MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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SS 2002
Sheet#9
June 18, 2002

Qualitative Theory of Ordinary Differential Equations

27. Assume that $A : I \subset \mathbb{R} \to \mathrm{End}\,(\mathbb{C}^n)$ is continuous, $\mu_0 \in I$, and that $W \subset \mathbb{C}$ is an open neighborhood of the spectrum $\sigma\,(A_{\mu_0})$. ($A_\mu := A\,(\mu)$.)

    Show that there exist $a, b \in \mathbb{R}, a < \mu_0 < b$ with $\sigma\,(A_\mu) \subset W$ for all $\mu \in [a, b] \cap I$.

    (4)

28. Assume that $I \subset \mathbb{R}$ is compact and $A : I \to \mathrm{End}\,(\mathbb{C}^n)$ is continuous. For $\mu \in I$ denote $A_\mu := A\,(\mu)$. Choose $\alpha > 0$ such that $Re\,(\lambda) \notin [-\alpha, \alpha]$ for all $\lambda \in \sigma\,(A_\mu)\,, \mu \in I$.

    Show that there exists $C_0 > 0$ such that for all $\mu \in I$:

    $$| \, e^{tA_\mu} x_0 \, | \leq C_0 e^{-\alpha t} \, | \, x_0 \, |$$

    for all $x_0 \in S\,(A_\mu)$ and $t > 0$, and

    $$| \, e^{tA_\mu} x_0 \, | \leq C_0 e^{\alpha t} \, | \, x_0 \, |$$

    for all $x_0 \in U\,(A_\mu)$ and $t < 0$.

    (8)

29. We consider the van der Pol's equation and $y_1 \in \mathbb{R}^+, \delta : \mathbb{R}^+ \to \mathbb{R}$ as in exc. 26.
    a) Show that δ is strictly decreasing on $[y_1, \infty[$ with $\lim_{y_0 \to \infty} \delta\,(y_0) = -\infty$.
    b) Show that the van der Pol's equation has exactly one periodic orbit.

    Hint (for a)): For $y_0 > y_1$ let $0 = s_0\,(y_0) < s_1\,(y_0) < s_2\,(y_0) < s_3\,(y_0) = t_2\,(y_0)$ with $\phi_1\,(s_1\,(y_0)\,, (0, y_0)) = \phi_1\,(s_2\,(y_0)\,, (0, y_0)) = 1$.
    Consider $\delta_j\,(y_0) := \int_{s_{j-1}(y_0)}^{s_j(y_0)} \phi_1^2\,(t, (0, y_0))\,(1 - \phi_1^2\,(t, (0, y_0)))\,dt$ for $j = 1, 2, 3$.

    (7)

Due: Tuesday, June 25, 2002; 11:15 a.m.