

Impact of capacity volatility and input substitutability on supply chain resilience

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Supply chains are intrinsically vulnerable to stochastic shocks due to their sequential production dependencies. Building on the Feld-Barthelemy framework, we investigate how capacity volatility and input substitutability determine critical demands in stochastic supply chains. By modeling production capacity with a truncated normal distribution, we show that in long supply chains, reducing capacity volatility is often more effective than increasing average capacity, emphasizing the need for firm-level synchronization. Furthermore, introducing a modified Leontief-type production function reveals that input substitutability effectively disperses stochastic shocks. Supplier diversification inherently raises critical demands, even under fixed maximum capacities, by introducing the effect of network topology that independently enhances the resilience of physical stock. Our findings demonstrate that mitigating capacity volatility and structurally diversifying supply routes are just as crucial to supply chain resilience as traditional inventory expansion.

I. INTRODUCTION

Supply chain management [1–3] aims to reduce costs and improve customer satisfaction by coordinating the flow of materials, products, information, and resources from raw-material procurements to final distributions. Efficiency, however, does not necessarily imply stability. Disruptions caused by natural disasters, financial crises, and geopolitical tensions have shown that optimized supply chains can still be vulnerable to systemic breakdowns, motivating extensive studies on supply chain networks and resilience [4–12].

A key source of fragility is the sequential dependence among firms: Delay or shortage at one node can propagate downstream and amplify across the network. The concept of *timeliness criticality* [13] shows that insufficient buffers can drive such systems close to a critical regime, where small local delays trigger delay avalanches. In the context of supply networks, Feld and Barthelemy [14] introduced a stochastic model in which firms experience random production capacities, and studied how stocks and network structure affect the maximum demand that the network can satisfy.

In this paper, we extend the Feld–Barthelemy (FB) model by relaxing two simplifying assumptions: First, instead of assuming uniformly distributed production capacity, we examine how the volatility of random production capacity affects critical demand. Second, we introduce a Leontief-type production function with substitutable inputs to study how supplier diversification changes supply chain performance. Our results show beyond stock capacity that both production-capacity volatility and the number of substitute suppliers are key determinants of critical demand.

The remainder of this paper is organized as follows: In Sec. II, we introduce the stochastic supply chain model and discuss how to generalize the assumptions. In Sec. III A, we analyze the effect of production-capacity volatility on critical demand. In Sec. III B, we examine the role of substitutable inputs and substitute suppliers. Finally, in Sec. IV, we summarize our findings and discuss their broader implications.

II. MODEL

The model consists of a single root node connected to external consumers, its suppliers, and the suppliers of those suppliers. The supply chain is represented by a directed acyclic graph (DAG) that begins at a root node ($i = 0$) and ends at leaf nodes. Each node i corresponds to a firm that produces its own product: The firm receives materials from its suppliers, $\mathcal{S}(i)$, and delivers its output to its customers, $\mathcal{C}(i)$. Leaf nodes have no suppliers, i.e., $\mathcal{S}(i) = \emptyset$, and can therefore produce independently of material inputs. The root node, located at the top of the supply chain, delivers its output to external consumers rather than to downstream firms.

At each time step, ordering, production, and delivery proceed in a prescribed sequence. First, external consumers add a constant demand r to the root node. The total demand $D_0(t)$ at the root node includes the unmet demand at the previous time step, which is given by

$$D_0(t) = u(t-1) + r. \quad (1)$$

Here $u(t-1)$ denotes the unmet demand carried over from the previous step. Given $D_0(t)$, the root node places orders with its suppliers, $l \in \mathcal{S}(0)$, and these orders become the demand of the supplier nodes.

For a general node $i (\neq 0)$ with demand $D_i(t)$, the order quantity $\Omega_{il}(t)$ placed to the supplier $l \in \mathcal{S}(i)$ and the

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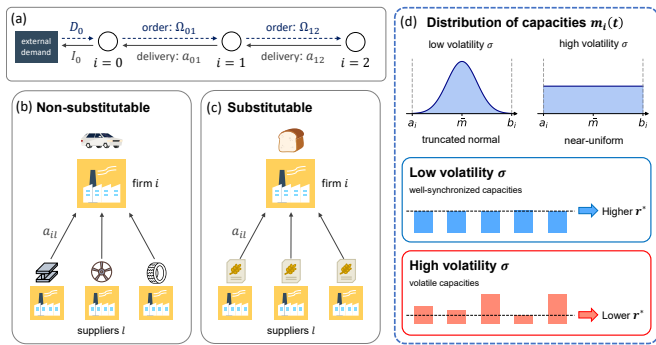


FIG. 1. Schematic illustration of order-delivery process and two different inputs: (a) Order propagation (dashed line arrows) and delivery (solid line arrows) processes in 1D supply chains with two levels from root to leaf node. Conceptual comparison of (b) non-substitutable production to (c) substitutable production. Automobile production requires distinct inputs, such as steel, steering wheels, and tires, from different suppliers, whereas bread production can use compatible inputs, such as flour, supplied by multiple firms. (d) Production-capacity distributions with different volatility σ . As σ increases, the truncated normal distribution (left) approaches a near-uniform distribution (right) in the upper panel. Low volatility in the middle panel synchronizes production capacities across firms and weakens bottleneck effects with higher r^* , whereas high volatility in the bottom panel makes weak nodes more likely and lowers r^* .

resulting demand $D_l(t)$ of node l are given by

$$\Omega_{il}(t) = \max[0, D_l(t) - k_{il}(t)], \quad (2)$$

$$D_l(t) = \sum_{i \in \mathcal{C}(l)} \Omega_{il}(t), \quad (3)$$

where $k_{il}(t)$ denotes the stock of product l held by firm i . Through this ordering process, demand propagates from a root node toward leaf nodes.

Once demand reaches the leaf nodes, production begins to propagate backward to the root node. The production quantity $I_i(t)$ of node i is determined by the following Leontief-type production function:

$$I_i(t) = \min_{l \in \mathcal{S}(i)} [m_i(t), D_l(t), a_{il}(t) + k_{il}(t)], \quad (4)$$

where $m_i(t)$ is the random production capacity of node i , and $a_{il}(t)$ is the amount of product delivered from supplier l to firm i at time t . For leaf nodes without suppliers, production is only determined by $m_i(t)$ and $D_i(t)$. The delivery amount $a_{il}(t)$ is defined as

$$a_{il}(t) = \begin{cases} 0 & \text{if } D_l(t) = 0 \text{ or } l \notin \mathcal{S}(i), \\ \frac{\Omega_{il}(t)}{D_l(t)} I_i(t) & \text{otherwise.} \end{cases} \quad (5)$$

When several customer firms place orders with the same supplier l , the output $I_l(t)$ is allocated in proportion to their order quantities $\Omega_{il}(t)$, relative to the total demand

$D_l(t)$. The pairs $[I_i(t), a_{il}(t)]$ are then computed sequentially from leaf nodes to the root node [see Fig. 1 (a)].

After production and delivery are completed, both stocks and root's demand are updated. The stock update rule is

$$k_{il}(t+1) = \begin{cases} \min[s, a_{il}(t) + k_{il}(t) - I_i(t)] & \text{if } l \in \mathcal{S}(i), \\ 0 & \text{otherwise,} \end{cases} \quad (6)$$

where s is the stock capacity. Hence, firm i can store the unused quantity of product l after production, up to the capacity limit s . At time t , the root node satisfies external demand through its production $I_0(t)$, so the unmet demand is $u(t) = D_0(t) - I_0(t)$.

From Eq. (1), root's demand evolves as $D_0(t+1) = D_0(t) - I_0(t) + r$, or equivalently,

$$\Delta D_0(t) = r - I_0(t). \quad (7)$$

If the average output of the root node, $\langle I_0 \rangle$, is less than the incoming demand r , unmet demand accumulates over time, and the supply chain cannot satisfy external consumers. The maximum demand rate that can be sustained by the supply chain, referred to as critical demand, is therefore defined as $r^* \equiv \langle I_0 \rangle$.

Most recently, Feld and Barthelémy [14] have derived r^* analytically for a simple one-dimensional (1D) supply chain in the absence of stock capacity, i.e., $s = 0$, and showed that stock capacity can increase critical demand. While r^* is independent of the network structure for $s = 0$, positive stock capacity introduces a memory effect, making critical demand dependent on the topology of the supply chain. They focused on how stock capacity s and its interaction with network structure determine critical demand, but assumed uniformly distributed production capacity and non-substitutable inputs.

However, in real supply chains, both the magnitude of firm-level production shocks and the availability of substitute suppliers can vary substantially. Thus, we focus on the distribution of production capacity and the number of substitute suppliers and address how these two factors affect critical demand independently of stock capacity.

III. RESULTS

In this section, we present analytical derivations and numerical results for the effect of production capacity distributions and substitutable production functions on critical demand in supply chains.

A. Effect of production capacity distribution

Since the volume of production is non-negative, the capacity of production must also be non-negative, $m \geq 0$. We normalize the maximum external demand rate to

unity, so that the maximum production capacity required to satisfy this demand is also set to 1.

In the FB model [14], it is assumed that the production capacity $m_i(t)$ follows an independent and identically distributed (i.i.d.) uniform random variable on $[0, 1]$:

$$m_i(t) \sim \text{Uniform}(0, 1). \quad (8)$$

For this case, the variance of $m_i(t)$ remains fixed. However, in real supply chains, the volatility of production capacity can vary substantially depending on the nature of the supply chain, the industrial sector, and seasonality. To incorporate this effect, we consider the truncated normal distribution [see Fig. 1 (c)] as follows:

$$f(m; \mu, \sigma, a, b) = \frac{1}{\sigma} \frac{\phi\left(\frac{m-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}, \quad (9)$$

where ϕ and Φ are the probability density function (PDF) and the cumulative distribution function (CDF) of the normal distribution with the location parameter μ and the deviation parameter σ , respectively. a and b denote the lower and upper bounds of the support. For the special case $\mu = (a + b)/2$, we have $\mu = \mathbb{E}[m]$.

Throughout our study, we set $[a, b] = [0, 1]$. This choice ensures that f approaches $\text{Uniform}(0, 1)$ in the limit $\sigma \rightarrow \infty$. For the case of $\mu = 1/2 = \mathbb{E}[m]$, which has the same

average production capacity as Eq. (8), the only control parameter is the volatility of production capacity, σ .

For a 1D supply chain without stock capacity, i.e., $s = 0$, stock is always $k_{i,i+1}(t) = 0$ and demand at every node is equal to root's demand, $D_i(t) = D_0(t)$. For chain length N , applying the delivery rule $a_{i,i+1}(t)$ recursively to Eq. (4) gives $I_i(t) = \min[m_i(t), \dots, m_N(t), D_0(t)]$. Let $L = N - i$ be the distance from node i to leaf nodes. We define $M_L \equiv \min[m_i, \dots, m_N]$ as the minimum production capacity over this segment. Then, we get $I_i(t) = \min[M_L(t), D_0(t)]$. When demand is sufficiently large, $D_0(t) \geq M_L(t)$, the output of node i is determined by the minimum-capacity random variable M_L .

Because the production capacity m_i is i.i.d., the CDF of M_L can be obtained from that of individual capacities. The derivative yields the corresponding PDF. The average production of node i then follows from this PDF. If m_i is uniformly distributed in $[0, 1]$, $\langle I_i \rangle$ is given by the Beta integral, resulting in the closed form $\langle I_i \rangle = 1/(1 + N - i)$. Since critical demand is the average output of the root node, we obtain $r^* = \langle I_0 \rangle = 1/(N + 1)$.

Similarly, for m_i that follows a truncated normal distribution, critical demand can be derived from the CDF of the minimum-capacity variable. The result is given by [see Appendix A]

$$r^*(N, \sigma) = \frac{N}{\left[2\text{erf}\left(\frac{1}{2\sqrt{2}\sigma}\right)\right]^N} \sqrt{\frac{2}{\pi\sigma^2}} \int_0^1 m e^{-\frac{(2m-1)^2}{8\sigma^2}} \left[\text{erf}\left(\frac{1}{2\sqrt{2}\sigma}\right) - \text{erf}\left(\frac{2m-1}{2\sqrt{2}\sigma}\right)\right]^{N-1} dm. \quad (10)$$

Figure 2 shows theoretical predictions in Eq. (10) that agree well with the numerical simulation results for the various values of (N, σ) . For sufficiently large volatility $\sigma = 1$, critical demand becomes close to $1/(N + 1)$ as expected from the convergence of the capacity distribution to the uniform distribution in the limit $\sigma \rightarrow \infty$. Thus, increasing σ drives the critical demand toward the theoretical lower bound set by the uniform-capacity case.

From now on, we consider the case in which m_i follows a truncated normal distribution with $\mu \neq 1/2$. The average production capacity is

$$\bar{m} = \mu + \sigma \frac{\phi(\alpha) - \phi(\beta)}{\Phi(\beta) - \Phi(\alpha)}, \quad (11)$$

where $\alpha = (a - \mu)/\sigma$ and $\beta = (b - \mu)/\sigma$. Figure 2(b) shows the crossover points between supply chains with the larger average production capacity but higher volatility and those with the smaller average production capacity but lower volatility. This indicates a trade-off between production performance and stability that depends on the supply chain length N . For large N , a supply chain

with lower average capacity but smaller volatility can sustain larger critical demand.

Even when firms have the same average production capacity, high volatility σ induces a strong bottleneck effect and causes critical demand r^* to decrease rapidly. This effect becomes more pronounced as the number of firms (length) N in the supply chain increases. These results suggest that in supply chains where a failure or anomaly at a single node can create a system-wide bottleneck, synchronizing the production capacities of participating firms at a given time is more important than simply increasing their average production capacity.

B. Effect of substitutable production function

The production function in Eq. (4) describes a case in which all suppliers l of firm i provide non-substitutable inputs. However, in real supply chains, multiple firms may produce compatible inputs. To examine the role of substitute suppliers, we consider that all suppliers

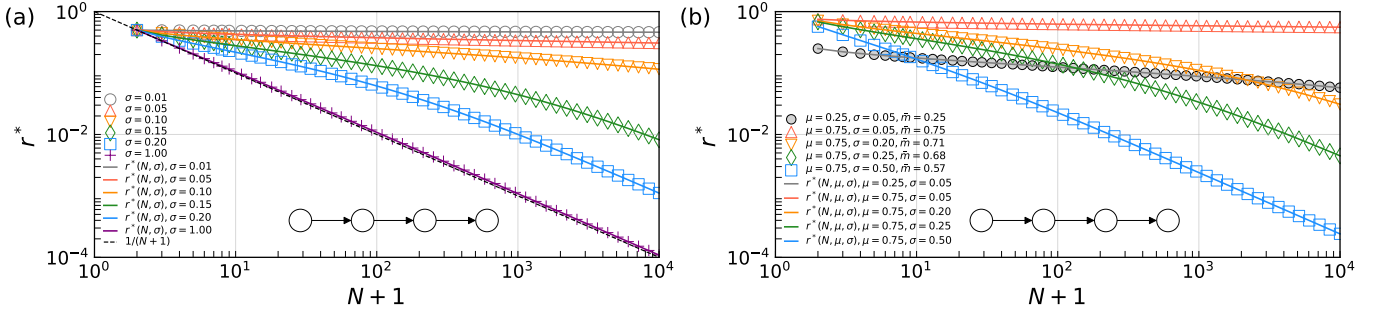


FIG. 2. Critical demand $r^*(N, \sigma)$ of 1D supply chains with no stock capacity and truncated-normal production capacities as a function of supply chain length N and volatility σ . Dots represent numerical simulation results and solid lines represent the analytical expressions in Eqs. (10) and (A3), evaluated by numerical integration. (a) Capacity distributions with the same location parameter $\mu = 1/2$ and different values of σ . For small σ , production is nearly deterministic and r^* depends weakly on N . In the large- σ limit, it approaches the baseline $r^* = 1/(N+1)$ for a uniform capacity distribution. (b) Different values of μ . Gray circle symbols represent $\mu = 0.25$ and other open symbols represent $\mu = 0.75$, which have the larger average production capacity \bar{m} in Eq. (11) than that of the gray ones.

$l \in \mathcal{S}(i)$ of firm i provide substitutable inputs. The production function is then given by

$$I_i(t) = \min \left[m_i(t), D_i(t), \sum_{l \in \mathcal{S}(i)} a_{il}(t) + k_i(t) \right]. \quad (12)$$

Because inputs are substitutable, stocks are no longer distinguished by supplier and indexed only by firm i . The stock update rule becomes

$$k_i(t+1) = \min \left[s, \sum_{l \in \mathcal{S}(i)} a_{il}(t) + k_i(t) - I_i(t) \right]. \quad (13)$$

The demand $D_i(t)$ and the delivery quantity $a_{il}(t)$ are defined in Eqs. (3) and (5), respectively.

However, the order quantity $\Omega_{il}(t)$ differs from the non-substitutable case. The firm i divides the required input quantity $D_i(t) - k_i(t)$ among its suppliers, so that $\sum_{l \in \mathcal{S}(i)} \Omega_{il}(t) = D_i(t) - k_i(t)$. If the order is divided equally among its suppliers, the order quantity becomes

$$\Omega_{il}(t) = \max \left(0, \frac{D_i(t) - k_i(t)}{|\mathcal{S}(i)|} \right), \quad (14)$$

where $|\mathcal{S}(i)|$ is the number of suppliers of firm i .

We denote the support of the production capacity of node i as $m_i \in [a_i, b_i]$, where a_i and b_i are non-negative real numbers. It is assumed that the total production capacity of all firms is bounded by the maximum external demand rate r_{\max} : $\sum_{i \in \mathcal{P}(j)} m_i \in [0, r_{\max}]$, where $\mathcal{P}(j)$ is the set of firms producing product j . We normalize this maximum demand rate to $r_{\max} \equiv 1$. For the non-substitutable case, $|\mathcal{P}(j)| = 1$ for every product j , and therefore all nodes have the same capacity range $[a_i, b_i] = [0, 1]$.

Let d denote the shortest-path distance from a node to the root, and call the set of nodes with the same depth a

layer. In a supply chain with fully substitutable inputs, all nodes in the same layer produce qualitatively identical products. Hence,

$$\max \left[\sum_{i \in \mathcal{V}(d)} m_i \right] = r_{\max} \equiv 1, \quad (15)$$

where $\mathcal{V}(d)$ is the set of nodes in layer d . Although there are many ways to distribute the maximum production capacity while satisfying Eq. (15), we follow the allocation rule considered in which the maximum production capacity of a customer node is evenly divided among its suppliers [8]. Thus,

$$\max [m_i] = \sum_{i \in \mathcal{C}(l)} \frac{\max [m_i]}{|\mathcal{S}(i)|}. \quad (16)$$

This value can be obtained sequentially from the root node to the leaf nodes. For example, in a regular tree with a branching factor z , the maximum production capacity of a node in layer d is $b_i = 1/z^d$. In this subsection, we mainly consider that m_i is uniformly distributed over its support.

As the simplest case, in Fig. 3(a), we consider a regular-tree supply chain with height $h = 2$, branching factor z , and no stock capacity ($s = 0$). When demand does not create a bottleneck, i.e., $D_0(t)/z \geq m_l(t)$, the output of the root node is

$$\begin{aligned} I_0(t) &= \min[m_0(t), D_0(t), \sum_{l \in \mathcal{S}(0)} a_{0l}(t)] \\ &= \min[m_0(t), \sum_{l \in \mathcal{S}(0)} m_l(t)]. \end{aligned} \quad (17)$$

Applying Eqs. (15) and (16) to this network structure ($h = 2, z$), the production capacities of the root node

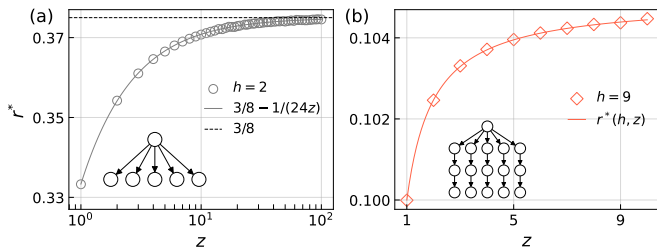


FIG. 3. Critical demand r^* against the number of substitute suppliers z . (a) Regular trees with $h = 2$. (b) Multiple-chain structure ($h = 9$), under no stock capacity, and a Leontief production function with substitutable inputs. Open symbols represent numerical simulation results, and the solid line represents the analytical result in Eqs. (19) and (20). r^* increases monotonically with a branching factor z . The insets show schematic illustrations of the network settings.

and its suppliers are given by

$$\begin{cases} m_0(t) & \sim \text{Uniform}(0, 1), \\ m_l(t) & \sim \text{Uniform}(0, 1/z), \\ \sum_{l=1}^z z m_l(t) & \sim \text{Irwin-Hall}(z). \end{cases} \quad (18)$$

For convenience, we define $X = m_0$ and $Y = \sum_l m_l$. The random variable X and each scaled supplier capacity $z m_l$ follow a standard uniform distribution. Thus, zY is the sum of z independent standard uniform random variables and follows an Irwin–Hall distribution. The mean output of the root node is $\mathbb{E}[I_0] = \mathbb{E}[\min(X, Y)]$. Since X and Y have the same support, the conditional mean for $Y = y$ is $\mathbb{E}[\min(X, Y) | Y = y] = \int_0^y x dx + (1 - y)y = y - y^2/2$. Therefore, $\mathbb{E}[\min(X, Y)] = \mathbb{E}[Y] - \mathbb{E}[Y^2]/2$. Using the moments of the Irwin–Hall distribution, we obtain

$$r^*(h = 2, z) = \frac{3}{8} - \frac{1}{24z}. \quad (19)$$

This behavior contrasts with the non-substitutable case, in which critical demand r^* decreases as the number of required suppliers increases. When all inputs to the root node are substitutable, r^* instead increases with the number of substitute suppliers z and approaches $3/8$ as z becomes large [see Fig. 3(a)].

In general, one considers a multiple-chain structure in which the root node is connected to a branching factor z independent 1D chains. Using the moments of the sum of beta-distributed random variables, critical demand is given by [see Fig. 3(b)]

$$r^*(h, z) = \frac{1}{h} \left(1 - \frac{1}{2h} \right) - \frac{(h-1)}{2h^2(h+1)z}. \quad (20)$$

For regular trees with $h > 1$, increasing h introduces more interlayer bottlenecks and therefore decreases r^* . In contrast, increasing z raises the number of substitute suppliers and is expected to increase r^* . Figure 4(a) shows the corresponding numerical simulation results.

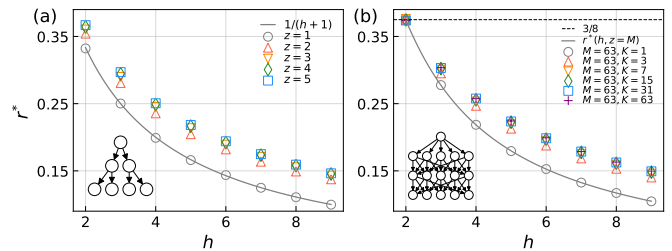


FIG. 4. Critical demand r^* against height h , under no stock capacity and a Leontief production function with substitutable inputs. Symbols represent numerical simulation results for (a) regular-tree structure with various z and (b) regular-lattice structure with the number of nodes and links, (M, K) in each lattice layer, respectively. The insets show schematic illustrations of the network settings.

Importantly, the increase in r^* occurs even though the total maximum production capacity of the supply chain is fixed at unity, regardless of the number of participating firms. This suggests that, even with the same total scale of production facilities, final output can be increased by subdividing production units subject to independent stochastic shocks. In this sense, supplier diversification mitigates the impact of individual production shocks within the supply chain.

Finally, in Fig. 4(b), we consider that all layers, except the root layer, form regular networks, where each non-root layer consists of M nodes. The root node has M links, and every other node has the network connectivity $K \leq M$ links. The case of $K = 1$ corresponds to the multiple-chain structure. Unlike a regular tree, where increasing z also increases the total number of nodes, this setting allows us to vary the network connectivity K while keeping the number of nodes fixed. Thus, we can isolate the effect of network structure on r^* independently of h and the number of participating firms N .

Figure 4(b) shows that the larger K gets the larger r^* . This result is notable because, in the non-substitutable case, network topology affects critical demand only when stock capacity is present ($s > 0$). With substitutable inputs, however, r^* depends on K even in the absence of stock capacity ($s = 0$).

IV. CONCLUSION

We investigated two extensions of the Feld–Barthelemy model, the effect of production-capacity volatility and substitutable inputs on critical demand r^* : First, we presented that the lower volatility in random production capacity increases r^* in the supply chain. This effect becomes more pronounced as the supply chain length N increases, because a local production shock is more likely to create a system-wide bottleneck in long sequential structures. Thus, in highly dependent supply chains, synchronizing production capacities across firms can be more important than simply increasing their average pro-

duction capacity. Second, we showed that substitutable inputs can increase critical demand even when the total maximum production capacity is fixed. When multiple suppliers provide compatible inputs, production shocks are dispersed across independent suppliers, thereby reducing the impact of individual fluctuations. In addition, the substitutable-input case exhibits a network-topology effect even in the absence of stock capacity. This contrasts with the non-substitutable case, where network topology affects critical demand mainly through the memory effect induced by stock capacity.

These results suggest that supply chain resilience can be improved not only by increasing stock capacity but also by reducing production-capacity volatility and diversifying substitute suppliers. This implication is particularly relevant for just-in-time production systems, where firms often avoid large inventories because of storage costs. In such systems, the reliable coordination of delivery cycles and the availability of alternative suppliers can serve as important mechanisms for mitigating stochastic production shocks.

The importance of alternative suppliers and supply routes is also evident in real-world disruptions. The paralysis of the Japanese automobile industry after the 2011 earthquake [15] and the disruption of global maritime logistics caused by the Suez Canal bottleneck in 2021 [16] illustrate how limited substitutability and concentrated supply routes can amplify local failures into system-wide disruptions. Our results provide a simple theoretical mechanism for this observation: Diversification across substitutable suppliers or routes disperses shocks and reduces the likelihood that a single bottleneck constrains final production.

Several extensions remain for possible future works: First, production-capacity shocks may be correlated across firms that share common infrastructure, such as power grids, communication systems, or transportation networks. Second, firms may adaptively allocate orders [8] or target production [17] according to supplier reliability or expected production capacity rather than dividing orders equally. Finally, incorporating financial constraints, prices, and competition among substitute suppliers would allow the model to address strategic behavior and profitability in addition to physical production constraints.

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Appendix A: Derivation of critical demand for one-dimensional (1D) chain with no stock

For a 1D chain with no stock capacity, $s = 0$, the critical demand r^* is the average output of the root node under sufficiently large demand. Applying $D_0 \geq m_i$ to Eq. (4) gives

$$r^* \equiv \langle I_0 \rangle = \mathbb{E}[\min(m_0, m_1, \dots, m_{N-1})], \quad (\text{A1})$$

where N is the number of nodes in the chain. Let m_i be i.i.d. random variables with CDF $F(m)$ and PDF $f(m)$. For $I_0 = \min_{i=0, \dots, N-1} m_i$, we have

$$\mathbb{P}(I_0 > m) = \mathbb{P}(m_0 > m, \dots, m_{N-1} > m) = [1 - F(m)]^N.$$

Thus, the CDF and PDF of I_0 are

$$\begin{aligned} F_{I_0}(m) &= 1 - [1 - F(m)]^N, \\ f_{I_0}(m) &= N f(m) [1 - F(m)]^{N-1}. \end{aligned}$$

The average root output is therefore

$$\langle I_0 \rangle = N \int_a^b m f(m) [1 - F(m)]^{N-1} dm, \quad (\text{A2})$$

where $[a, b]$ is the support of m_i .

For a truncated normal distribution,

$$\begin{aligned} f(m) &= \frac{\varphi\left(\frac{m-\mu}{\sigma}\right)}{\sigma \left[\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right) \right]}, \\ F(m) &= \frac{\Phi\left(\frac{m-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}. \end{aligned}$$

Substituting these expressions into Eq. (A2) yields

$$\begin{aligned} r^*(N; \mu, \sigma, a, b) &= \frac{N}{\left[\operatorname{erf}\left(\frac{b-\mu}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{a-\mu}{\sqrt{2}\sigma}\right) \right]^N} \sqrt{\frac{2}{\pi\sigma^2}} \\ &\int_a^b m e^{-\frac{(m-\mu)^2}{2\sigma^2}} \left[\operatorname{erf}\left(\frac{b-\mu}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{m-\mu}{\sqrt{2}\sigma}\right) \right]^{N-1} dm. \end{aligned} \quad (\text{A3})$$

The solid lines in Fig. 2 are obtained from numerically evaluating Eq. (A3).

Appendix B: Complementary numerical simulations

1. Effect of monopolistic layers

Some supply chains contain unavoidable intermediate nodes, such as logistics hubs, ports, or monopolistic suppliers. In network terms, these nodes have high betweenness centrality and can create bottlenecks even when inputs are otherwise substitutable. To examine this effect,

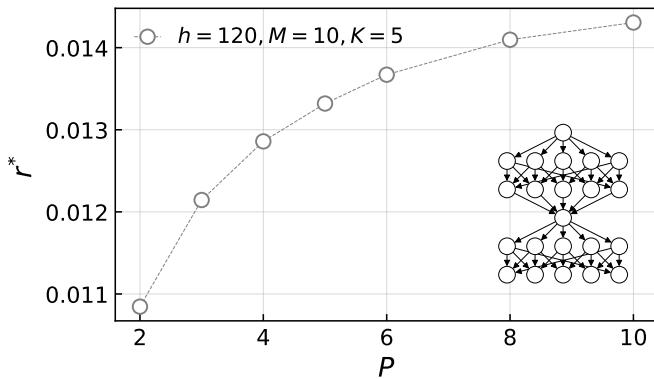


FIG. A1. r^* against monopolistic-layer period P in a repeated bottleneck structure. Open symbols represent numerical simulation results. The stock capacity is $s = 0$, h is the supply chain height, and M and K are the number of nodes and links in each non-monopolistic layer, respectively. Orders and capacities follow Eqs. (14) and (16).

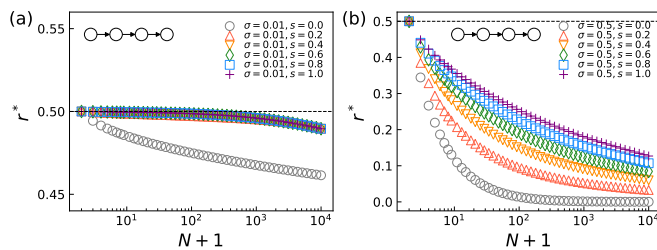


FIG. A2. r^* against $N + 1$ in 1D chain with truncated-normal production capacities. For all cases, we set $\mu = 1/2$ with (a) $\sigma = 0.01$ and (b) $\sigma = 0.5$. Symbols represent numerical simulation results for $s = \{0.0, 0.2, \dots, 1.0\}$.

we consider a structure in which the regular lattice used in the main text is repeated with monopolistic layers.

Figure A1 shows critical demand r^* as a function of the monopolistic-layer period P , with fixed supply chain height $h = 120$ and fixed number of non-monopolistic nodes $M = 10$ in each layer. More frequent monopolistic layers reduce critical demand, indicating that bottleneck frequency affects supply chain performance even when the total length h is fixed.

2. Effect of stock capacity

The main text focuses on the no-stock case, $s = 0$, to isolate the effects of production-capacity volatility and substitute suppliers. In this subsection, we briefly exam-

ine how stock capacity modifies these effects. Consistent with Feld and Barthelémy [14], increasing s generally increases critical demand and can introduce dependence on network topology.

Figure A2 shows r^* for a 1D chain with truncated-normal production capacities. When volatility is high, stock capacity substantially increases critical demand by buffering temporary mismatches between delivered inputs and production capacity. When volatility is low, production and delivery become nearly deterministic, so little stock accumulates, and critical demands are almost similar for sufficiently large s [see Fig. A2(a)].

Figure A3 shows r^* as a function of stock capacity in a regular tree with $h = 2$. As the number of substitute suppliers z increases, deliveries to the root node become more stable. Stock then mainly buffers the mismatch between the root production capacity $m_0(t)$ and the aggregate supplier output $\sum_{l \in S(0)} m_l(t)$. Therefore, even a large stock capacity cannot increase critical demand beyond the average production capacity of the root node, $\langle m_0 \rangle = 1/2$.

In the FB model [14], the external demand is served by a single root node, so fluctuations in unmet demand are strongly affected by root's production capacity. If the top layer contains many nodes instead of a single root node, and the external demand is also divided among them as in Eq. (14), the aggregate delivery to external consumers becomes increasingly deterministic as the number of top-layer nodes increases. In this limit, increasing the number of substitute supplier nodes plays a role analogous to reducing the production-capacity volatility, and the sum of product delivered follows a rescaled Irwin-Hall distribution that approximately converges to the truncated-normal distribution with $\sigma^2 \sim 1/(12z)$ for $z \rightarrow \infty$ limit by the central limit theorem.

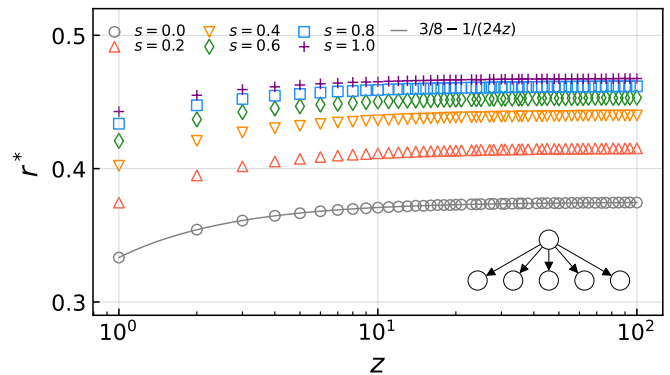


FIG. A3. r^* against z for regular tree with $h = 2$. Symbols represent numerical simulation results for $s = \{0.0, 0.2, \dots, 1.0\}$.

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