MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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SS 2002
Sheet#8
June 11, 2002

Qualitative Theory of Ordinary Differential Equations

24. Find all solutions of

$$\dot{x_1} = x_1$$

$$\dot{x_2} = -x_2 + x_1^3$$

and determine the stable and unstable manifold at $(0,0)$. (5)

25. Let $U \subset \mathbb{R}^n$ be open, $g \in C^2(U;\mathbb{R})$ and $f(x) := \operatorname{grad} g(x)$. Let $x_1 \neq 0, x_2 \neq 0$ be solutions of $\dot{x} = f(x)$ with $\alpha(x_1) = \omega(x_2) = \{0\}$, and assume that $f'(0)$ is non-singular. Show that 0 is a saddle point.
Hint: Prove first that $\sigma(f'(0)) \subset \mathbb{R}$.

(5)

26. Consider $W : \mathbb{R}^2 \to \mathbb{R}_0^+$, $W(z) := \frac{1}{2} \mid z \mid^2$.
(a) Show that $(W\psi)' = -\psi_1.(\psi_1^3 - \psi_1)$ holds for any solution $\psi = (\psi_1, \psi_2) : I \to \mathbb{R}^2$ of the van der Pol's equation

$$\dot{x} = y - x^3 + x$$

$$\dot{y} = -x.$$

(b) Consider $\delta : \mathbb{R}^+ \to \mathbb{R}, \delta(y_0) := \frac{1}{2}\left(\alpha(y_0)^2 - y_0^2\right)$. (Compare sheet 7.)
Show that for $y_0 \in \mathbb{R}^+$ and $\psi(t) := \phi(t, (0, y_0))$

$$\delta(y_0) = \int_0^{t_2(y_0)} \psi_1^2(t)\left(1 - \psi_1^2(t)\right) dt$$

holds.
(c) Prove $\delta(y_0) > 0$ for $0 < y_0 \leq y_1$. (y_1 defined in exc. 23.) (7)

Due: Tuesday, June 18, 2002; 11:15 a.m.