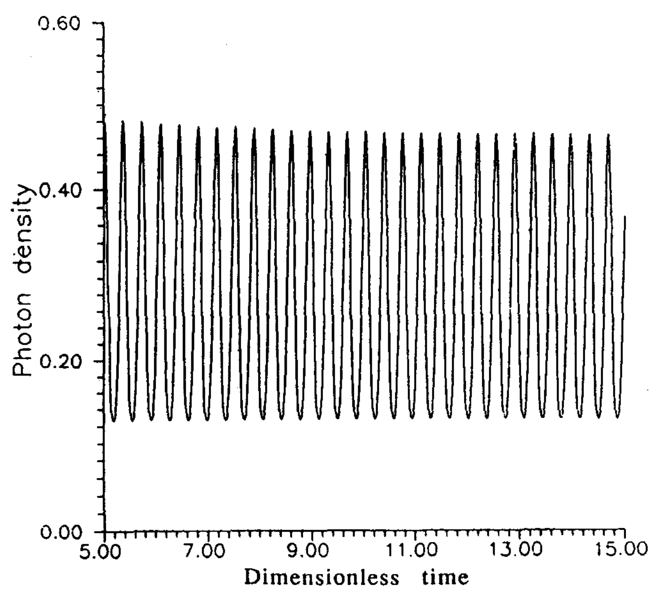

Fig. 5. Time-evolution of the photon density when the system initially suffers a slight displacement from the steady state with $\delta = 116.1473$ in the upper branch of the bistability curve in Fig. 3. Periodic self-oscillation.

and photon densities are plotted with $\delta = 116.1473$ corresponding to periodic self-oscillations. If we continue increasing the control parameter frequency detuning δ we will observe many different multi-periodic oscillations and a rich variety of phase portraits. When δ increases from 118.024 to 119.0371 we can observe a frequency detuning window inside which chaotic self-oscillations appear. In Figs. 6 and 7 chaotic time structure of exciton and photon densities are presented with $\delta = 118.6324$ in the meantime the rightmost part next to the chaos window has another kind of instability which appears a chain of multi-peaked pulses that is similar with the results in Refs. 8–10, and these are plotted in Figs. 8a and 9a with their phase portraits in Figs. 8b and 9b, respectively. The above results show that in the interacting excitons and photons system governed by a resonantly pumping laser field, we can control the chaotic dynamics of the system not only by the intensity as in Refs. 8–10 but also by the frequency detuning as in this work. In both cases, the system has very rich information on instability and chaotic dynamics.

4. Conclusion

We have carried out a linear stability analysis for a set of nonlinear dynamics equations of the interacting excitons and photons system. Then we have also performed a computer experiment on the lower (upper) branch of the photonic (excitonic) frequency-density optical bistability curve for a chosen set of parameters as in Ref. 9. For other parameter sets the physical picture may be dramatically changed. The

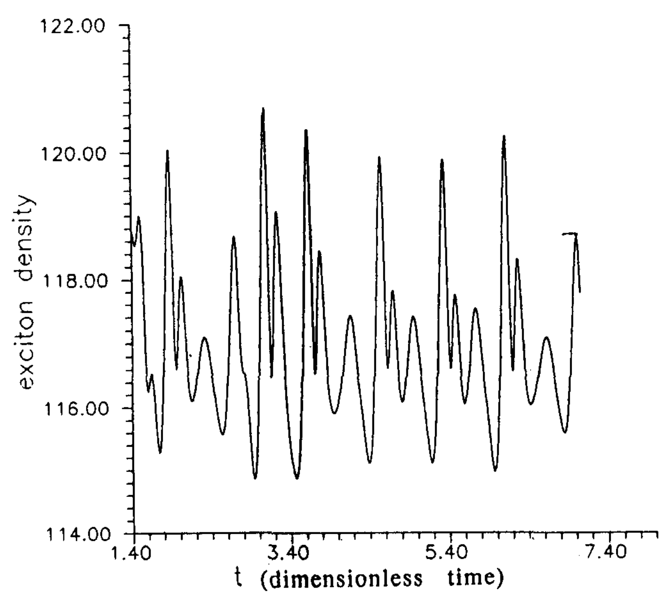

Fig. 6. The same as in Fig. 4 but $\delta = 118.6324$. Chaotic self-pulsation.

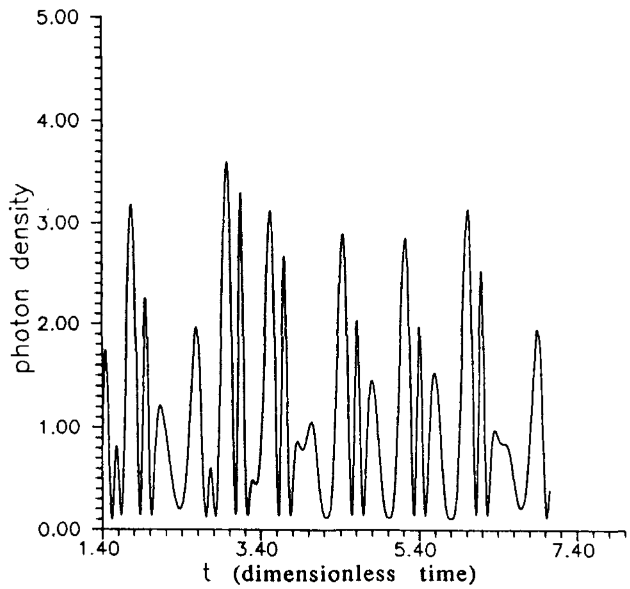

Fig. 7. The same as in Fig. 5 but $\delta = 118.6324$. Chaotic self-pulsation.

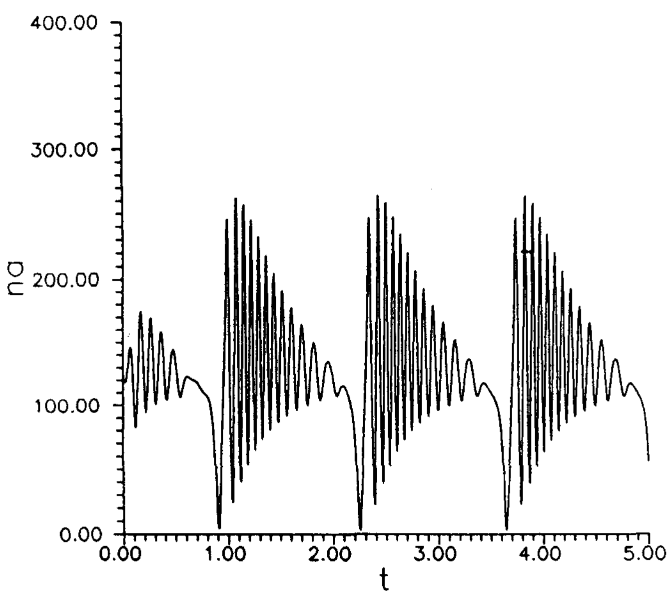

Fig. 8a. The same as in Fig. 4 but $\delta = 119.7781$. A chain of 16-peaked pulses.

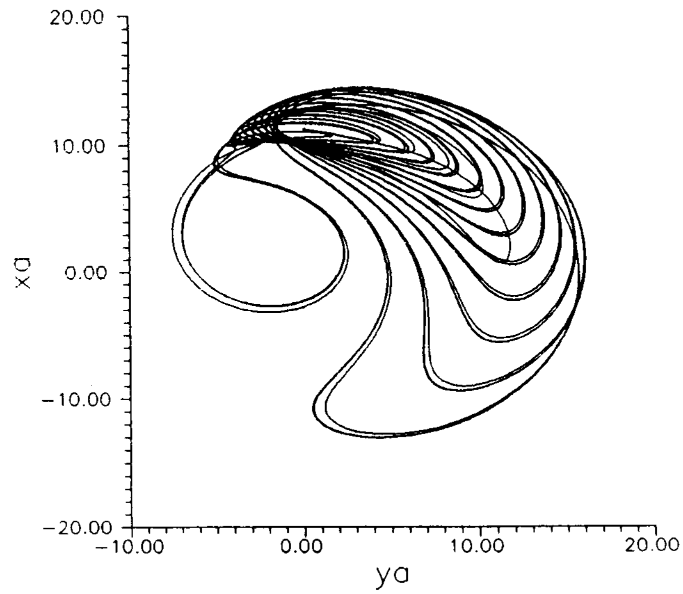

Fig. 8b. Phase portrait corresponding to Fig. 8a.

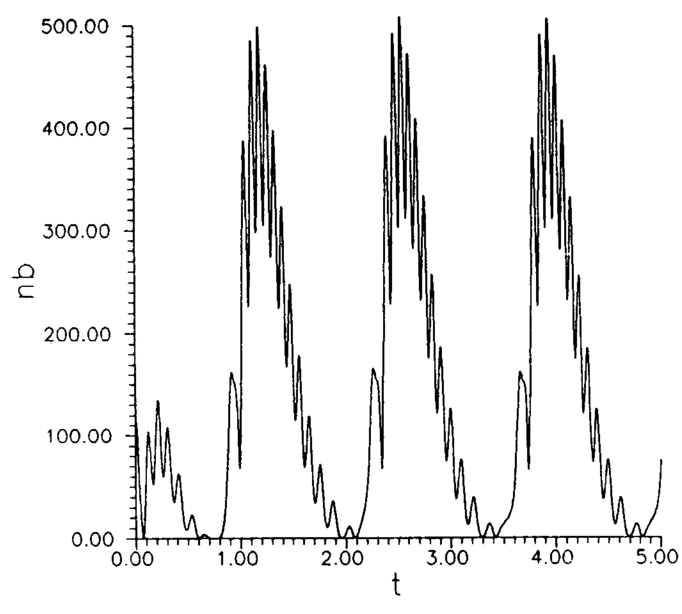

Fig. 9a. The same as in Fig. 5 but $\delta = 119.7781$. A chain of 16-peaked pulses.

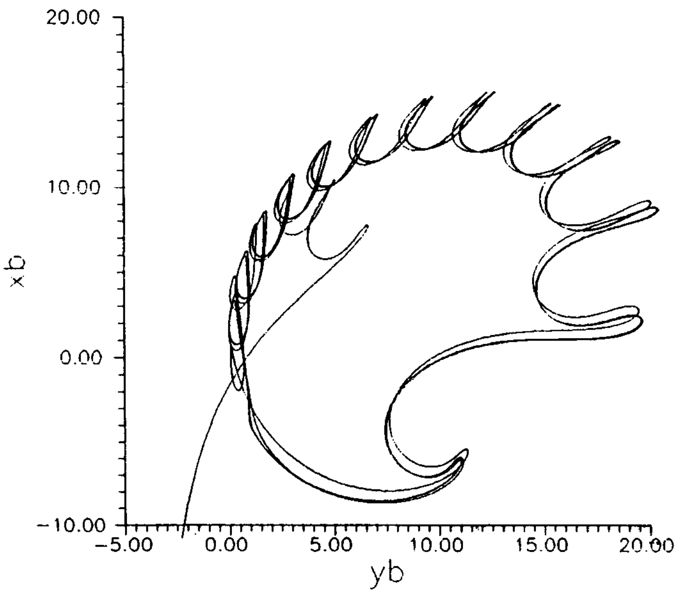

Fig. 9b. Phase portrait corresponding to Fig. 9a.

results show that in the system (and other ones) governed by a laser field one can control the dynamical behaviour not by intensity but also frequency of the field. Moreover, if we consider the influence of the free carriers that coexist with the exciton as in Refs. 10, 11 we can obtain other control parameters of the dynamical system. According to our new result, it is worth to suggest a possible way to control the dynamical system by using simultaneously some control parameters, for example the laser intensity, free carrier density and frequency, for practical purposes.

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