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Light squeezing via exciton–exciton interaction in semiconductors

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Abstract. In the excitonic polariton picture light squeezing is studied as a function of the exciton–exciton interaction. The influence of the photon frequency detuning, the initial photon intensity and exciton density as well as the magnitude of both the exciton–exciton and exciton–photon interactions on the squeezing process of light are investigated in detail.

1. Introduction

Among various non-classical effects, the squeezing phenomenon seems to be the most interesting one [1, 2] which is not limited only to quantum optics but has been expanded to many other areas such as elementary particle physics, quantum field theory and condensed matter physics. In the last few years terminology such as soliton squeezing [3], polariton squeezing [4], phonon squeezing [5], exciton squeezing [6], biexciton squeezing [7], etc has begun to appear in the literature. Concerning the mechanisms of producing squeezed states of light, it has been shown [8] that an intense laser beam can squeeze itself by means of its interaction with the so-called Kerr-type medium via an interaction Hamiltonian $\propto \chi^{(3)} b^{+2} b^2$ (where $\chi^{(3)}$ is the *third-order susceptibility* of the medium and b, b^+ denote the photon operators). A more general model has been suggested in [9] in which higher-order non-linearities were considered by using an interaction Hamiltonian $\propto \chi^{(2\kappa-1)} b^{+\kappa} b^{\kappa} (\kappa = 3$ or $4)$. Recently, the authors of [10] have studied the roles of different simultaneously present higher-order susceptibilities, i.e. they have investigated an interaction Hamiltonian of the form $\propto \sum_{\kappa=2}^{4} \chi^{(2\kappa-1)} b^{+\kappa} b^{\kappa}$. Nevertheless, interaction Hamiltonians of the above-mentioned forms are in fact valid only in the adiabatic limit when the frequency of light is far from the energy level of the quasiparticle in the medium (the units used throughout are $\hbar = c = 1$). In this limit the medium just plays a passive role with only virtual optical transitions involved. Due to the non-resonant nature, higher-order susceptibilities are often very small and therefore, for possible observation of the squeezing effect, the light intensity must be sufficiently high which may cause damage of the sample or prevent the quantum-mechanical description of light. In the other limit, when the frequency of light is resonant or nearly resonant with one of the energy levels of a specific quasiparticle in the medium, light is able to create real

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quasiparticles which in turn can radiatively recombine emitting again photons and so on. That is, we have a strong mutual coupling between the light and the quasiparticle. Then the finite-order susceptibility formalism which is by its nature based on perturbation theory will no longer be applicable. Furthermore, a dense system of interacting quasiparticles in highly excited media should cause a non-linearity and thus make the problem a genuine many-body one.

In the present paper, for definiteness, we shall consider a single-mode light field propagating through a highly excited semiconductor modelled as an interacting one-level excition. Thus we have to account for both the exciton–photon and exciton–exciton interactions which are respectively labelled by g and f. The system total Hamiltonian can be formulated as follows:

$$H = Ea^+a + \omega b^+b + g(a^+b + b^+a) + fa^+a^+aa \tag{1}$$

where a and a^+ are the operators of the one-level exciton of energy E. Note that (1) has been used in [11] for the study of the collapse and revival effect of light in a Kerr-type medium. In the case of $f=0$ there is a periodic exchange of statistical properties between excitons and photons [6]. However, squeezing can occur if and only if the exciton–photon system initially possesses some degree of squeezing [6], i.e. squeezing in the output light is impossible when the input light is, e.g., coherent. We are here applying the polariton theory to show that squeezing in the output light becomes possible for non-squeezed inputs if the exciton–exciton interaction is taken into account ($f \neq 0$).

2. Output light squeezing degree

Suppose an input light enters a semiconductor at $z=0$ and leaves it at $z \neq 0$ (z serves as the semiconductor width which is controllable). The semiconductor is considered as an excited one in a generalized sense, i.e. it can by one or another means initially possess a finite exciton density. The semiconductor initial state then may be the coherent ground state [12] with renormalized exciton levels or the usual ground state with no excitons present. To measure the squeezing degree of the output light we introduce two Hermitian z-dependent operators $X(z)$ and $P(z)$, called quadrature operators:

$$X(z) = \tfrac{1}{2}[b^+(z) + b(z)] \qquad P(z) = \tfrac{1}{2}\mathrm{i}[b^+(z) - b(z)]. \tag{2}$$

As is well known, the output light will be squeezed if either of the normally ordered quadrature variances $\langle:(\Delta X(z))^2:\rangle$ or $\langle:(\Delta P(z))^2:\rangle$ becomes negative. Since z is related to t by $z = v_g t$ with v_g being the group velocity of light in the semiconductor, we can instead of the z dependence, consider the corresponding t dependence. The t-dependent normally ordered variance of, e.g., the quadrature $X(t)$ is defined by:

$$\langle:(\Delta X(t))^2:\rangle = \tfrac{1}{4}[\langle b^+(t)b(t)\rangle + \mathrm{Re}\langle b^2(t)\rangle - 2(\mathrm{Re}\langle b(t)\rangle)^2]. \tag{3}$$

Hence, we need to know the explicit t dependence of the operators $b(t)$ and $b^+(t)$. Unfortunately, for the model Hamiltonian given by (1) an exact analytical solution is not possible. In the next section we will go to the polariton representation and try to derive closed analytical expressions for the variance (3) within an approximation.

3. Polariton representation

Before coming in and after going out the semiconductor light and excitons are independent. However, inside the semiconductor the light is coupled to the exciton leading to a renormalization of their energy spectra. By Bogolubov transformations

$$\alpha_\nu = u_\nu b + v_\nu a \qquad \alpha_\nu^+ = u_\nu b^+ + v_\nu a^+ \tag{4}$$

with $\nu = 1, 2$ and real functions

$$u_\nu = \frac{1}{[1 + g^2/(\Omega_\nu - E)^2]^{1/2}} \qquad v_\nu = \frac{g u_\nu}{|\Omega_\nu - E|} \tag{5}$$

where

$$\Omega_\nu = \tfrac{1}{2}\{E + \omega - (-1)^\nu[(E-\omega)^2 + 4g^2]^{1/2}\} \tag{6}$$

one can diagonalize the quadratic part of (1) into the two-branch polariton operators α_ν, α_ν^+ and transmit its last term into that describing the interaction between the polaritons. Namely, Hamiltonian (1) is now

$$H = \sum_\nu \Omega_\nu \alpha_\nu^+ \alpha_\nu + f \sum_{\nu\mu\zeta\xi} v_\nu v_\mu v_\zeta v_\xi \alpha_\nu^+ \alpha_\mu^+ \alpha_\zeta \alpha_\xi \tag{7}$$

with ν, μ, ζ and $\xi = 1$ or 2. Note that the multiple sum over ν, μ, ζ, ξ in (7) contains two kinds of terms. To the first kind belong four terms which are proportional to $\alpha_1^+\alpha_1^+\alpha_1\alpha_1$, $\alpha_1^+\alpha_2^+\alpha_2\alpha_1$, $\alpha_2^+\alpha_1^+\alpha_1\alpha_2$, $\alpha_2^+\alpha_2^+\alpha_2\alpha_2$ and do not oscillate. They are referred to as resonant terms. All the remaining terms belong to the second kind. We call them non-resonant terms in the sense that they oscillate with non-zero frequencies that are not smaller than $2g$. In the following we shall resort to the so-called secular approximation which allows us to neglect all terms of second kind and thus we shall be left only with the resonant terms:

$$H \rightarrow H = \Omega_1\alpha_1^+\alpha_1 + \Omega_2\alpha_2^+\alpha_2 + f_{11}\alpha_1^+\alpha_1^+\alpha_1\alpha_1 + \tfrac{1}{2}f_{12}\alpha_1^+\alpha_2^+\alpha_2\alpha_1 + \tfrac{1}{2}f_{21}\alpha_2^+\alpha_1^+\alpha_1\alpha_2 + f_{22}\alpha_2^+\alpha_2^+\alpha_2\alpha_2 \tag{8}$$

where $f_{11} = fv_1^4$, $f_{12} = f_{21} = 2fv_1^2v_2^2$ and $f_{22} = fv_2^4$. Note that the Hamiltonian (8) resembles the one in [13] describing two modes of light coupled to each other via lossless non-linear media. From (8) we can set up the Heisenberg equations of motion for the operators $\alpha_1(t)$ and $\alpha_2(t)$. Since the numbers of polaritons of each branch are constants of motion (within the secular approximation), the Heisenberg equations can be easily solved yielding explicit t-dependent solutions:

$$\alpha_\nu(t) = \exp\{-\mathrm{i}[\Omega_\nu + 2f_{\nu\nu}\alpha_\nu^+(0)\alpha_\nu(0) + f_{\nu\mu}\alpha_\mu^+(0)\alpha_\mu(0)]t\}\alpha_\nu(0) \tag{9}$$

$$\alpha_\nu^+(t) = \alpha_\nu^+(0)\exp\{\mathrm{i}[\Omega_\nu + 2f_{\nu\nu}^*\alpha_\nu^+(0)\alpha_\nu(0) + f_{\nu\mu}^*\alpha_\mu^+(0)\alpha_\mu(0)]t\} \tag{10}$$

where $\mu \neq \nu$.

The explicit t dependence (9) and (10) and the following transformations (which are inverse with respect to (4))

$$a = \sum_\nu v_\nu \alpha_\nu \qquad a^+ = \sum_\nu v_\nu \alpha_\nu^+ \qquad b = \sum_\nu u_\nu \alpha_\nu \qquad b^+ = \sum_\nu u_\nu \alpha_\nu^+ \tag{11}$$

allow one to calculate the photon quadrature variance (3) in the polariton representation. Anticipate that at $t = 0$ both the exciton and the photon are in their coherent states characterized respectively by complex numbers m_a and m_b, i.e. the initial state of the system reads

$$|m_a, m_b\rangle = D_a(m_a)D_b(m_b)|0,0\rangle \tag{12}$$

where $D_a(m_a)$ and $D_b(m_b)$ are the well known displacement operators with respect to the exciton and the photon:

$$D_c(m_c) = \exp(-\tfrac{1}{2}|m_c|^2)\exp(+m_c c^+)\exp(-m_c^* c) \tag{13}$$

with $c=a$ or b. When $m_b \neq 0$ one has the coherent ground state of the semiconductor [12] and $m_b = 0$ corresponds to the usual semiconductor vacuum. The averaging denoted by $\langle\ldots\rangle$ in (3) must in principle be carried out at a moment $t>0$ when light leaves the semiconductor. Fortunately, as seen from (9) and (10), operators $\alpha_1(t)$, $\alpha_2(t)$ and via (11) and (4) also $a(t)$, $b(t)$ are fully determined by their behaviours at $t=0$. Then, in fact, we have

$$\langle\ldots\rangle \equiv \langle 0,0|D_b^+(m_b)D_a^+(m_a)\ldots D_a(m_a)D_b(m_b)|0,0\rangle \tag{14}$$

or in terms of displacement operators of the polariton

$$\langle\ldots\rangle \equiv \langle 0,0|D_{\alpha_1}^+(u_1 m_b)D_{\alpha_2}^+(u_2 m_b)D_{\alpha_1}^+(v_1 m_a)D_{\alpha_2}^+(v_2 m_a) \\ \ldots D_{\alpha_2}(v_2 m_a)D_{\alpha_1}(v_1 m_a)D_{\alpha_2}(u_2 m_b)D_{\alpha_1}(u_1 m_b)|0,0\rangle. \tag{15}$$

(The vacuum is of course the same in both (a, b) and (α_1, α_2) representations.) Further, with the aid of (11) we can transform the RHS of (3) into the polariton representation as follows

$$\langle:(\Delta X(t))^2:\rangle = \tfrac{1}{2}u_1^2[\langle\alpha_1^+(t)\alpha_1(t)\rangle + \mathrm{Re}\langle\alpha_1^2(t)\rangle - 2(\mathrm{Re}\langle\alpha_1(t)\rangle)^2] \\ + \tfrac{1}{2}u_2^2[\langle\alpha_2^+(t)\alpha_2(t)\rangle + \mathrm{Re}\langle\alpha_2^2(t)\rangle - 2(\mathrm{Re}\langle\alpha_2(t)\rangle)^2] \\ + u_1u_2[\mathrm{Re}\langle\alpha_1^+(t)\alpha_2(t)\rangle + \mathrm{Re}\langle\alpha_1(t)\alpha_2(t)\rangle - 2\mathrm{Re}\langle\alpha_1(t)\rangle\mathrm{Re}\langle\alpha_2(t)\rangle]. \tag{16}$$

Now, utilizing some properties of displacement operators such as (c, arbitrary bosonic operator; λ, arbitrary real number; α, β, γ, δ, arbitrary complex numbers):

$$D_c^+(\alpha)cD_c(\alpha) = c + \alpha \tag{17}$$

$$D_c^+(\alpha)c^+D_c(\alpha) = c^+ + \alpha^* \tag{18}$$

$$\mathrm{e}^{\mathrm{i}\lambda c^+c}D_c(\alpha)\,\mathrm{e}^{-\lambda c^+c} = D_c(\alpha\mathrm{e}^{\mathrm{i}\lambda}) \tag{19}$$

$$G(\alpha;\beta) \equiv \langle 0|D_c^+(\alpha)D_c(\beta)|0\rangle = \exp[-\tfrac{1}{2}|\alpha-\beta|^2 + \mathrm{i}\mathrm{Im}(\alpha^*\beta)] \tag{20}$$

$$G(\alpha,\beta;\gamma,\delta) \equiv \langle 0|D_c(\alpha)D_c^+(\beta)D_c(\gamma)D_c(\delta)|0\rangle = \exp[-\tfrac{1}{2}|\alpha+\beta-\gamma-\delta|^2 \\ + \mathrm{i}\mathrm{Im}(\alpha^*\gamma + \alpha^*\delta + \beta^*\gamma + \beta^*\delta + \beta^*\delta + \beta^*\alpha + \delta^*\gamma)]. \tag{21}$$

We can analytically derive expressions for various averaged values of operators α_1, α_2 and their products that enter (16). Namely, they are of the forms

$$\langle\alpha_1^+(t)\alpha_1(t)\rangle = |v_1m_a + u_1m_b|^2 A(v_1m_a, u_1m_b, 2(f_{11}-f_{11}^*)t)A(v_2m_a, u_2m_b, (f_{12}-f_{12}^*)t) \tag{22}$$

$$\langle\alpha_1(t)\rangle = \mathrm{e}^{-\mathrm{i}\Omega_1 t}(v_1m_a + u_1m_b)A(v_1m_a, u_1m_b, 2f_{11}t)A(v_2m_a, u_2m_b, f_{12}t). \tag{23}$$

$$\langle\alpha_1^2(t)\rangle = \mathrm{e}^{-2\mathrm{i}(\Omega_1+f_{11})t}(v_1m_a + u_1m_b)^2 A(v_1m_a, u_1m_b, 4f_{11}t)A(v_2m_a, u_2m_b, 2f_{12}t) \tag{24}$$

$$\langle\alpha_2^+(t)\alpha_2(t)\rangle = |v_2m_a + u_2m_b|^2 A(v_2m_a, u_2m_b, 2(f_{22}-f_{22}^*)t)A(v_1m_a, u_1m_b, (f_{21}-f_{21}^*)t) \tag{25}$$

$$\langle\alpha_2(t)\rangle = \mathrm{e}^{-\mathrm{i}\Omega_2 t}(v_2m_a + u_2m_b)A(v_2m_a, u_2m_b, 2f_{22}t)A(v_1m_a, u_1m_b, f_{21}t) \tag{26}$$

$$\langle a_2^2(t)\rangle = e^{-2i(\Omega_2+f_{22})t}(v_2m_a+u_2m_b)^2A(v_2m_a, u_2m_b, 4f_{22}t)A(v_1m_a, u_1m_b, 2f_{21}t) \tag{27}$$

$$\langle a_1^+(t)a_2(t)\rangle = e^{-i(\Omega_2-\Omega_1)t}(v_1m_a^*+u_1m_b^*)(v_2m_a+u_2m_b) \times A(v_1m_a, u_1m_b, (f_{12}-2f_{11}^*)t)A(v_2m_a, u_2m_b, (2f_{22}-f_{12}^*)t) \tag{28}$$

$$\langle a_1(t)a_2(t)\rangle = e^{-i(\Omega_1+\Omega_2+f_{12})t}(v_1m_a+u_1m_b)(v_2m_a+u_2m_b) \times A(v_1m_a, u_1m_b, (f_{12}+2f_{11})t)A(v_2m_a, u_2m_b, (2f_{22}+f_{12})t) \tag{29}$$

where

$$A(x, y, \tau) \equiv G(xe^{i\tau}, ye^{i\tau}; y, x). \tag{30}$$

Taking the real parts of (23), (24) and (26) to (29) then substituting them into (16) we shall get for the normally ordered quadrature variance $\langle:(\Delta X(t))^2:\rangle$ a lengthy (but quite easy to handle numerically) analytical formula which will be illustrated graphically in the next section.

4. Graphical illustration

To demonstrate whether light squeezing is possible we shall in this section graphically illustrate our formulae derived analytically in the previous section. We shall use concrete material and initial state parameters E, ω, g, f, m_a and m_b. Typically, for semiconductors $E \gg g > f$. For example, CdS has $E \simeq 2550$ meV, $g \simeq 51.85$ meV and $f \simeq 5.4$ meV, i.e. $g/E \simeq 0.02$ and $f/E \simeq 0.002$. We shall therefore perform the numerical calculation for g/E (f/E) ranging between 0.001 (0) and 0.1 (0.002). In figure 1 we plot $\langle:(\Delta X(t))^2:\rangle$ (written on the graphs as X-quadrature variance) against Et (written on the graphs as scaled time) for the perfect resonance case ($\omega = E$) and $g/E = 0.02$, $f/E = 0.002$. Two interesting points could be remarked: (i) from (22) to (29) it is evident that $m_a(m_b)$ always goes together with $v_\nu(u_\nu)$. As can be checked easily from

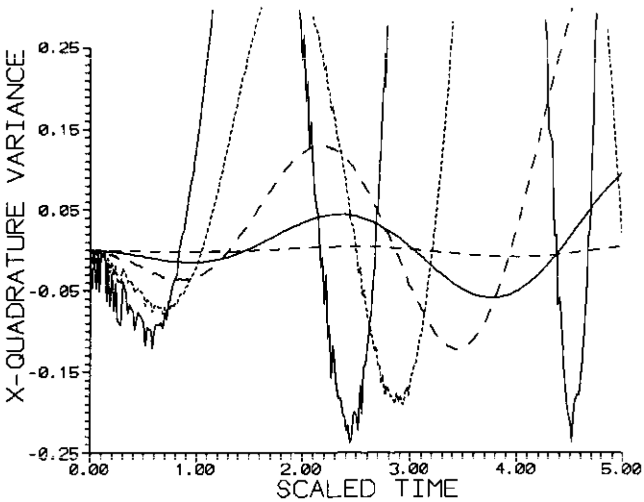

Figure 1. The normally ordered X-quadrature variance of the photon plotted against the scaled time Et for $\omega/E = 1$, $g/E = 0.02$, $f/E = 0.002$, $m_a + m_b = 2$, 6, 10, 16 and 21 corresponding to increasing antipeaks' depth.

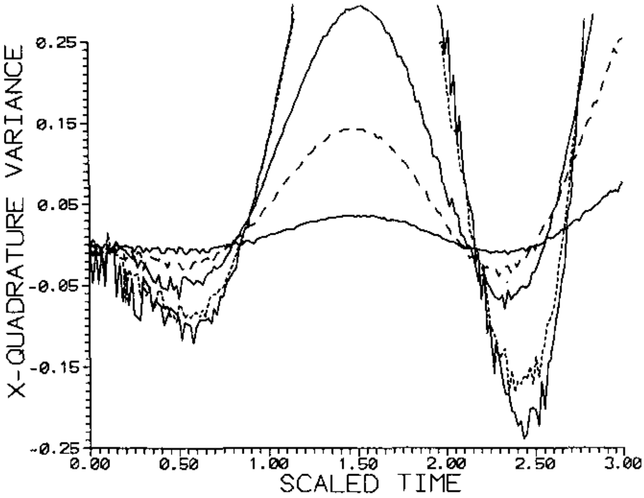

Figure 2. The same as in figure 1 but $m_a = m_b = 10.5$ and ω/E varies. Corresponding to increasing antipeaks' depth we have $\omega/E = 0.9$, 0.95, 0.97, 0.99 and 1.

(5) and (6), the coefficients u_ν, v_ν as functions of ω/E vary distinctly in general. However, at $\omega = E$ all they become coincident and equal to $\sqrt{1/2}$. So, for $\omega = E$ we can put $u = u_1 = u_2 = v_1 = v_2$ and pull out u as a common multiplier in the formulae (22)–(29). Then these formulae will depend only on the sum of $(m_a + m_b)$ but not on either of them separately. That is why figure 1 is plotted for different values of $m = m_a + m_b$; (ii) the larger the initial exciton (m_a) or/and photon (m_b) numbers the deeper the antipeaks, i.e. the higher the degree of obtainable squeezing of light. Figure 2 represents the same relationship as in figure 1 but with regard to the dependence on detuning. m_a and m_b are fixed equal to 10.5. The best squeezing (the deepest antipeak) is observed when $\omega = E$. If one is leaving the perfect resonance

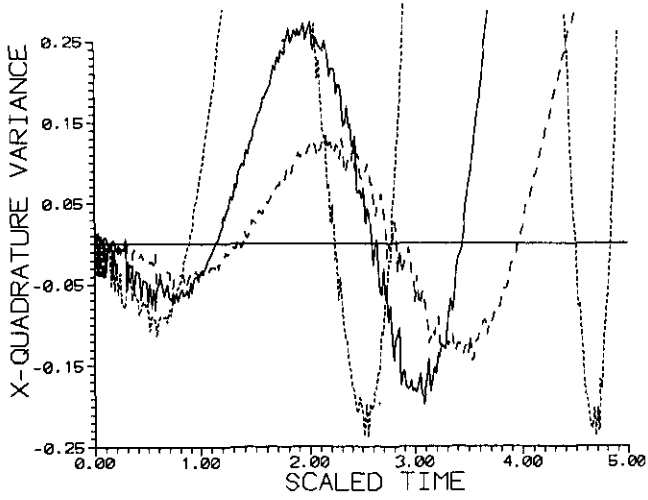

Figure 3. The same as in figure 1 but $m_a = m_b = 10$ and $f/E = 0$ (horizontal line), 0.0005 (long-broken curve), 0.001 (full curve) and 0.002 (short-broken curve).

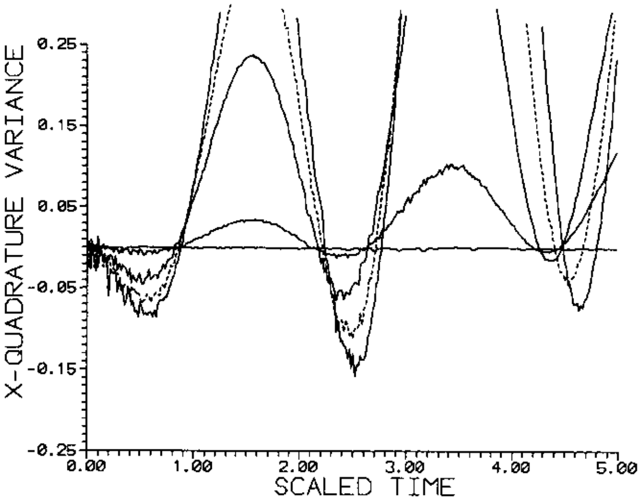

Figure 4. The same as in figure 1 but $m_a = m_b = 10$, $\omega/E = 0.95$, $g/E = 0.001$ (nearly horizontal line), 0.01 (long-broken curve), 0.03 (full curve with shallower antipeak), 0.05 (short-broken curve) and 0.1 (full curve with deeper antipeak).

region the squeezing decreases rapidly. For $\omega/E = 0.9$ the squeezing almost disappears. For $\omega/E < 0.9$ the squeezing turns out to be negligible. Further, to assess the role of the exciton–exciton interaction we change the parameter f/E while keeping $m_a = m_b = 10$, $g/E = 0.02$. For $f = 0$ we get a straightforward horizontal line showing as expected that no squeezing can occur. Namely, if in (1) $f = 0$ then, as shown in [6], no output squeezing happen for coherent input. For $f \neq 0$, however, the output squeezing becomes possible even for coherent input and the squeezing manifests itself as more pronounced for larger f (see figure 3). Finally, we pay attention to the exciton–photon interaction g. Of course, g plays a very important role, too. g connects the photon with the exciton making the polariton picture meaningful. However, the influence of g on the light squeezing degree is almost quenched in the case of perfect resonance ($\omega = E$). This can be understood from the following argument. Mathematically, g just enters the expressions of u_ν, v_ν and Ω_ν. When $\omega = E$, as already mentioned above, u_ν and v_ν are g independent. Then g may affect light squeezing only through Ω_ν. Since at $\omega = E$ the polariton energies $\Omega_\nu = E - (-1)^2 g$ and $g \ll E$, one can then ignore the role of g. On the other side, for $\omega < E$ the coefficients u_ν and v_ν become strongly g dependent and thus light squeezing might be heavily influenced by g. This can be seen from figure 4 where ω/E is taken to be equal to 0.95. We see that larger values of g favour light squeezing.

5. Conclusion

We have analytically derived a closed expression for X-quadrature variance of photons in the interacting exciton–photon system. Figures graphed from the expression have illustrated that squeezing of output light is possible for a coherent input light if both g and f differ from zero. To detect the output light with a large degree of squeezing one may adjust the input light intensity and prepare beforehand a finite concentration of excitions as well as control the length of the semicondcutor slab through which the light passes. Concerning the semiconductor length suitable for observing a good squeezing degree two points are in order. First, the evolution of the X-quadrature variance is in general a quasi-periodic function of time. Thus there is a 'discrete spectrum' of the values of semiconductor length corresponding to deep antipeaks of the X-quadrature variance, i.e. to good squeezing. Second, the shortest length (thinnest width) must correspond to the first deep antipeak which as seen from the figures occurs at about Et_1 (scaled time) $\simeq 2.5$. In the case of $E=\omega$ the polariton group velocity is $v_g \simeq 1/2\sqrt{\varepsilon}$ with ε being the semiconductor static dielectric constant. If one takes $E \simeq 2.5$ eV and $\varepsilon \simeq 9$, the shortest semiconductor length z_1 is then defined by $z_1 = t_1 v_g \simeq 3\times 10^{-6}$ cm which is of the size of a semiconductor quantum well. Finally, as compared with the mechanisms treated in [8–10], in our case a good degree of squeezing is already obtainable when the number of the initial input photons is of the order of 10 (whereas in a Kerr-type medium [10] the input photon number must be as large as 10^6 (10^{12} or 10^{18}) to expect the squeezing effects connected with $\chi^{(3)}$ ($\chi^{(5)}$ or $\chi^{(7)}$)).

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References

[1] Loudon R and Knight P L 1987 *J. Mod. Opt.* **34** 709
[2] Teich M C and Saleh B E A 1989 *Quantum Opt.* **1** 153
[3] Carter S J and Drummond P D 1987 *Phys. Rev. Lett.* **58** 1841
[4] Artoni M and Birman J L 1989 *Quantum Opt.* **1** 91
[5] Feinberg D, Ciuchi S and Pasquale F de 1990 *Int. J. Mod. Phys.* B **4** 1317
[6] Nguyen Ba An 1991 *Mod. Phys. Lett.* **5B** 587
[7] Nguyen Ba An 1992 *Int. J. Mod. Phys.* B **6** 395
[8] Tanas R and Kielich S 1983 *Opt. Commun.* **45** 351; 1984 *Opt. Acta* **31** 81
[9] Gerry C C 1987 *Phys. Lett.* *124* 237
[10] Tanas R and Kielich S 1990 *Quantum Opt.* **2** 23
[11] Agarwal G S and Puri R R 1989 *Phys. Rev.* A **39** 2969
[12] Schmitt-Rink S, Chemla D S and Haug H 1988 *Phys. Rev.* B **37** 941
[13] Buzek V and Jex I 1990 *Int. J. Mod. Phys.* B **4** 659