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Effect of Blockage on Driven Flow with Periodic Boundary Conditions

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We provide a detailed discussion of the role of boundary conditions in the totally asymmetric simple exclusion process (TASEP) with a localized blockage, located at the middle of the system. This quenched disorder is a bond defect, either a slow bond (SB) or a fast bond (FB), where the transport probability is different from those of other bonds in the system. Using numerical simulations, we found that for the SB case, a queuing transition occured in the periodic TASEP as it does in the open one. A density depletion was found for the FB case as well, irrespective of boundary conditions. Queuing and depletion are related to a power-law decay of the density profile from the defect bond. Finally, we argue ensemble equivalence for the effect of the quenched disorder in the TASEP, which implies that the same class of critical behaviors of the queuing transition are observed and that the critical bond strength is exactly the same for both boundary conditions.

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I. INTRODUCTION

In many physical and biological systems, stochastic driven flow through narrow channels with a blockage are often observed, where the most interesting phenomenon is queuing or traffic jam. There are several ways to study on its characteristics. Among them, the simplest model approach of the driven diffusive flow is to employ the asymmetric simple exclusion process (ASEP) in a one-dimensional lattice [1,2]. The ASEP is well established to mimics driven flow, which belongs to the same universality class as the Kardar-Parisi-Zhang (KPZ) type growth of one-dimensional interfaces [3].

The original purpose of ASEP was to describe biopolymerization [4,5]. Later on, it is known that the ASEP could also cover gel electronics [6], directed polymers in a random medium [7], traffic jams [8,9], and the fluctuations of shock fronts [10–13]. In the case of periodic boundary conditions without any blockage, the time development of the ASEP is exactly soluble by the Bethe ansatz [1,2] while in several other setups the exact stationary state has been constructed as well [12,14,15].

The effect of a localized blockage on driven flow has been widely studied as an example of quenched disorder phase transitions. A localized slow bond (SB) was first introduced in the totally ASEP (TASEP) to study on shock fluctuations and its finite-size scaling (FSS) [13], where periodic boundary conditions (PBCs) were employed and mean-field (MF) solutions were used. MF solutions says that the periodic TASEP with a SB has always macroscopic queue at any SB strength, so that no queuing transition occurs. However, it turns out that MF solutions are not correct. In the earlier work by the author and coworkers [16], the role of the SB in TASEP was considered with open boundary conditions, of which main results are compared with the experimental results of stochastic flameless combustion of paper. In the experiment, the paper was impregnated with KNO_3 in order to provide a steady oxygen source. The burning speed could thus be controlled by the KNO_3 concentration. As a result, the KNO_3 concentration could be enhanced or reduced in a narrow strip along the burning direction.

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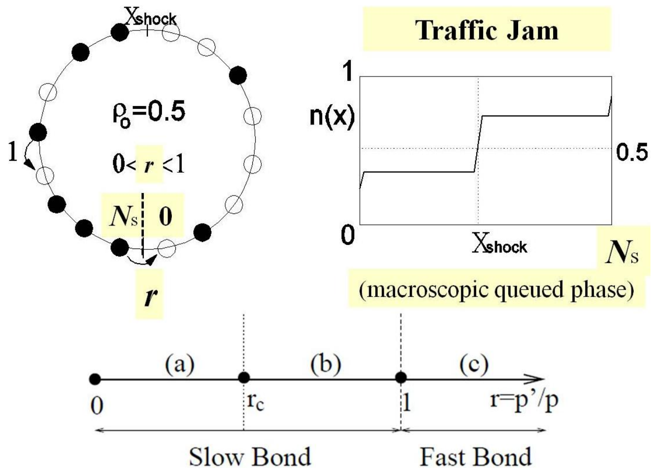

Fig. 1. (Color online) Periodic TASEP with a blockage in a one-dimensional ring and its phase diagram:(a) queued slow-bond (SB) phase (b) non-queued SB phase (c) non-queued fast-bond (FB) phase

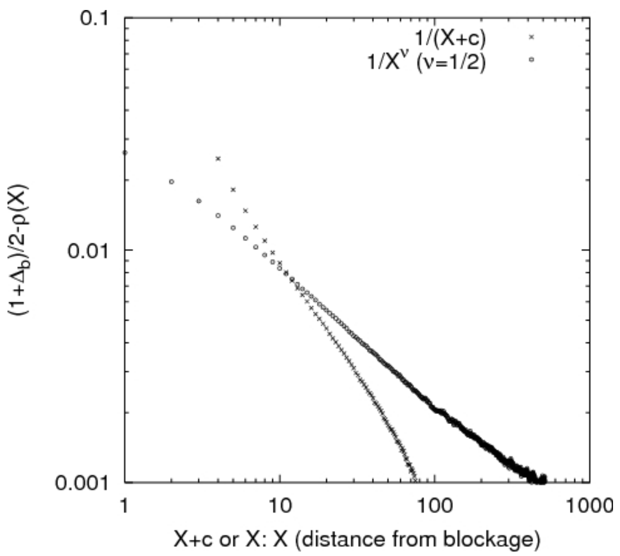

Fig. 2. The decay of the density profile from the SB is fitted by two different ways, namely JL-fit (see Fig. 2 in Ref. [13] for the details) versus our power-law fit ($\nu \simeq 1/2$). The data are obtained at $r = 0.4$ for $N_s = 1200$ with PBCs.

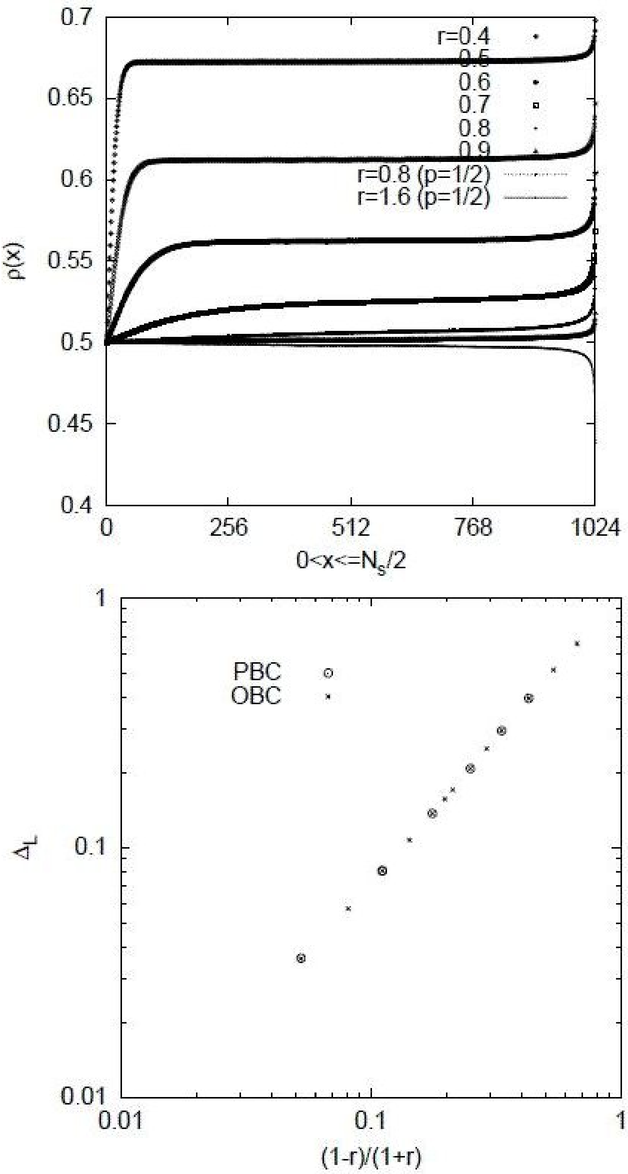

Fig. 3. In the left panel, MC simulation data of density profiles for various r are obtained at $N_s = 2048$. Note that the location of the SB/FB is changed to the middle of the system. In the right panel, $\Delta_L = 2(\langle n_L \rangle - 1)$ versus $(1-r)/(1+r)$.

Experiment results nicely illustrate the presence of nonlinear terms in the equation of motion; the burning front facets for enhanced concentrations but not for reduced concentrations. The detail of these experiments and the matching of the experimental data with our numerical results for the SB in the TASEP were published as well as the fast bond (FB) case, separately [17].

The existence of such a transition, scaling properties, the shape of the density profile near the SB (above, below, and at the transition), and also how and whether information transports through the SB, are the most important issues.

II. MODEL AND MAIN RESULTS

We here focus only on the half-filled TASEP system, *i.e.*, $\rho_o = 1/2$, which is equivalent to our boundary-free setup $\alpha = \beta = 1/2$ in the open TASEP [16]. Using the power-law best-fit method for various r, we can find the

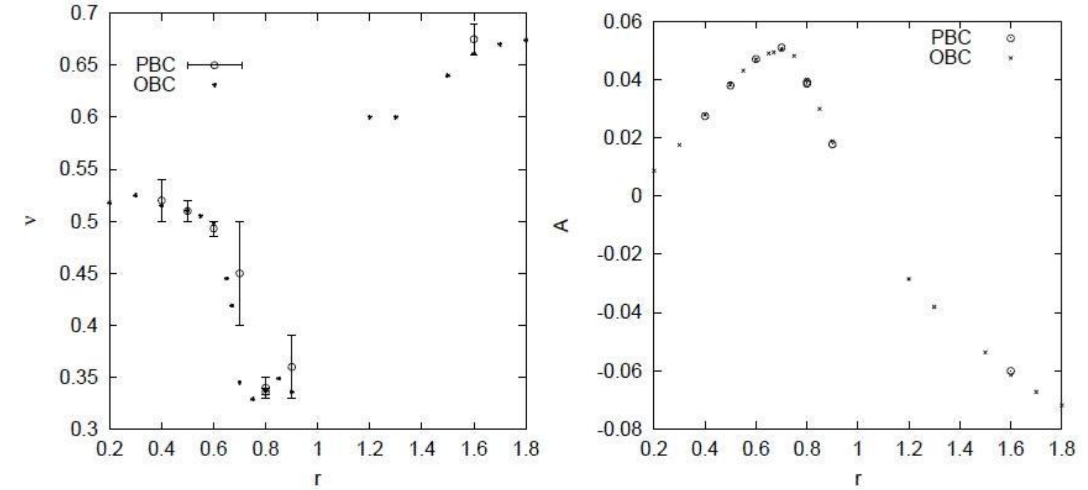

Fig. 4. The best-fit values of (left panel) the exponent ν and (right panel) the amplitude A for various r in Eq. (1). Again, we also plot the OBC results for comparison.

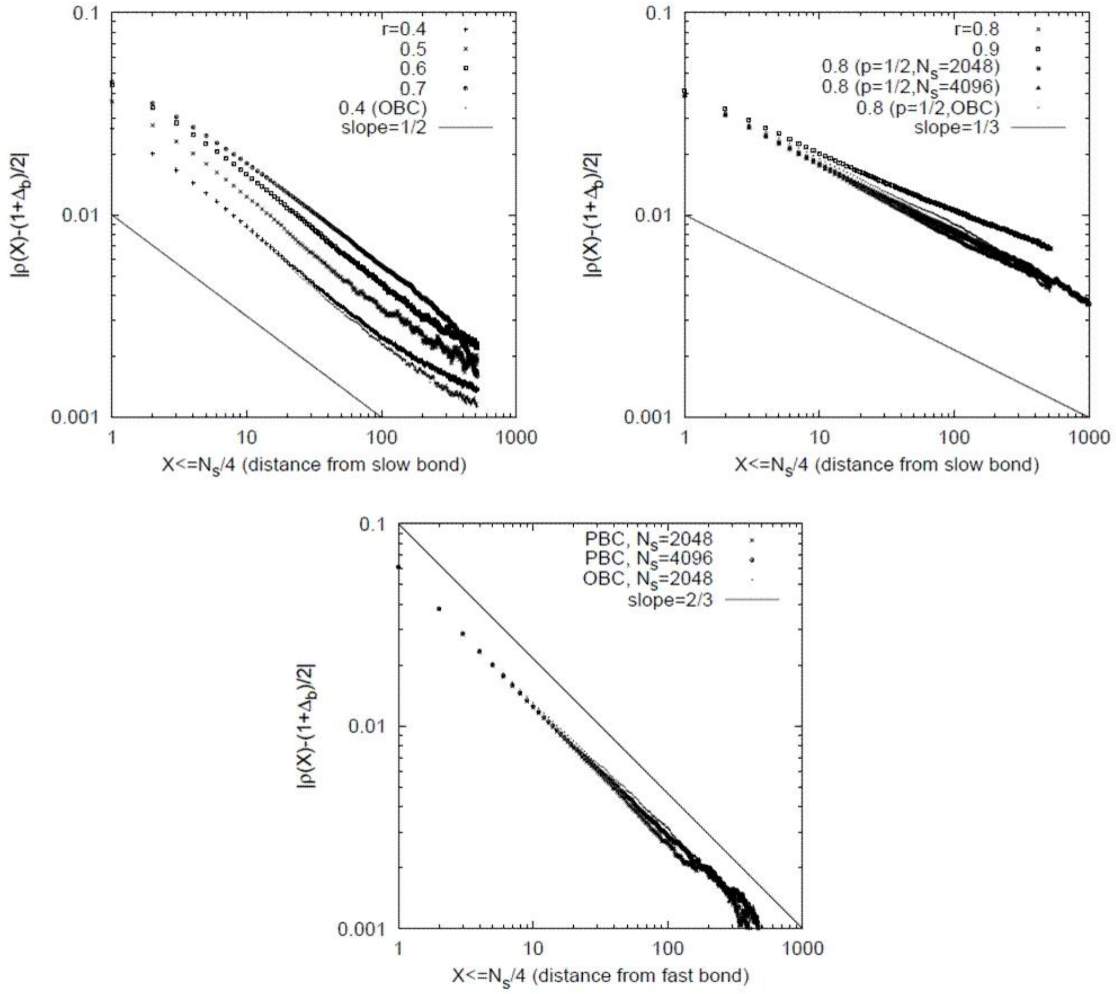

Fig. 5. Double logarithmic plots for the power-law decay of density profiles from the SB/FB with $N_s = 2048$: (upper-left panel) $r = 0.4 - 0.7$ (queued phase), (upper-right panel) $r = 0.8,\ 0.9$ (non-queued slow bond phase), and (lower panel) $r = 1.6$ (non-queued fast bond phase). The data of $N_s = 4096$ are only shown for $r = 0.8$ and $r = 1.6$. For comparison, we also plot the OBC data.

critical value of r_c in terms of the decay exponent ν. If the SB strength is smaller than r_c, the system yields a

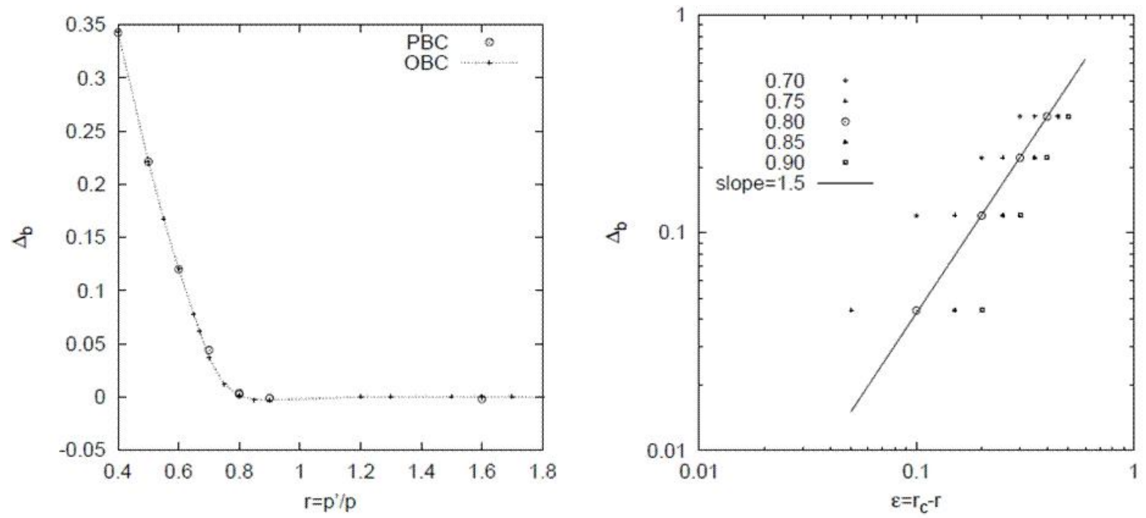

Fig. 6. In the left panel, the order parameter Δ_b versus the strength r is compared to that in OBC. In the right panel, double logarithmic plots indicate the value of r_c and the critical exponent β in $\Delta_b \sim \epsilon^{\beta}$, where $\epsilon = r_c - r$.

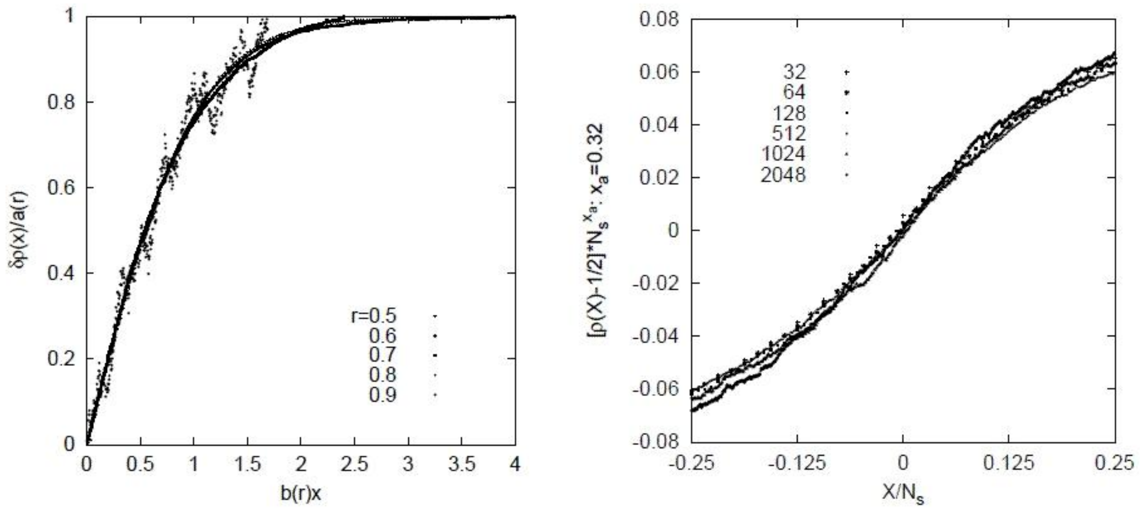

Fig. 7. Data collapse with the best-fit values of a and b for the shock profile by Eq. (2).

queued phase with $J = J(r) < J_{\max}$ with a shock wave. For $r_c < r < 1$ ($r > 1$), there is a non-queued SB (FB) phase. For all r, the density profile from the blockage decays as a power law:

$$\rho(X) \simeq \frac{1}{2}(1 + \Delta_b + AX^{-\nu}), \tag{1}$$

where $X = \frac{N_s}{2} + 1 - x$ is the distance from the blockage.

We denote that the order parameter is Δ_b, but the exponent ν also indicates the value of r_c. As a starting point of a detailed numerical analysis, we address that excess density profile from the blockage decays as a the same power law as that found in [16]. The earlier result claimed by Janowsky and Lebowitz (JL) was $1/|X|$ without any further investigation [13]. However, their result looks only valid for small X. By two different function forms, $|X|^{-1}$ and $|X|^{-\nu}$ with $\nu < 1$, we compare ours to JL-fitted results. Figure 2 confirms that the density profile indeed decays as a power law with $\nu \simeq 1/2$ rather than $1/|X|$ when including a larger X region.

Our scenario including phase diagrams is confirmed by Figs. 3 and 4, which show how density profiles changes as increasing r at $N_s = 2048$. Since there is particle-hole symmetry, we plot them only up to $\frac{N_s}{2}$ (just in front of the SB/FB). As expected, the decay of density profiles from the SB/FB with PBCs is exactly the same as that

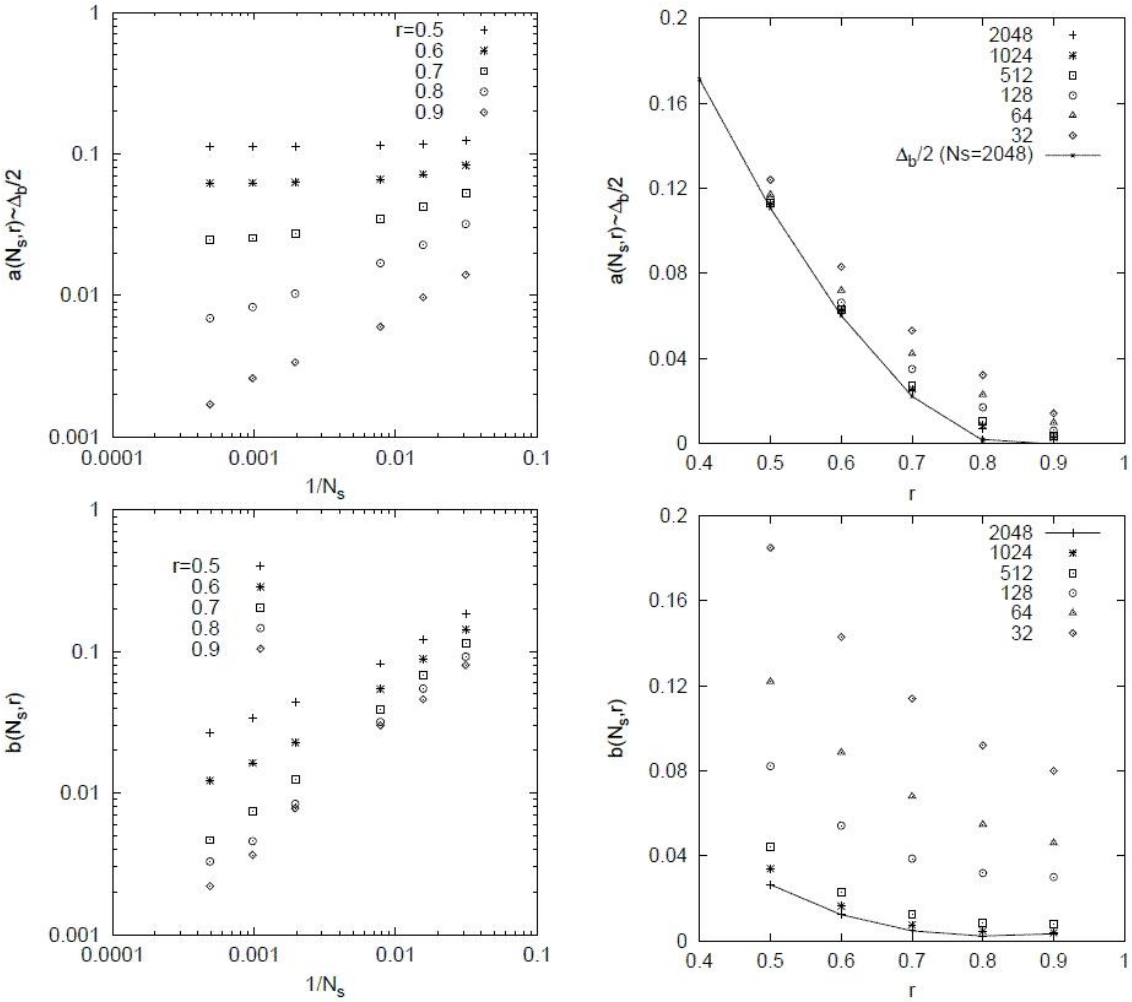

Fig. 8. Shock profiles are fitted by Eq. (2) for various r and various system sizes N_s.

with OBCs, even including the average value of the density just in front of the SB/FB, see Fig. 3 (right panel). Through three-parameter fits for each density profile in terms of Eq. (1), we get the order parameter Δ_b, the amplitude A, and the exponent ν. In Fig. 5, we show the power-law decay of the density profile from the SB/FB. The best-fit values of A and ν for various r are shown in Fig. 4 and the plot of the order parameter versus r in Fig. 6 (left panel). The OBC data (from Ref. [16]) are also plotted for comparison. Figure 6 (right panel) indicates that $r_c \simeq 0.8$ and $\Delta_b \sim \epsilon^\beta$ with $\beta \sim 1.5$, where $\epsilon = r_c - r$. This is almost the same results as what we obtained with OBCs.

On the other hand, the edge density profile is quite different from each other, since PBCs generate a shock wave for $r < 1$, see [8] for a detailed discussion. We find that it is well-fitted by the form:

$$\rho(x) \simeq \frac{1}{2} + a\tanh(bx.) \tag{2}$$

Here the amplitude a is related to the order parameter Δ_b, *i.e.*, $a \simeq \Delta_b/2$ since $\rho(x) \simeq \frac{1}{2}(1+\Delta_b)$ for $0 \ll x \ll N_s/2$. Through two-parameter fit, we get the best-fit values of a and b for various r and for various system sizes. Due to less statistical averages for large system sizes, we cannot get such a good FSS results. However, we can show data collapse by using the best values of a and b at the largest system size, $N_s = 2048$, for various r, and at $r_c = 0.8$ for various N_s. Figure 7 confirms the validity of Eq. (2) for the shock profile in the presence of the blockage. In particular, we reconfirm the FSS at $r_c = 0.8$ in Fig. 7 (right panel), where we used $b \sim N_s^{-y_b}$ and $a \sim N_s^{-x_a}$ with $y_b \simeq 1$ and $x_a \simeq 0.32(4)$. Recall that $\Delta_b \sim N_s^{-x_\Delta}$ with $x_\Delta = 0.360(5)$. The detailed analysis is shown in Fig. 8.

III. CONCLUSION AND REMARK

In summary, we studied on ensemble equivalence between the periodic TASEP for $\rho_o = 1/2$ and open TASEP for $\alpha = \beta = 1/2$ in the presence of the localized blockage (bond defect-type disorder) at the middle of the system. It was observed that the quenched disorder in the TASEP yields the queuing transition for the SB case at the critical bond strength, which is exactly the same in both boundary conditions. For the FB case, we also checked out the density depletion with the power-type decay in both boundary conditions. Our results implies that MF solutions are incorrect even in the periodic TASEP with the SB.

Finally, we should mention that some of the contents and plots in this paper are taken from Appendix of my Ph.D. thesis at the University of Washington [18], which are not published anywhere.

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