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Effect of Thermal Noise on Conserved Lattice Gas

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We present a modified conserved lattice gas (CLG) model in two dimensions with a thermal noise parameter, $P = \exp(-1/T)$, where T is the effective temperature. In particular, we focus on the interplay of thermal noise P and the total density of particles ρ in the CLG model. The universality class of the case of $0 < P < 1$ can be compared with $P = 0$ and $P = 1$ cases. We discuss the critical behavior of absorbing phase transitions in the generalized CLG (GCLG) model by two well-defined order parameters: one is the density of active particles (a pair of consecutive particles) and the other is the density of energy-loss particles. While the former is useful at $P = 1$ ($T = \infty$) as the typical one in nonequilibrium absorbing phase transitions, the latter is suggested as the best at $P = 0$ ($T = 0$) due to the oscillatory behavior in the localized active phase. Based on extensive numerical tests, we find that the density of energy-loss particles can also be a good indicator even for finite P values. Finally, we propose a schematic phase diagram of the GCLG as P varies.

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I. INTRODUCTION

Lattice gas models have been extensively studied for understanding dynamic phase transitions and critical phenomena in nonequilibrium systems [1, 2]. Among them, the conserved lattice gas (CLG) model is well known as the simplest one that exhibits absorbing phase transitions (APTs) with a conserved field. APTs are dynamic phase transitions from dynamically active phases to absorbing (dynamically dead) phases where the active particle density is the order parameter from nonzero finite values to zero. For last decades, the universality class of APTs has been intensively discussed with theoretical scenarios and numerical confirmations, so that it has been well established depending on the symmetry of absorbing states [3]. It was also discussed that the criticality of APTs could explain the critical tuning of the self-organized critical (SOC) [4–7] sandpile models. Hence the CLG model can be considered as the stochastic variant of a lattice gas model in the study of $1/f$ noise in driven systems [8].

Since Rossi and coworkers [9] first introduced the CLG model with short range stochastic microscopic dynamic rules to discuss the universality class of APTs with the conserved field, Lübeck and coworkers had continued the investigation of the CLG model from the critical tuning parameter to critical exponents for various dimensions and discussed the upper critical dimension issues [10,11]. In order to check the conjecture for the universality class of APTs in the absence of additional symmetries [12], a variety of stochastic models with infinite absorbing states were suggested, where dynamical processes are coupled to a non-diffusive conserved field (see Refs. 9–18 and references therein).

During the relaxation process, the one-dimensional (1D) CLG model can only minimize the total number of particle bonds, so that the critical density is exactly 1/2. To find the same condition in two-dimensional (2D) CLG model, the noise parameter was first suggested by Yang and coworkers [17]. In the 2D generalized CLG (GCLG) model, the noise parameter plays a role of an effective temperature in particle dynamics, which restricts the movement of active particles depending on the number of the nearest neighbors before and after the change, such as temperature in equilibrium systems. Yang and coworker intended to design that the noiseless case of the 2D GCLG model exhibits two symmetric absorbing states with the half-filled checker-board type patterns.

However, for the noiseless case, any critical behaviors cannot be detected by the order parameter of the original CLG model at any density conditions. Through careful inspections, it was attributed to irrelevant contributions

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of the immobility and the oscillation of particle movement. To resolve this, Yang and coworkers proposed a new order parameter in the 2D GCLG model. Based on numerical tests for the two-limiting cases, it was found that this new order parameter was able to identify the criticality of two-limiting cases evenly well. The noiseless case only becomes critical at the half-filled system and the ordinary case does at the same critical density found by the original order parameter. Moreover, two-limiting cases belongs to the different universality classes. In our earlier work [18], we studied the GCLG dynamics in uncorrelated random networks to confirm the mean-field results of the GCLG criticality as the noise parameter varies, with two distinct order parameters.

In this paper, we revisit the 2D GCLG model for all levels of the thermal noise, and discuss the effect of the thermal noise on the universality class of the 2D GCLG model in the context of two distinct order parameters. In particular, we focus on glassy behaviors at the intermediate level of the thermal noise, which is observed in static simulations. Based on extensive numerical tests, we argue which order parameter can identify the criticality of the 2D GCLG model better. Finally, we suggest the schematic phase diagram with the noise parameter and the total density of particles as well as the universality classes of the 2D GCLG model.

II. MODEL

Consider the 2D GCLG model in a periodic $L \times L$ square lattice, where each site can be occupied by at most one particle. Each particle can be categorized as either active if it has at least one neighbor or inactive one. Moreover, active ones can be colored with by of four colors (red, yellow, blue, and black). The color indexing depends on the possibility of the move and the change before and after the particle move. Red particles can decrease the number of active bonds N_b, blue ones can only increase N_b, and yellow ones cannot change N_b, while black ones are immobile and cannot move anywhere.

The configuration energy can be specified as follows:

$$\varepsilon(\sigma) = J \sum_{\langle i,j \rangle} \sigma_i \sigma_j, \tag{1}$$

where $J > 0$ denotes the repulsive binding energy, $\langle i,j \rangle$ means the nearest neighboring pairs, and $\sigma_i = 0, 1$ is the occupation number at site i. The tuning parameter of the 2D GCLG model is the total density of particles $\rho = \sum_i \sigma_i / L^2$ and the noise parameter is $P = \exp(-1/T)$. Here T is an effective temperature from 0 ($P = 0$) to ∞ ($P = 1$). The Metropolis-Hastings algorithm is implemented to the movement of active particles in the GCLG dynamics with the following acceptance (transition) probability function:

$$p_a(\Delta\varepsilon) = \min\{1, P^{\Delta\varepsilon}\}, \tag{2}$$

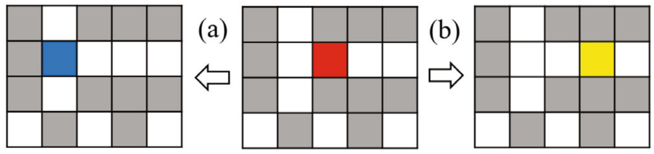

Fig. 1. (Color online) Schematic illustrations of three-colored active particles, energy-gain (blue), energy-loss (red), and the energy-conserving (yellow) ones, which can move to an empty site with an acceptance probability $p_a = \min\{1, P^{\Delta N_b}\}$, where P is the thermal noise parameter, and ΔN_b is the change of the number of active bonds: In the middle panel, a red particle can be moved to either (a) the left with $\Delta N_b = -1$ or (b) the right with $\Delta N_b = 0$.

where the configuration energy difference between two configurations, σ_{before} and σ_{after}, is exactly the same as the number of changed active bonds, $\Delta\varepsilon = \Delta N_b$. With the transition probability p_a, active particles move to the targeted empty sites if the move can reduce the number of active bonds as P decreases ($T \to 0$). Hence the original CLG model corresponds to the GCLG model with $P = 1$.

The GCLG model dynamics with P is summarized as follows:

1. Choose a site at random from the active list of particles, of which the total number is $N_{\text{act}} = N - \delta(\sum_{\langle i,j \rangle} \sigma_i \sigma_j)$. Here $\delta(x) = 1$ if $x = 0$; 0 otherwise, and $0 \le N_{\text{act}}(t) \le \rho L^2$. This is the rejection-free algorithm of the CLG model.

2. Unless the color of the chosen particle is black, it moves a randomly chosen empty site among the nearest neighboring sites with p_a. After this trial, the simulation time is updated from t to $t + \Delta t$ with $\Delta t = 1/N_{\text{act}}(t)$, and the active list is updated as well as N_{act} if the trial is accepted. The time update is independent of the success of the trial.

3. Keep the above procedures in order until t is not smaller than $t_{\max}$ (the maximum simulation time) or there are no mobile particles in the active list of the system, which is either empty or black colored only. It is noted that black colored ones are counted to the number of active particles but they cannot move anywhere since all of the nearest neighboring sites are already occupied by other active particles.

Figure 1 represents the GCLG model dynamics, where we show that a red particle in the middle panel can be either (a) blue or (b) yellow, provided that the move is accepted. We employ two order parameters: one is the density of active particles, ρ_{act}, and the other is the density of red particles, ρ_{red}. In previous studies [17,18], the latter one turns out to be the best indicator for the zero-temperature GCLG model. In the next section, we show how to numerically investigate the critical tuning density as the thermal noise parameter P varies.

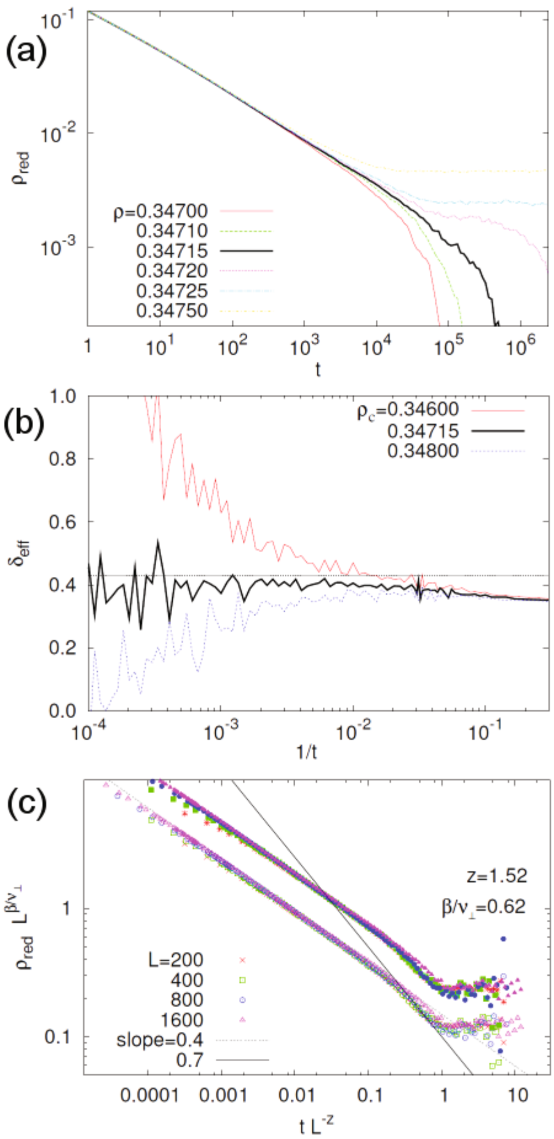

Fig. 2. (Color online) For the 2D CLG model (the case of $P = 1$, *i.e.*, $T = \infty$), (a) the red-particle (red, from now on) density, $\rho_{\rm red} \sim t^{-\delta}$ are double-logarithmically plotted against t as the total density of particles ρ varies, which indicates $\rho_{c,\infty(P=1)} = 0.34715(4)$; (b) the effective decay exponents are plotted as the function of $1/t$ in semi-logarithmic scales, which implies that $\delta = 0.42(5)$; (c) the FSS theory of the red density (below) is numerically tested by plotting the scaling function $\mathcal{F}(x) = \rho_{\rm red}L^{\alpha}$ as a function of $x = tL^{-z}$ with $\alpha = \beta/\nu_\perp = 0.62(5)$ and $z = 1.52(5)$ for $L = \{200,\ 400,\ 800,\ 1600\}$. Here numerical data with symbols are averaged over all samples, $\mathcal{O}(10^2)$, and the upper data collapse correspond to $\mathcal{F}(x)$ of $\rho_{\rm act}$.

III. NUMERICAL RESULTS

We perform Monte-Carlo simulations in the 2D GCLG model in order to investigate its criticality as ρ and P vary. In particular, we focus on the dynamic simulation of APTs and employ the rejection-free algorithm of the ordinary CLG model with the active list of particles. The system initially starts with random configurations of the total density of particles ($0 < \rho \leq 0.5$), so that $\rho_{\rm act} \leq \rho$ and $\rho_{\rm red} \leq \rho_{\rm act}$. At the critical tuning density, we expect either that both algebraically decay as $\rho_{\rm act} \sim \rho_{\rm red} \sim t^{-\delta}$.

To check out the validity of our numerical methods and the finite-size scaling (FSS) theory before the investigation of the thermal effect on the 2D GCLG model, we first reproduce all the results of the ordinary 2D CLG model in terms of the red density $\rho_{\rm red}$ by setting $P = 1$, which are presented in Fig. 2. As shown in Fig. 2, $\rho_{\rm red}$ can also indicate exactly the same results of the ordinary CLG model as those of the ordinary order parameter at the same critical tuning density.

At the criticality ($\rho = \rho_c$), the order parameter of APTs exhibits the FSS behaviors:

$$\rho_{\rm act}(t, L) = L^{-\alpha}\mathcal{F}(tL^{-z}) = \begin{cases} t^{-\delta} & \text{if } t \ll L^z, \\ L^{-\alpha} & \text{if } t \gg L^z, \end{cases} \quad (3)$$

where $\alpha = \beta/\nu_\perp$, $\delta = \beta/\nu_\parallel$, and $z = \nu_\perp/\nu_\parallel$. The static

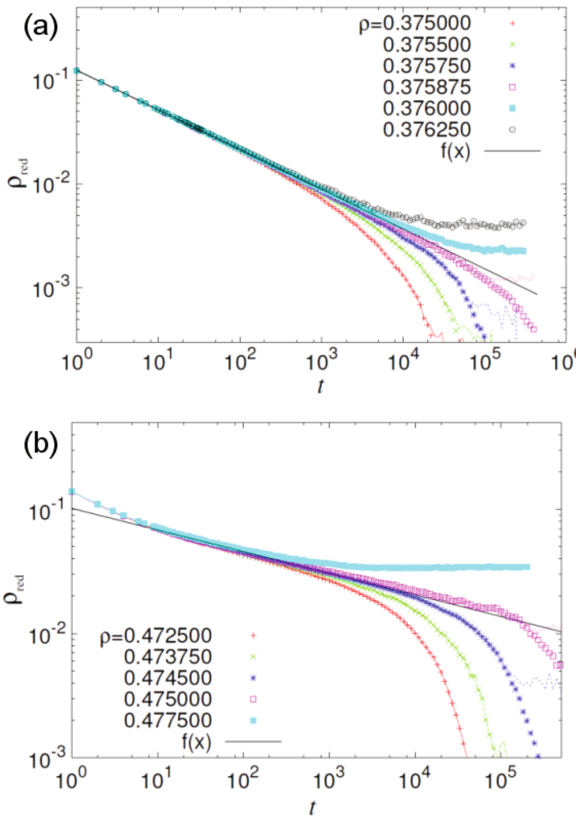

Fig. 3. (Color online) For the cases of (a) $P = 0.75$ and (b) $P = 0.5$, $\rho_{\rm red}$ is double-logarithmically plotted against t, which indicates critical scaling behaviors as $f(x) \sim t^{-\delta}$ at $\rho_{c,P=0.75} \approx 0.375875$ with $\delta = 0.42$ and $\rho_{c,P=0.5} \approx 0.4750$ with $\delta = 0.17$. Numerical data are averaged over 120 samples for $L = 1600$, where symbols (lines) are averaged over all (surviving) samples.

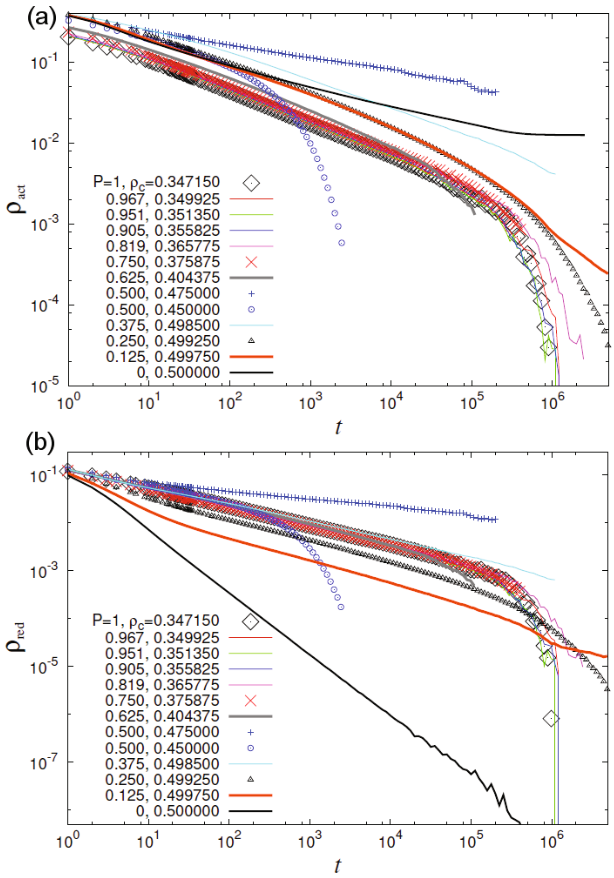

Fig. 4. (Color online) The critical behaviors of (a) ρ_{act} and (b) ρ_{red} are double-logarithmically plotted against t at $\rho = \rho_{c,P}$ for the cases of $0 \le P \le 1$, where numerical data are averaged over 120 samples for $L = 1600$.

exponent β is originated from the off-critical behavior: $\rho_{\text{act}} \sim \epsilon^{\beta}$ with $\epsilon = \rho - \rho_c$ for $\rho > \rho_c$ in the steady-state limit as $L \to \infty$. Near the criticality, the correlation time and the correlation length satisfy the scaling relation of $\tau \sim \xi^z$, derived from $\tau \sim \epsilon^{-\nu_\parallel}$ and $\xi \sim \epsilon^{-\nu_\perp}$. It is noted that we only focus on the exponent δ for the cases of $0 < P < 1$ in this paper.

In Fig. 3 and Fig. 4, we present extensive numerical tests of the GCLG model with P for the indication of the critical tuning density $\rho_{c,P}$, in terms of dynamic simulations, where we employ both ρ_{act} and ρ_{red} as the order parameter. In Fig. 3, we find that the GCLG model with $P = 0.75$ belongs to the same university class of the ordinary one at the different critical tuning density $\rho_{c,P=0.75} \simeq 0.375875 > \rho_{c,\infty}$. However, the critical behavior of the GCLG model with $P = 0.5$ quite different at $\rho_{c,P=0.5} \simeq 0.4750$ from the cases of $P \neq 0.5$ (see Fig. 3 and Fig. 4). As compared to all other cases in Fig. 4, numerical data can be categorized as three-distinct scaling behaviors at $P = 0,\ 0.5,$ and 1 with the different decaying exponent $\delta = 1,\ 0.17,$ and, 0.42, respectively.

For the 2D original CLG model [9] at $\rho_{c,\infty} \simeq 0.34715$,

Table 1. The critical tuning density of the 2D GCLG model and its decaying exponent of the red density are summarized: $\rho_{\text{red}} \sim t^{-\delta}$ at $\rho = \rho_{c,P}$.

P	1^a	0.75	0.5	0.25	0^b
ρ_c	0.34715(4)	0.375875(4)	0.4750(4)	0.499250(4)	0.5
δ	0.42(5)	0.42(4)	0.17(2)	0.42(7)	$1.0(2)^c$

[a] The original CLG model [9].
[b] The zero-temperature GLCG model [17].
[c] It seems to have some logarithmic corrections.

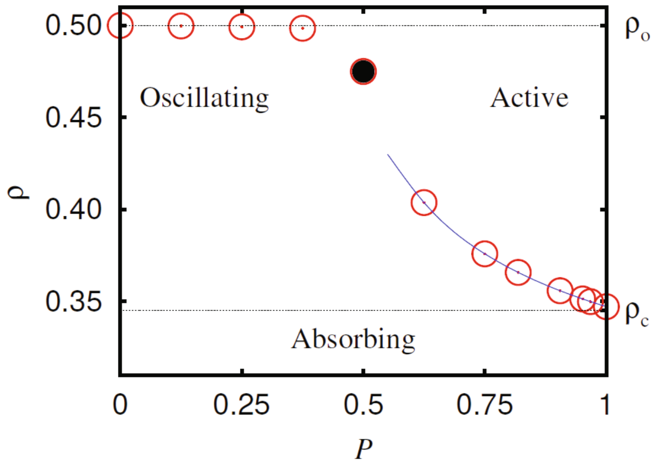

Fig. 5. (Color online) The phase diagram of the 2D GCLG model is schematically illustrated.

critical exponents are $\delta \simeq 0.43$, $\alpha \simeq 0.81$, and $z \simeq 1.52$, while for the zero-temperature GLCG model [17] at $\rho_{c,0} = 0.5$, $\delta \simeq 1.28$, $\alpha \simeq 2.0$, and, $z \simeq 1.55$. While both ρ_{act} and ρ_{red} work well for the case of $P > 0.5$, ρ_{red} seems to indicate the criticality of the system better than ρ_{act} for the case of $P < 0.5$. As $P \to 0$, the oscillation of yellow particles becomes relevant in the active density.

Based on the extensive numerical results of P values presented in Fig. 4 and Table 1, we suggest a schematic phase diagram of the 2D GCLG model with P. In Fig. 5, the lower guided line corresponds to $\rho = \rho_{c,\infty(P=1)}$ at which the ordinary CLG model ($P = 1$) yields nonequilibrium absorbing phase transitions, whereas the upper one corresponds to $\rho = 0.5$ at which the zero-temperature GCLG model ($P = 0$) exhibits the critical decay of $\rho_{\text{red}} \sim t^{-\delta_0}$ as $\delta_0 \approx 1$.

To provide a comprehensive picture for the origin of the anomalous critical behavior near $P = 0.5$, we numerically investigate the effect of the thermal noise on the temporal behaviors of two distinct indicators, ρ_{act} and ρ_{red}. Overall, the critical tuning density gets larger as the thermal noise decreases, $\rho_c = \rho_{c,\infty} \to \rho_{c,0}$. For $\rho < \rho_{c,T}$, the system becomes inactive (oscillating) if $P > 0.5$ ($P < 0.5$), while for $\rho > \rho_{c,T}$, the system becomes active (oscillating) if $P > 0.5$ ($P < 0.5$).

Figure 6 shows that ρ_{red} can identify the anomalous temporal scaling of the half-filled system ($\rho = 0.5$) for

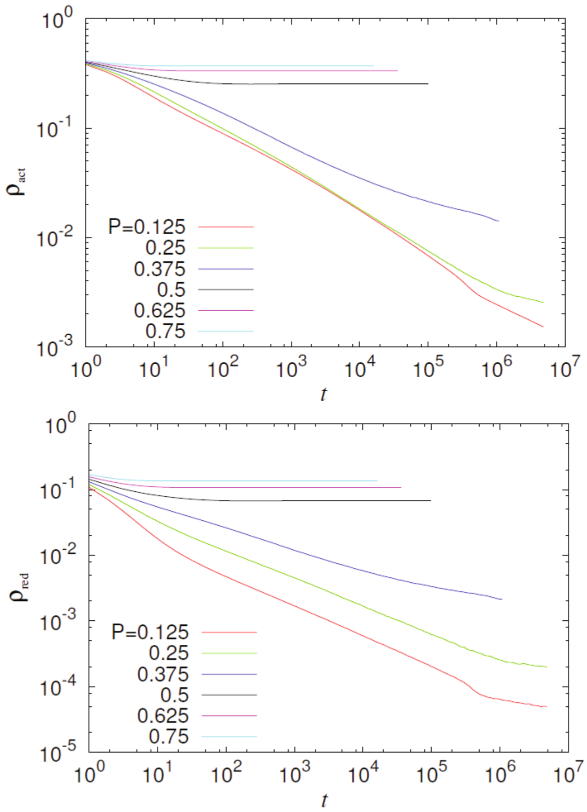

Fig. 6. (Color online) At $\rho = \rho_{c,0} = 0.5$, we numerically speculate how thermal noise affects the temporal behaviors of $\rho_{\rm act}$ and $\rho_{\rm red}$ for the cases of $0 < P < 1$, where numerical data are averaged over 120 samples for $L = 1600$.

$P < 0.5$, compared to the temporal behavior of $\rho_{\rm act}$. This implies that the red density can be an optimal order parameter of the GCLG model with the thermal noise, as discussed in Ref. 18.

IV. SUMMARY WITH REMARKS

We have studied the interplay of the noise parameter $P = \exp(-1/T)$ by an effective temperature T and the total density of particles ρ in the two-dimensional (2D) conserved-lattice gas (CLG) model. As thermal noise decreases, the critical tuning density ρ_c increases up to 1/2 for the noiseless case. In the small-noise ($T = 0$) limit, $\rho_{c,T} \to 1/2$, $\rho_{\rm red} \sim t^{-\delta_0}$ with $\delta_0 \approx 1$, while in the large-noise ($T = \infty$) limit, $\rho_{\rm red} \sim t^{-\delta_\infty}$ with $\delta_\infty \approx 0.4$. In the intermediate level of thermal noise, we observed anomalous scaling, which is maximized at $P = 0.5$ as $\rho_{\rm red} \sim t^{-\delta}$ with $\delta \approx 0.17$ (neither δ_0 nor δ_∞).

To sum up, it is found that the 2D generalized CLG (GCLG) model exhibits three-distinct critical scaling in temporal behaviors. A possible future work is the finite-size scaling analysis in the presence of thermal noise, which enables us to conclude the universality issue of the 2D GCLG model.

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