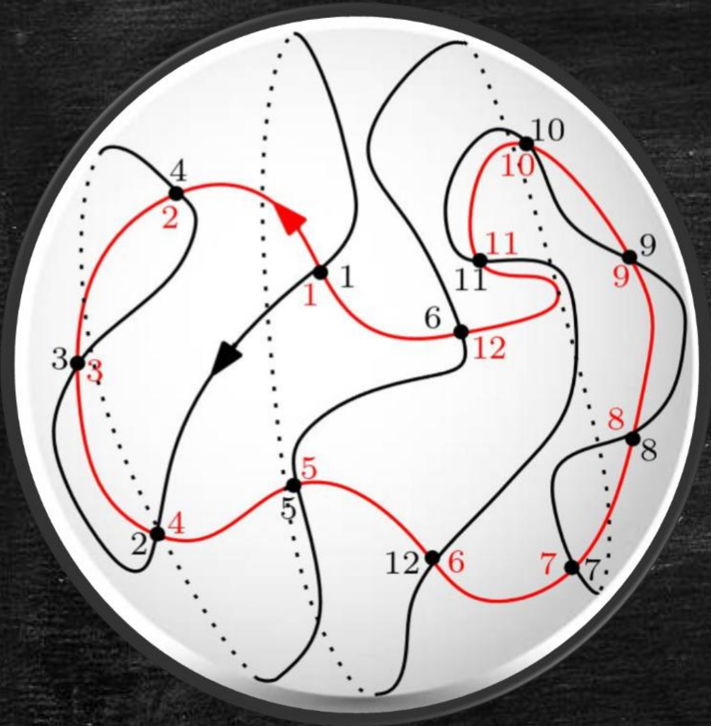 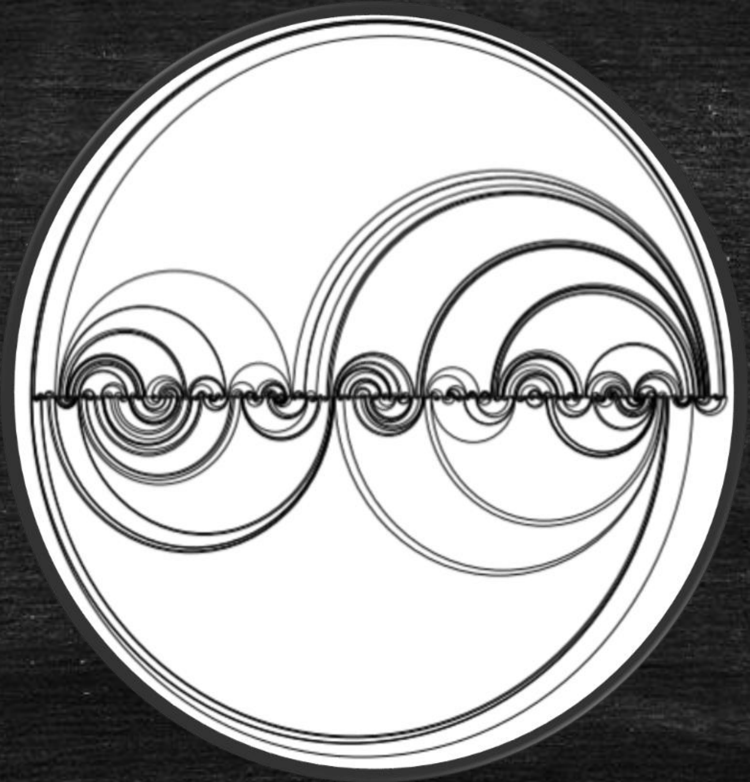 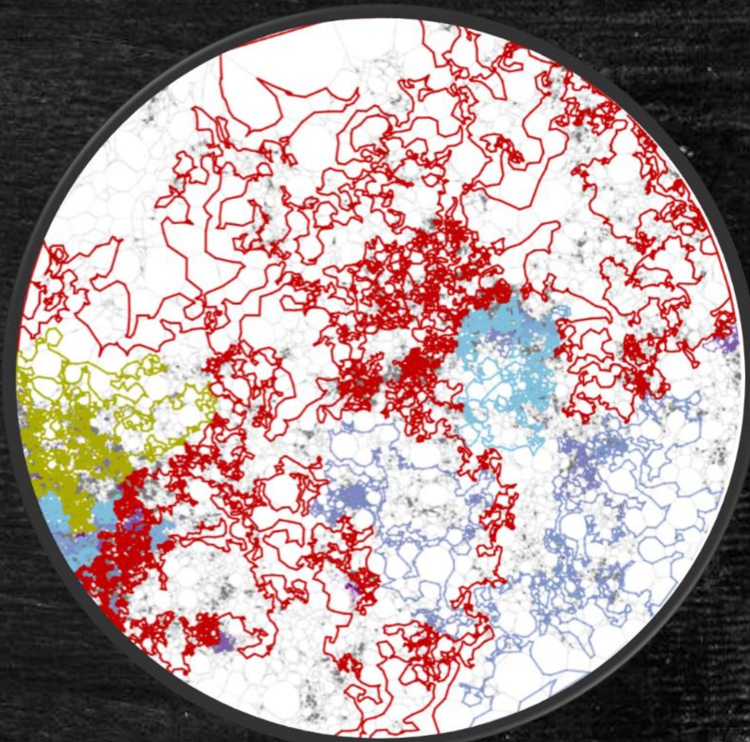

Meanders & Meandric Systems

(joint works with E. Gwynne, M. Park, and X. Sun)

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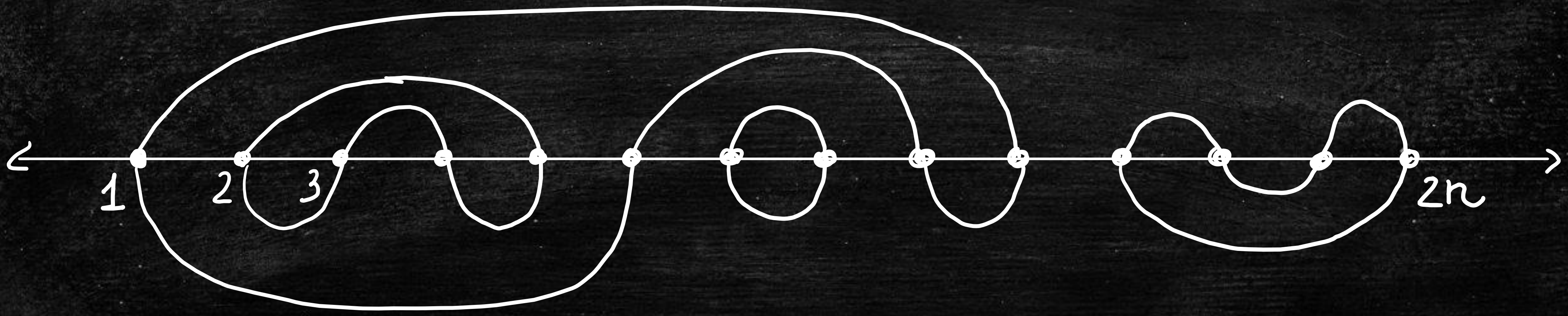
Jeju, June 6, 2023

Meandric systems

Def: A MEANDRIC SYSTEM of size $n \in \mathbb{N}$ is a collection of disjoint simple loops in $\mathbb{R}^2$ which orthogonally cross $\mathbb{R}$, precisely at the points $\{1, \dots, 2n\}$. Configurations are viewed modulo homeomorphisms fixing $\mathbb{R}$.

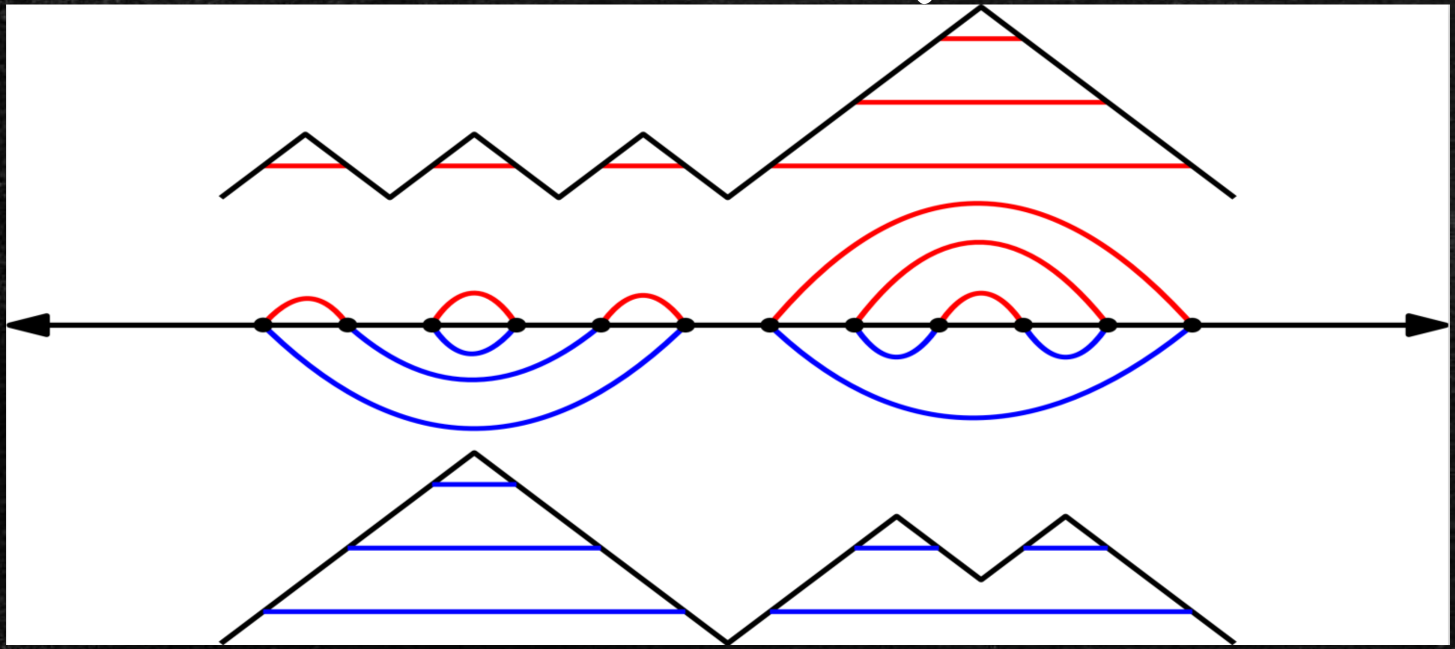


- This model is equivalent to:

  CROSSING FULLY PACKED O(O×1) loop model on PLANAR MAPS

- STUDIED BY : Korgin, Féray-Thévenin, Curien-Kozma-Sidoravicious-Tournier
  Golden-Nica-Puder, Fukuda-Nechita, Janson-Thévenin, etc...

- How can we sample a uniform meandric system of size 2n?



ISSUE: Loops are a very complicated functional of the 2 walks

BASIC QUESTIONS are still OPEN:

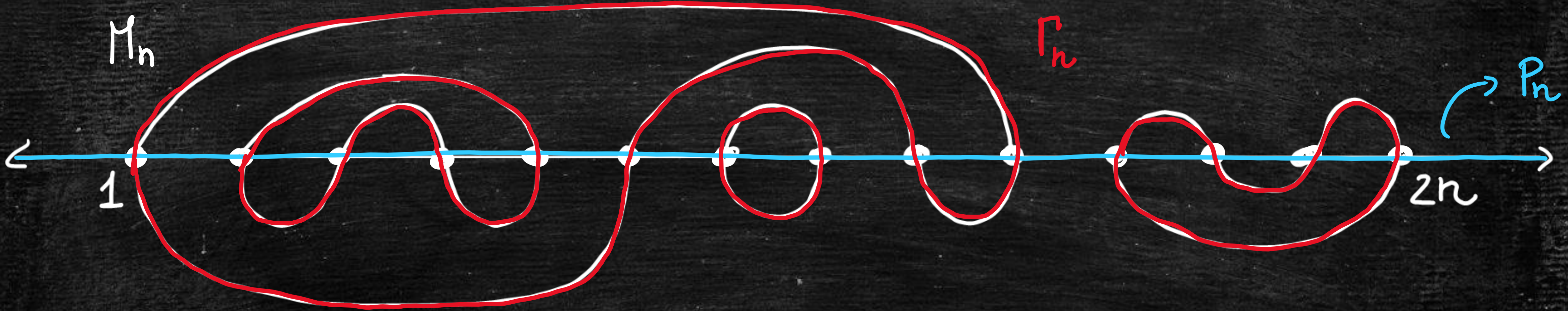
Is there an infinite loop?

How does a large uniform meandric system look like?

We view a meandric system as a

PLANAR MAP + HAMILTONIAN PATH + LOOPS

M_n $\qquad$ P_n $\qquad$ Γ_n



CONJECTURE: (B., Gwynne, Park, '22)

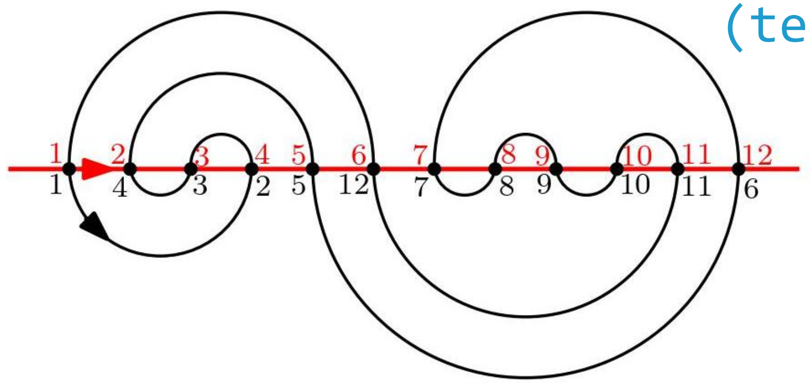
(M_n, P_n, Γ_n) converges under an appropriate scaling limit to a

$\sqrt{2}$ - LQG - measure + SLE_8 + CLE_6

What happens if we require to have a single loop?

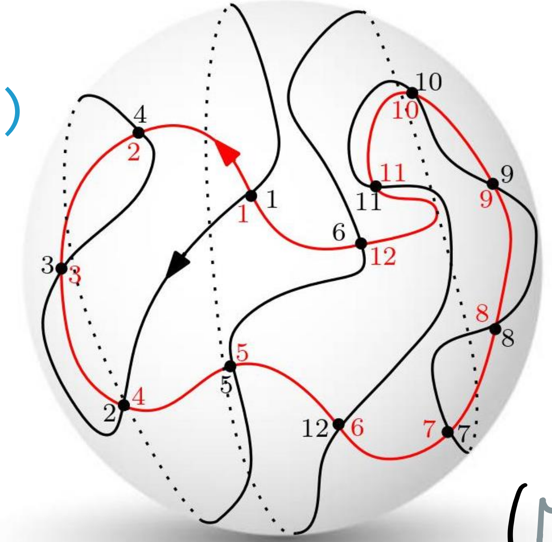
Meanders

(term coined by V.Arnold)



$\sigma_n = 1\ 4\ 3\ 2\ 5\ 12\ 7\ 8\ 9\ 10\ 11\ 6$

There is no "efficient" algorithm to sample uniform meanders



(M_n, P_n^1, P_n^2)

LITERATURE: (Survey papers)

- Zvonkin, 2021, "Meanders: A personal perspective"
- La Croix, 2003, "Approaches to the enumerative theory of meanders"

Connections to many different subjects: COMBINATORICS, THEORETICAL PHYSICS, GEOMETRY of MODULI SPACES, ...

CONJECTURE (B., Gwynne, Sun, '22)

$$(M_n, P_n^1, P_n^2) \xrightarrow{d} \gamma\text{-LQG} + 2 \perp\!\!\!\perp \underset{}{\overset{(c=-2)}{SLE_8}}$$

$$\gamma = \sqrt{\tfrac{1}{3}\left(17 - \sqrt{145}\right)} \quad (c=-4)$$

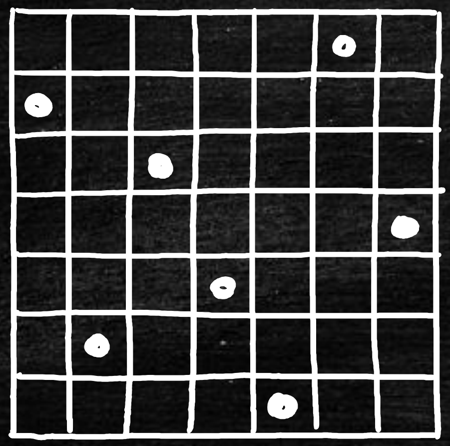
Permutons from Liouville quantum gravity & Schramm-Loewner evolutions

Permutons

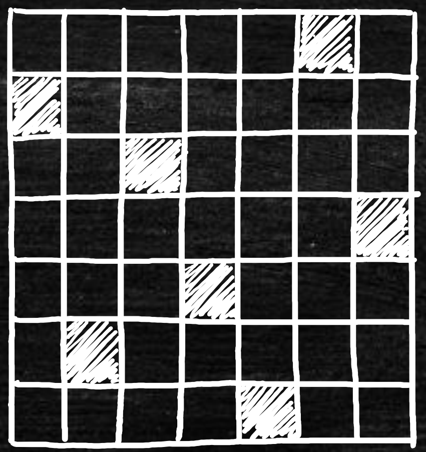
Definition : A PERMUTON is a probability measure on $[0,1]^2$ with uniform marginals.

Consider the permutation $\sigma = 6\ 2\ 5\ 3\ 1\ 7\ 4$

$\sigma =$



$\rightsquigarrow$ $\mu_\sigma =$



The ingredients:

- Fix $\gamma \in (0,2)$ and $\kappa_1, \kappa_2 \geq 8$.
- Let μ be a γ-LQG area measure with $\mu(\hat{\mathbb{C}}) = 1$.

$\perp\!\!\!\perp$

- Let (η_1, η_2) be a pair of space-filling SLEs of parameters (κ_1, κ_2).
  ↳ coupling is unspecified.

Directions:

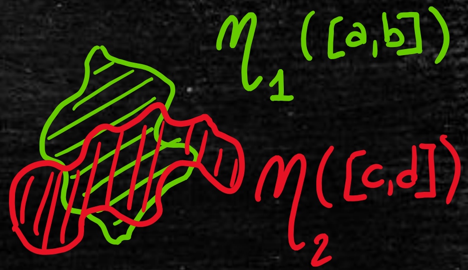
$\eta_i(0) = \eta_i(1) = \infty$, $\mu(\eta_i([0,t])) = t \quad \forall t \in [0,1]$

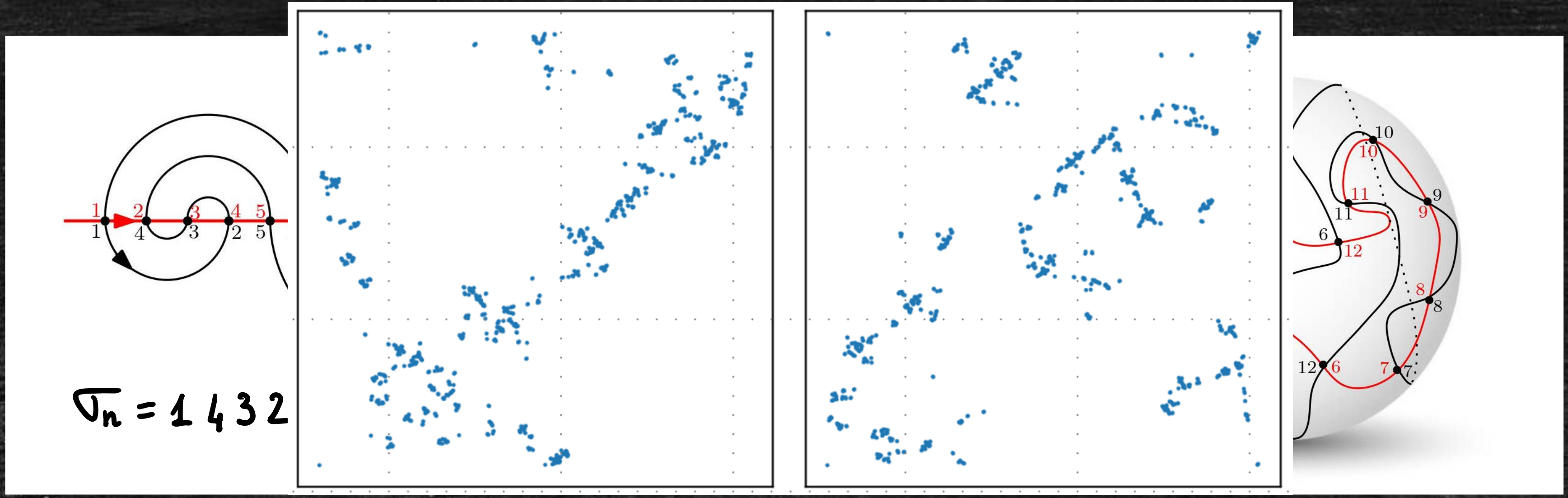
- For $i \in \{1,2\}$, parametrize η_i by μ.

⚠ This is a RANDOM PERMUTON & its law depends on $(\gamma, \kappa_1, \kappa_2)$ and the coupling of (η_1, η_2)

The permuton π associated with (μ, η_1, η_2) is defined by

$$\pi\big([a,b] \times [c,d]\big) = \mu\Big(\eta_1([a,b]) \cap \eta_2([c,d])\Big)$$





MEANDRIC PERMUTON (MP): $\kappa_1 = \kappa_2 = 8$, $\gamma = \sqrt{\frac{1}{3}\left(17 - \sqrt{145}\right)}$, $\eta_1 \perp\!\!\!\perp \eta_2$

CONJECTURE (B., Gwynne, Sun, '22)

$$\mu_{\sigma_n} \xrightarrow{d} \text{MP} \quad (\text{w.r.t weak topology})$$

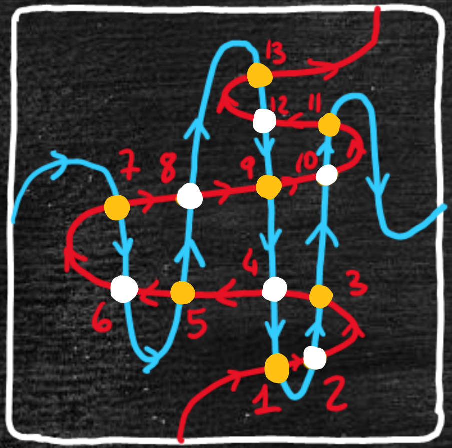
⚠ The γ-LQG and the two $\perp\!\!\!\perp$ SLE_8 are a.s. given by MP.

GOAL (Ongoing project): Can we find a "NATURAL" CHARACTERIZATION of the MP?

- REROOTING INVARIANCE (TRUE $\forall\, \gamma \in (0,2)$ but only when $\kappa_1 = \kappa_2 = 8$) theorem
- RESHUFFLING INVARIANCE (if $\kappa_1 = \kappa_2 = 8$, TRUE ONLY when $\gamma = \sqrt{\frac{1}{3}\left(17 - \sqrt{145}\right)}$) work in progress

Definition: A <u>MONOTONE MEANDER</u> of size $2n+1$ is:

[Connected with other combinatorial objects]



$$\overset{\psi}{\rightsquigarrow} \quad \sigma := \overbrace{7\ 6\ 5\ 8\ 13\ 12\ 9\ 4\ 1\ 2\ 3\ 10\ 11}^{4\ 3\ 7\ 5\ 1\ 2\ 6}$$

4 3 7 5 1 2 6 <u>BAXTER PERMUTATION</u>

<u>MONOTONE MEANDRIC PERMUTATION</u>

(This is a bijection)

THEOREM (B., Maazoun, AOP '21) + (B., PLMS '23)

Let σ_n be a <u>uniform Baxter</u> <u>permutation</u> of size n, then

$$\mu_{\sigma_n} \xrightarrow{d} \mu_B \rightsquigarrow$$

<u>BAXTER PERMUTON</u>:

[IMAGINARY GEOMETRY of Miller & Sheffield]

Obtained from two <u>coupled</u> SLE_{12} $(c=-7)$

& an independent $\sqrt{4/3}$ – LQG $(c=-7)$

감사합니다