A Complex Analysis Conference

Everything is Complex

March 6 – 11, 2016

Saas-Fee, Switzerland

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Ilia Binder (Univ. of Toronto)
Nam-Gyu Kang (Korea Institute for Advanced Study)
Alexei Poltoratski (Texas A&M Univ.)
Stanislav Smirnov (Univ. de Genève)

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Dmitry Krachun (St. Petersburg State Univ.)
Antti Kupiainen (Univ. of Helsinki)
Gregory Lawler (Univ. of Chicago)
Jean-François Le Gall (Univ. Paris-Sud Orsay)
Seung-Yeop Lee (Univ. of South Florida)
Yurii Lyubarskii (NTNU)
Mikhail Lyubich (SUNY Stony Brook)
Nikolai Makarov (Caltech)
Mishko Mitkovski (Clemson Univ.)
Paul Müller (Johannes Kepler Univ. Linz)
Nikolai Nikolski (Univ. de Bordeaux I)
Maria Podkopaeva (Univ. de Genève)
Alexei Poltoratski (Texas A&M Univ.)
Feliks Przytycki (Polish Academy of Sciences)
Juan Rivera-Letelier (Univ. of Rochester)
Steffen Rohde (Univ. of Washington)
Rishika Rupam (Univ. de Lille 1)
Marianna Russkikh (Univ. de Genève)
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Hee-Joon Tak (SNU)
Sergei Treil (Brown Univ.)
Fredrik Viklund (KTH)
Alexander Volberg (Michigan State Univ.)
Aron Wennman (KTH)
Wendelin Werner (ETH Zürich)
Paul Wiegmann (Univ. of Chicago)
Hao Wu (Univ. de Genève)
Dapeng Zhan (Michigan State Univ.)
Michel Zinsmeister (Univ. d'Orléans)

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
09:15 - 10:00	Bishop	Lyubich		Borodin	Nikolski
10:00 - 10:30	Coffee Break		Discussion	Coffee Break	
10:30 - 11:15	Rohde	Kupiainen		Werner	Sheffield
11:20 - 12:05	Volberg			Lawler	Duplantier
	Lunch and Discussion				
16:45 - 17:15	Coffee Break				
17:30 - 17:55	Treil	Lee	Hongler	Rupam	
17:55 - 18:20	Hedenmalm	Benedicks	Gruzberg	Mitkovski	
18:20 - 18:45	Wiegmann	Rivera-Letelier	Zhan	Ameur	
18:45 - 19:10	Le Gall	Epstein	Ivrii	Müller	
19:30 -	Dinner		Gala Dinner	Fondue Night	

A click on speaker's name leads to his or her abstract.
If you want to come back to this page, click on any text in abstract.

Monday

	Speaker	Title
09:15 - 10:00	Bishop	Snowflakes and trees
10:00 - 10:30		Coffee Break
10:30 - 11:15	Rohde	Trees and conformal laminations
11:20 -	Volberg	Harmonic measure absolutely continuous with respect to Hausdorff measure of codimension one is rectifiable
		Lunch and Discussion
16:45 - 17:15		Coffee Break
17:30 - 17:55	Treil	Singular integrals, rank one perturbations and Clark's model in general situation
17:55 - 18:20	Hedenmalm	Coulomb gas ensembles, Laplacian growth, and orthogonal polynomials
18:20 - 18:45	Wiegmann	Coulomb gas on Riemann surfaces: geometry vs analysis
18:45 - 19:10	Le Gall	Harmonic measure on random trees
19:30 -		Dinner

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Snowflakes and trees

Christopher Bishop

SUNY Stony Brook

Abstract

I will survey a variety of problems that involve comparing harmonic measure on two different sides of a domain boundary. First I will discuss harmonic measures on two sides of closed Jordan curve; this involves conformal welding and Makarov's theorem on the dimension of harmonic measure. Next, we will consider finite planar trees where the harmonic measures on the two sides of each edge are equal; such trees are related to Grothendieck's "dessins d'enfants". Which combinatorial trees can have this property? What possible shapes can they have? If time permits I will discuss the analogous questions for infinite planar trees and some applications to holomorphic dynamics.

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Trees and conformal laminations

Steffen Rohde

University of Washington

Abstract

I will describe a conformally natural way to draw finite planar trees (as "balanced trees" in Chris Bishop's terminology) and will give an overview of what is known about these drawings in both the deterministic and the random setting. Similarly, every planar dendrite has a "combinatorial description" via a conformal map to its complement, encoded by a conformal lamination. I will describe a characterization of the laminations of a large class of dendrites (namely complements of John domains), and will speculate about the Brownian lamination and Collet-Eckman Julia sets.

Singular integrals, rank one perturbations and Clark's model in general situation

Sergei Treil

Brown University

Abstract

For a unitary operator the family of its unitary perturbations by rank one operators with fixed range is parametrized by a complex parameter γ, $|\gamma| = 1$. Namely all such unitary perturbations are the operators $U_\gamma := U + (\gamma - 1)bb_1^*$, where $b \in \mathcal{H}$, $\|b\| = 1$, $b_1 = U^{-1}b$, $|\gamma| = 1$. For $|\gamma| < 1$ the operators U_γ are contractions with one-dimensional defects.

Restricting our attention on the non-trivial part of perturbation we assume that b is a cyclic vector for U, i.e. that $\mathcal{H} = \overline{\operatorname{span}}\{U^n b : n \in \mathbb{Z}\}$. In this case the operator U_γ, $|\gamma| < 1$ is a completely non-unitary contraction, and thus unitarily equivalent to its functional model $\mathcal{M}_\gamma$, which is the compression of the multiplication by the independent variable z onto the model space $\mathcal{K}_{\theta_\gamma}$, where θ_γ here is the characteristic function of the contraction U_γ.

The Clark operator Φ_γ is a unitary operator intertwining the contraction U_γ, $|\gamma| < 1$ (in the spectral representation of the operator U) and its model $\mathcal{M}_\gamma$, $\mathcal{M}_\gamma\Phi_\gamma = \Phi_\gamma U_\gamma$. In the case when the spectral measure of U is purely singular (equivalently, the characteristic function θ_γ is inner) the operator Φ_γ was described from a slightly different point of view by D. Clark. The case when θ_γ is an extreme point of the unit ball in H^∞ was treated by D. Sarason using the sub-Hardy spaces $\mathcal{H}(\theta)$ introduced by L. de Branges.

We treat the general case and give a systematic presentation of the subject. We first find a formula for the adjoint operator Φ_γ^* which is represented by a singular integral operator, generalizing in a sense the normalized Cauchy transform studied by A. Poltoratskii. We first present a "universal" representation that works for any transcription of the functional model. We then give the formulas adapted for specific transcriptions of the model, such as Sz.-Nagy–Foiaş and the de Branges–Rovnyak transcriptions, and finally obtain the representation of Φ_γ.

The talk is based on a joint work with C. Liaw.

Coulomb gas ensembles, Laplacian growth, and orthogonal polynomials

Håkan Hedenmalm

KTH Royal Institute of Technology

Abstract

This reports on joint work with N. Makarov. We review the macroscopic behavior of Coulomb gas, in the semiclassical limit as the confining potential gets proportionally stronger with the number of particles. The behavior in the bulk of the droplet is now quite well understood in the $\beta = 2$ regime (inverse temperature). The boundary behavior is not quite as well understood, but we believe this can be resolved using a new asymptotic expansion for the orthogonal polynomials. Once the boundary behavior can be analyzed to high accuracy, we expect to obtain strong asymptotics for the free energy (the logarithm of the partition function).

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Coulomb gas on Riemann surfaces: geometry vs analysis

Paul Wiegmann

University of Chicago

Abstract

Coulomb gas is one of the most celebrated problems of statistical mechanics. The mathematical theory of Coulomb gas has been developed by N. Makarov with friends. In this talk I discuss geometric aspects of the problem. Placing the gas on a Riemann surface one encounter a number of important concepts of Riemann geometry, such as Riemann-Roch theorem, cohomology of Teichmü ller space, and more.

Tuesday

	Speaker	Title
09:15 - 10:00	Lyubich	Geometry of Feigenbaum Julia sets
10:00 - 10:30		Coffee Break
10:30 - 11:15	Kupiainen	Constructive conformal field theory
		Lunch and Discussion
16:45 - 17:15		Coffee Break
17:30 - 17:55	Lee	Topology of quadrature domains and the valence of harmonic polynomials
17:55 - 18:20	Benedicks	Almost sure continuity along curves traversing the Mandelbrot set
18:20 - 18:45	Rivera-Letelier	Thermodynamic formalism of rational maps
18:45 - 19:10	Epstein	Transversality principles in holomorphic dynamics
19:30 -		Dinner

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Geometry of Feigenbaum Julia sets

Mikhail Lyubich

SUNY Stony Brook

Abstract

Feigenbaum maps are infinitely renormalizable quadratic polynomials with bounded combinatorics. We have analysed the Basic Trichotomy for the Julia sets of this class:

- Lean Case: $HD(J) < 2$
- Balanced Case: $HD(J) = 2$ but area $J = 0$
- Black Hole Case: area $J > 0$

and showed that all three cases can be realized (contrary to the intuition suggested by the Dictionary with Kleinian groups).

The black hole Julia sets are locally connected and visible in the parameter plane (making them quite different from the Buff-Cheritat examples of positive area Julia sets). The lean construction is based upon a new recursive estimate for the Poincare series in various renormalization scales.

I can describe the ideas of these constructions (based upon four renormalization theories).

This is based on my joint work with Artur Avila.

Constructive conformal field theory

Antti Kupiainen

University of Helsinki

Abstract

Liouville conformal field theory is a basic building bloc of $2d$ gravity, the scaling limit of discrete random surfaces. We review a probabilistic construction of the theory, discuss its local conformal invariance and speculate about its exact solution. Joint work with David, Rhodes and Vargas.

Topology of quadrature domains and the valence of harmonic polynomials

Seung-Yeop Lee

University of South Florida

Abstract

A quadrature domain is the domain whose Schwarz reflection has the analytic continuation over the domain. The topology of the quadrature domains can be studied by considering the dynamics under the iteration of the Schwarz reflections. The problem is related to the open problem of finding the maximal number of roots for a harmonic polynomial of a given degree. This is a joint work with Nikolai Makarov.

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Almost sure continuity along curves traversing the Mandelbrot set

Michael Benedicks

KTH Royal Institute of Technology

Abstract

We study continuity properties of dynamical quantities while crossing the Mandelbrot set through typical smooth curves. In particular, we prove that for almost every parameter c_0 in the boundary of the Mandelbrot set M with respect of the harmonic measure and every smooth curve $\gamma : [-1, 1] \mapsto \mathbb{C}$ with the property that $c_0 = \gamma(0)$ there exists a set $\mathcal{A}_\gamma$ having 0 as a Lebesgue density point and such that $\lim_{x\to 0} \mathrm{HDim}(J_{\gamma(x)}) = \mathrm{HDim}(J_{c_0})$ for the Julia sets J_c. This is joint work with Jacek Graczyk.

Thermodynamic formalism of rational maps

Juan Rivera-Letelier

University of Rochester

Abstract

About 20 years ago, Makarov and Smirnov started their study of the thermodynamic formalism of rational maps and its connection to the integral means spectrum. For an arbitrary rational map, they gave a complete characterization of phase transitions in the negative spectrum. The purpose of this talk is to survey the recent progress on the more involved case of the positive spectrum, leading to the classification of phase transitions for quadratic maps with real coefficients. We will also discuss the occurrence of phase transitions at infinity.

Transversality principles in holomorphic dynamics

Adam Epstein

University of Warwick

Abstract

A generalization of Thurston's rigidity theorem yields smoothness and transversality of dynamically defined subloci of natural deformation spaces.

Wednesday

	Speaker	Title
09:15 - 11:50		Discussion
		Lunch and Discussion
16:45 - 17:15		Coffee Break
17:30 - 17:55	Hongler	Ising model and conformal field theory: an overview
17:55 - 18:20	Gruzberg	Network models, quenched quantum gravity, and critical behavior at quantum Hall transitions
18:20 - 18:45	Zhan	Complex analysis in backward SLE
18:45 - 19:10	Ivrii	On Makarov's principle in conformal mapping
19:30 -		Gala Dinner

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Ising model and conformal field theory: an overview

Clément Hongler

École polytechnique fédérale de Lausanne

Abstract

The convergence of statistical mechanics and quantum field theory in two dimensions is one of the most beautiful chapters of mathematical physics, which has unfortunately remained largely conjectural. For the Ising model, we are now close to having a completely rigorous picture. I will discuss some recent results in this direction. Based on joint works with S. Benoist, D. Chelkak, A. Glazman, K. Izyurov, K. Kytölä and S. Smirnov

Network models, quenched quantum gravity, and critical behavior at quantum Hall transitions

Ilya Gruzberg

Ohio State University

Abstract

We consider network models for quantum Hall transitions, and incorporate a geometric disorder previously overlooked. We argue that in the continuum this leads to an effective description in terms of a conformal field theory coupled to quenched $2D$ quantum gravity. This coupling changes critical behavior at the transition and may explain discrepancy between the values of critical exponents obtained in experiments and in numerical simulations of network models without the geometric disorder.

Complex analysis in backward SLE

Dapeng Zhan

Michigan State University

Abstract

Backward SLE is defined using the backward chordal or radial Loewner equation, which does not naturally generate a growing curve or increasing family of hulls. One needs to understand the conformal transformation of a backward Loewner chain. For this purpose, a framework is developed to study the effect of analytic perturbations of weldings on the corresponding hulls. With this tool, we were able to describe how two backward Loewner processes interact with each other. Then we used a stochastic coupling technique from the study of forward SLE to construct a commutation coupling between two backward SLEs, and proved the time reversal symmetry of the welding generated by backward chordal SLE_κ for $\kappa \leq 4$. The talk is based on the joint work with Steffen Rohde.

On Makarov's principle in conformal mapping

Oleg Ivrii

University of Helsinki

Abstract

We examine several characteristics of conformal maps that resemble the variance of a Gaussian: asymptotic variance, the constant in Makarov's law of iterated logarithm and the second derivative of the integral means spectrum at the origin. While these quantities need not be equal in general, they agree for domains whose boundaries are regular fractals such as Julia sets or limit sets of quasi-Fuchsian groups. We show that these characteristics have the same universal bounds, and furthermore, the extremals satisfy a central limit theorem. Our method is based on analyzing the local variance of dyadic martingales associated to Bloch functions. Combined with the recent work of Hedenmalm, these ideas show that quasicircles of dimension $1 + k^2$ do not exist.

Thursday

	Speaker	Title
09:15 - 10:00	Borodin	The six vertex model and randomly growing interfaces in (1+1) dimensions
10:00 - 10:30		Group Photo and Coffee Break
10:30 - 11:15	Werner	Some loop-soup stories
11:20 - 12:05	Lawler	Natural parametrization for SLE and the loop-erased random walk
		Lunch and Discussion
16:45 - 17:15		Coffee Break
17:30 - 17:55	Rupam	Inner functions and mixed spectral problems
17:55 - 18:20	Mitkovski	Determinacy problem for measure
18:20 - 18:45	Ameur	Analysis of eigenvalue ensembles using Ward identities
18:45 - 19:10	Müller	Davis and Garsia inequalities for Hardy martingales
19:30 -		Fondue Night

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The six vertex model and randomly growing interfaces in (1+1)dimensions

Alexei Borodin

Massachusetts Institute of Technology

Abstract

The goal of the talk is to explain how the six vertex model gives rise to models of $(1+1)d$ random growth in the KPZ universality class, and how the Yang-Baxter integrability of the former leads to solvability of the latter.

Some loop-soup stories

Wendelin Werner

ETH Zürich

Abstract

A complex structure on a two-dimensional real manifold automatically defines a natural measure on Brownian loops with conformal invariance properties. I will survey some features of Brownian loop-soups, the relation between some of these features and those of SLE, CLE and the Gaussian Free Field. This will be partly based on recent joint work with Wei Qian.

Natural parametrization for SLE and the loop-erased random walk

Gregory Lawler

University of Chicago

Abstract

I will discuss the construction of the natural (length) parametrization of the Schramm-Loewner evolution as well as a new result showing that the loop-erased random walk (that is, Laplacian growth at the tip) parametrized by the number of steps converges to SLE in the natural parametrization. This will include work with F. Viklund, M. Rezaei, B. Werness, C. Benes.

Inner functions and mixed spectral problems

Rishika Rupam

Univ. de Lille 1

Abstract

Given a family of exponentials $\{e^{i\lambda_n x}\}_\Lambda$, is it complete in some $L^2(0,a)$ space? This classical problem was the object of study for many analysts in the early 1900s, including Levinson, Koosis and Kahane. The problem was finally resolved in the 1960s by Beurling and Malliavin. In 2005, Makarov and Poltoratski introduced the language of Toeplitz kernels and model spaces to provide a different proof of the B-M theorem, in their paper *Meromorphic Inner Functions, Toeplitz kernels and the Uncertainty Principle*. In their work, they also set up a general framework to connect completeness problems of certain families of functions to spectral problems of differential operators.

In this talk, we will discuss the connection between Schrödinger operators and model spaces. We will also look at some recent results that use this framework to characterize the solutions of the so-called 'mixed spectral problems'- where part of the potential and part of the spectrum is used to determine the entire potential. This is joint work with Mishko Mitkovski.

Determinacy problem for measures

Mishko Mitkovski

Clemson University

Abstract

A finite positive measure μ is said to be a-determinate if there is no other finite positive measure ν such that the Fourier transforms of μ and ν agree on some interval of length a. For a given measure μ we show how to estimate the largest a for which μ is a-determinate by looking only at the support of μ. Our approach is partly based on the de Branges-Naimark extreme point method. We use the same method to improve the important result of Eremenko and Novikov concerning oscillations of measures with a spectral gap. I will also present some more recent results about the determinacy part of the classical moment problem.

Analysis of eigenvalue ensembles using Ward identities

Yacin Ameur

Lund University

Abstract

The talk will focus around some results on two-dimensional random matrix models, especially microscopic properties near the edge of the spectrum. Our approach uses rescaling in Ward identities on the mesoscopic scale. This leads to a non-linear equation for the one-point function, which we call "Ward's equation".

In this way, questions about the local distribution of eigenvalues (or more general systems of repelling point charges in a plane) translate to question about unique solution to Ward's equation, under suitable physically plausible conditions. Joint work with Makarov and Kang.

Davis and Garsia inequalities for Hardy martingales

Paul Müller

Johannes Kepler Universität Linz

Abstract

We present Davis decompositions for Hardy martingales with values in Banach spaces, and use them to derive their Davis and Garsia inequalities (DGI).

We discuss the relation of (DGI) to the open problem of characterizing complex Banach spaces with non-trivial Hardy-martingale-cotype.

The talk is based on

P.F. X. Müller: A decomposition for Hardy martingales Part II, Math. Proc. Camb. Philos. Scoc (2014), **157**, 189–207

P.F. X. Müller: A decomposition of Hardy martingales Part III, ArXiv 1504.06513v1

Friday

	Speaker	Title
09:15 - 10:00	Nikolski	Well and bad posed inversion problems
10:00 - 10:30		Coffee Break
10:30 - 11:15	Sheffield	TBA
11:20 - 12:05	Duplantier	TBA
		Lunch

Well and bad posed inversion problems

Nikolai Nikolski

Univ. de Bordeaux I

Abstract

This is a survey on the norm control of inverses in terms of the lower spectral parameter for a variety of operators - convolutions/Fourier multipliers, large matrices, H^∞ quotient algebras. Some new integral operators are added to the list.