A Probability Conference

Random Matrices and Related Topics

May 6 – 10, 2019

KIAS, Seoul, Korea

Organizers

Paul Jung (KAIST)
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Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
10:00 - 11:00	Hedenmalm	Baik	Shirai	Chafaï	Akemann
11:00 - 12:00	Nadakuditi	Ameur	Lee	Duits	Schnelli
			Lunch		
13:30 - 14:30	Fan	Feng	Free Time	Bao	Sosoe
14:30 - 15:30	Ghosh	Wennman	Free Time	Heiny	Ji
	Coffee Break and Discussion		Free Time	Coffee Break and Discussion	
15:50 - 16:50	Lodhia	Wang	Free Time	Seo	
17:30 - 19:30		Banquet			

Monday

10:00 - 11:00	Håkan Hedenmalm	Off-spectral analysis of Bergman kernels
11:00 - 12:00	Raj Rao Nadakuditi	Free Component Analysis
		Lunch and Discussion
13:30 - 14:30	Zhou Fan	Principal components and linear mixed models
14:30 - 15:30	Subhro Ghosh	Two manifestations of rigidity in point sets: forbidden regions and maximal degeneracy
		Coffee Break and Discussion
15:50 - 16:50	Asad Lodhia	Harmonic Means of Wishart Matrices

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Off-spectral analysis of Bergman kernels

Håkan Hedenmalm

KTH Royal Institute of Technology

Abstract

Standard methods of asymptotic expansion of Bergman kernels are local. Here we develop a nonlocal method of a novel kind, inspired by recent progress on orthogonal polynomials. This reports on joint work with A. Wennman.

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Free Component Analysis

Raj Rao Nadakuditi

University of Michigan

Abstract

We describe a method for unmixing mixtures of freely independent random variables in a manner analogous to the independent component analysis (ICA) based method for unmixing independent random variables from their additive mixture. Random matrices play the role of free random variables in this context so the method we develop, which we call Free component analysis (FCA), unmixes matrices from an additive mixture of matrices. We describe the theory – the various 'contrast functions', computational methods and compare FCA to ICA on data derived from real-world experiments. This is joint work with Hao Wu.

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Principal components and linear mixed models

Zhou Fan

Yale University

Abstract

In this talk, I'll describe some applications of random matrix theory to the study of covariance estimators in high-dimensional random effects linear models. These models are commonly used in statistical genetics to model different "levels" of variation in quantitative phenotypic traits. I'll discuss some of the applied and statistical interest in these models, and a few general results in random matrix theory which we developed in the course of studying spectral behavior in this application.

This is joint work with Iain M. Johnstone, Yi Sun, and Zhichao Wang.

Two manifestations of rigidity in point sets: forbidden regions and maximal degeneracy

Subhroshekhar Ghosh

National University of Singapore

Abstract

A point process is said to be "rigid" if its local observables are completely determined (as deterministic functions of) the point configuration outside a local neighbourhood. For example, it has been shown in earlier work that, in the Ginibre ensemble (a.k.a. the $2D$ Coulomb gas at inverse temperature $\beta = 2$), the point configuration outside any bounded domain determines, almost surely, the number of points in the domain.

In this talk, we will explore two recent manifestations of such rigidity phenomena. For the zeros of the planar Gaussian analytic function, we show that outside every large "hole", there is a "forbidden region" which contains a vanishing density of points. This should be seen in contrast with the corresponding situation for classically understood models (e.g. random matrix ensembles), where no such effects are known to occur.

In the second part of the talk, we will consider "stealthy" hyperuniform systems, which are systems whose structure function (i.e., the Fourier transform of thetwo-point correlation) vanishes near the origin. We show that such systems exhibit "maximal degeneracy", namely the points outside a bounded domain determine, almost surely, the entire point configuration inside the domain. En route, we establish a conjecture of Zhang, Stillinger and Torquato that such systems have (deterministically) bounded holes.

Based on joint works with Joel Lebowitz and Alon Nishry.

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Harmonic Means of Wishart Matrices

Asad Lodhia

University of Michigan

Abstract

Given a sequence of n Wishart matrices of aspect ratio P by N we show that the harmonic mean of these matrices is closer in operator norm to the population than the arithmetic mean so long is n is small enough. Joint work with Keith Levin and Liza Levina.

Tuesday

10:00 - 11:00	Jinho Baik	Spherical Sherrington-Kirkpatrick model and random matrix
11:00 - 12:00	Yacin Ameur	On the interface between Hermitian and normal eigenvalue ensembles
		Lunch and Discussion
13:30 - 14:30	Renjie Feng	Extreme gap problems in random matrix theory
14:30 - 15:30	Aron Wennman	Planar orthogonal polynomials and boundary universality for determinantal Coulomb gas
		Coffee Break and Discussion
15:50 - 16:50	Ke Wang	Random perturbation of low-rank matrices
17:30 - 19:30		Banquet

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Spherical Sherrington-Kirkpatrick model and random matrix

Jinho Baik

University of Michigan

Abstract

The Spherical Sherrington-Kirkpatrick (SSK) model is defined by the Gibbs measure on a high-dimensional sphere with a random Hamiltonian given by a symmetric quadratic function. The free energy at the zero temperature is same as the largest eigenvalue of the random matrix associated to the quadratic function. Even for the finite temperature, there is a simple relationship between the free energy and the eigenvalues. We will discuss how one can study the fluctuations of the free energy using this relationship and results from random matrix theory. We will also discuss the distribution of the spin sampled from the Gibbs measure.

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On the interface between Hermitian and normal eigenvalue ensembles

Yacin Ameur

Lund University

Abstract

The weakly Hermitian regime provides a natural bridge between Hermitian and normal random matrix theories. This interface was studied in an important paper of Fyodorov, Khoruzhenko and Sommers from 1998, and has later been the subject of a number of further investigations. In my talk I will discuss ensembles of a similar appearance, in particular from the point of view of Ward identities. The talk concerns work in progress, and is based on discussions with Sungsoo Byun, Gernot Akemann, and others.

Extreme gap problems in random matrix theory

Renjie Feng

Peking University

Abstract

The spacing between eigenvalues of classical random matrices is one of the most essential problems in random matrix theory. We will present three results we solved recently on the extreme spacing problems, including the rescaling laws of the smallest spacing of CβE and GOE, the largest spacing of CUE and GUE. We will also talk about some open problems. This is the joint work with Dongyi Wei and Gang Tian.

Planar orthogonal polynomials and boundary universality for determinantal Coulomb gas

Aron Wennman

Tel Aviv University

Abstract

The main objects of this talk are the planar orthogonal polynomials associated to an exponentially varying weight in the plane. The associated (weighted) reproducing kernel is the correlation kernel for two-dimensional Coulomb gas in an external field, in the special case when the inverse temperature equals two. This Coulomb gas model is well studied and well-understood in the bulk regime. The boundary behavior, however, has proven more difficult to obtain for general weights.

In this talk, I will attempt to explain how to obtain the asymptotics of the ONPs as the degree tends to infinity along with the weight parameter, and how this asymptotic information may be used to obtain boundary universality of the above random processes.

Of particular interest is a flow of simple curves, which induces a foliation of a region containing the boundary into smooth simple curves. The key property of the curve family is that one and the same polynomial is, in a strong approximate sense, orthogonal to lower order polynomials with respect to a family of measures on these curves, weighted by the normal velocity of the flow. This allows us to foliate planar orthogonality into a one-dimensional problem.

This is based on joint work with Håkan Hedenmalm.

Random perturbation of low-rank matrices

Ke Wang

Hong Kong University of Science and Technology

Abstract

Computing the singular values and singular vectors of a large matrix is a basic task in high dimensional data analysis with many applications in computer science and statistics. In practice, however, data is often perturbed by noise. In this talk, we consider the matrix model $Y = S + X$ where S is a low-rank deterministic matrix, representing the signal, and X is random noise. We give a precise description of the limiting distribution of the angles between the outlier singular vectors of Y with their counterparts, the leading singular vectors of S. It turns out that the limiting distribution depends on the structure of S and the distribution of X, and thus it is non-universal. This talk is based on joint work with Zhigang Bao and Xiucai Ding.

Wednesday

10:00 - 11:00	Tomoyuki Shirai	Limit theorems for determinantal point processes
11:00 - 12:00	Seung-Yeop Lee	Asymptotics of planar orthogonal polynomials corresponding to Ginibre ensemble perturbed by finite point charges
		Lunch

Limit theorems for determinantal point processes

Tomoyuki Shirai

Kyushu University

Abstract

The unitary group of size n together with the Haar probability measure is called Circular Unitary Ensemble of size n. All the eigenvalues lie on the unit circle in the complex plane and they can be regarded as a determinantal point process on the unit circle. It is also known that the scaled point processes converge weakly to the determinantal point process associated with the so-called sine kernel as n tends to ∞. In this talk, we discuss a generalization of this fact and also some related topics on determinantal point processes. This talk is based on a joint work with Makoto Katori (Chuo University).

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Asymptotics of planar orthogonal polynomials corresponding to Ginibre ensemble perturbed by finite point charges

Seung-Yeop Lee

University of South Florida

Abstract

The limiting distribution of the roots of planar orthogonal polynomials, is unknown except a few cases. Among the few known cases is the case when the Gaussian weight is perturbed by a single logarithmic singularity in the external potential. In such case the limiting zeros are supported on the Szegö curve. We extend this result to a family of external potentials with an arbitrary, finite number of singularities. We found that the limiting zero locus forms a certain combination of Szegö curves, that we may call Multiple Szegö curve. This work is a joint work with Meng Yang.

Thursday

10:00 - 11:00	Djalil Chafaï	Dynamics of gases of particles with singular repulsion
11:00 - 12:00	Maurice Duits	Periodically weighted tilings and (matrix) orthogonal polynomials
		Lunch and Discussion
13:30 - 14:30	Zhigang Bao	Spectral rigidity for addition of random matrices at the regular edge
14:30 - 15:30	Johannes Heiny	Assessing the dependence of high-dimensional time series via autocovariances and autocorrelations
		Coffee Break and Discussion
15:50 - 16:50	Seong-Mi Seo	Random normal matrix models with a soft edge of the spectrum

Dynamics of gases of particles with singular repulsion

Djalil Chafaï

CEREMADE / U. Paris-Dauphine / PSL

Abstract

We will present stochastic dynamics associated to gases of interacting particles, issued or inspired from random matrix theory and statistical physics. These dynamics are natural objects of interest and in the same time tools for sampling. High dimensional phenomena play an important role.

Periodically weighted tilings and (matrix) orthogonal polynomials

Maurice Duits

KTH Royal Institute of Technology

Abstract

In this talk I will discuss an approach for studying random tilings of planar domains that is particularly useful for asymptotic studies of periodically weighted tilings, such as the two periodic Aztec diamond. The backbone is a double integral formula for the correlation functions for determinantal point processes defined by products of minors of (block) Toeplitz matrices. In general, the integrand is expressed in terms of (matrix) orthogonal polynomials. In case of infinite minors the asymptotics of these (matrix) orthogonal polynomials can be computed explicitly and the double integrals can be further simplified, leading to explicit formulas for models such as the two periodic Aztec diamond.

Spectral rigidity for addition of random matrices at the regular edge

Zhigang Bao

Hong Kong University of Science and Technology

Abstract

In this talk, we consider the sum of two large Hermitian matrices A and B with a Haar unitary conjugation bringing them into a general relative position. The pioneering work of Voiculescu showed that the spectral distribution of this random matrix model is asymptotically given by the free additive convolution of the laws of A and B as the dimension of the matrix increases. Assuming the typical square-root behavior at the edge of the limiting free additive convolution, we show that the Voiculescu's law holds locally on the scale slightly above the local eigenvalue spacing at the edge. Together with our previous result in the bulk, this implies the optimal rigidity of the eigenvalues and optimal rate of convergence of Voiculescu's law. The key element of our proof is to compensate the deterioration of the stability of the subordination equations by sharp error estimates that properly account for the local density near the edge. This talk is based on the joint work with László Erdős and Kevin Schnelli.

Assessing the dependence of high-dimensional time series via autocovariances and autocorrelations

Johannes Heiny

Ruhr-University Bochum

Abstract

In the first part of this talk, we provide asymptotic theory for certain functions of the sample autocovariance matrices of a high-dimensional time series with infinite fourth moment. The time series exhibits linear dependence across the coordinates and through time. Assuming that the dimension increases with the sample size, we provide theory for the eigenvectors of the sample autocovariance matrices and find explicit approximations of a simple structure, whose finite sample quality is illustrated for simulated data. We also obtain the limits of the normalized eigenvalues of functions of the sample autocovariance matrices in terms of cluster Poisson point processes. In turn, we derive the distributional limits of the largest eigenvalues and functionals acting on them.

In the second part, we consider the sample correlation matrix R associated to n observations of a p-dimensional time series. In our framework, we allow that p/n may tend to 0 or a positive constant. If the time series has a finite fourth moment, we show that the sample correlation matrix can be approximated by its sample covariance counterpart for a wide variety of models. This result is very important for data analysts who use principal component analysis to detect some structure in high-dimensional time series. From a theoretical point of view, it allows to derive a plethora of ancillary results for functionals of the eigenvalues of R. For instance, we determine the almost sure behavior of the largest and smallest eigenvalues, and the limiting spectral distribution of R.

The optimal condition for the convergence of the empirical spectral distributions turns out to be slightly weaker than normal domain of attraction. In the case of time series with infinite $(2-\epsilon)$-moments, a new class of Marchenko–Pastur type laws appears as limiting spectral distributions of R.

The talk is based on joint work with Thomas Mikosch (University of Copenhagen), Jianfeng Yao (University of Hong Kong) and Richard Davis (Columbia University New York).

Random normal matrix models with a soft edge of the spectrum

Seong-Mi Seo

KIAS

Abstract

In this talk, I will introduce random normal matrix ensembles with a new type of boundary conditions. This model interpolates the free boundary case, the hard edge case, and the weakly confining case. Adapting the asymptotic expansions of planar orthogonal polynomials due to Hedenmalm and Wennman, we obtain the scaling limit of the ensemble at the edge of the spectrum and prove edge universality when the underlying potential is radially symmetric. This is based on joint work with Yacin Ameur and Nam-Gyu Kang.

Friday

10:00 - 11:00	Gernot Akemann	Gegenbauer and other orthogonal planar polynomials on an ellipse: orthogonality and asymptotic results
11:00 - 12:00	Kevin Schnelli	Tracy-Widom limit for sample covariance matrices
		Lunch and Discussion
13:30 - 14:30	Philippe Sosoe	Fluctuations of the overlap in 2-spin spherical spin glasses
14:30 - 15:30	Hong Chang Ji	Properties of Free Multiplicative Convolution

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Gegenbauer and other orthogonal planar polynomials on an ellipse: orthogonality and asymptotic results

Gernot Akemann

Bielefeld University

Abstract

We show that Gegenbauer polynomials also called ultraspherical form a family of planar orthogonal polynomials on an ellipse, enlarging the known families of planar Hermite, Laguerre or Chebyshev polynomials. This leads to a family of determinantal point process of N charged particles inside an ellipse, subject to a certain potential. In the large-N limit at weak non-Hermiticity in the bulk and at the edge of the spectrum we find several new universality classes, extending the known deformations of the universal sine- and Bessel-kernel of Hermitian random matrix theory.

Tracy-Widom limit for sample covariance matrices

Kevin Schnelli

KTH Royal Institute of Technology

Abstract

We consider the edge statistics of sample covariance matrices in the asymptotic regime when both sample size and dimensionality tend to infinity. We discuss a model where the entries are rescaled Bernoulli random variables with parameter p and show, under a suitable assumption on p, that the rescaled largest eigenvalues exhibit GOE Tracy-Widom fluctuations provided that a deterministic shift of the spectral edge is included. Joint work with Jong Yun Hwang and Ji Oon Lee.

Fluctuations of the overlap in 2-spin spherical spin glasses

Philippe Sosoe

Cornell University

Abstract

In a series of recent papers, J. Baik and J. O. Lee have used a contour integral representation for the partition function of the 2-spin spherical Sherrington-Kirkpatrick model to analyze its fluctuations via random matrix theory, including in the low temperature phase. Such results remain largely inaccessible for general p-spin models and the SK model with Ising spins.

Using a similar representation for functions of overlaps between replicas, we study the fluctuations of the overlap between two replicas in the 2-spin spherical SK model. We prove a quenched CLT for the overlaps in the high temperature phase but close to the critical temperature. We also show that the fluctuations of the overlap in the low temperature phase are of order $N^{-1/3}$ and are given by a simple, explicit function of the eigenvalues of GOE matrix. We show that this quantity converges and describe its limit in terms of quantities from random matrix theory. Joint work with B. Landon and V.L. Nguyen.

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Properties of Free Multiplicative Convolution

Hong Chang Ji

KAIST

Abstract

For given two Borel probability measures μ and ν on the positive half line, we will present some properties of the free multiplicative convolution $\mu \boxtimes \nu$ via its Cauchy-Stieltjes transform. In particular we proved that $\mu \boxtimes \nu$ always has no singular continuous part and, under certain conditions, that the density of its absolutely continuous part is bounded by x^{-1}. We will also consider a special case in which μ and ν are compactly supported Jacobi measures on $(0, \infty)$ having power law behavior with exponents in $(-1, 1)$. In this case, we proved that $\mu \boxtimes \nu$ is another such Jacobi measure whose density has square root decay at the edges of its support.