MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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SS 2002
Sheet#7
June 4, 2002

Qualitative Theory of Ordinary Differential Equations

22. We continue to consider the van der Pol's equation

$$\dot{x} = y - x^3 + x$$
$$\dot{y} = -x.$$

Let $A_1 := \{0\} \times \mathbb{R}^+, A_2 := \{(x, x^3 - x) : x > 0\}$,
$A_3 := \{0\} \times R^-, A_4 := \{(x, x^3 - x) : x < 0\}$,
$B_1 := \{(x, y) : x > 0, y > x^3 - x\}$,
$B_2 := \{(x, y) : x > 0, y < x^3 - x\}$,
$B_3 := \{(x, y) : x < 0, y < x^3 - x\}$,
$B_4 := \{(x, y) : x < 0, y > x^3 - x\}$.
Let $(x, y) : I \to \mathbb{R}^2$ be a maximal solution of the van der Pol's equation with $0 \in I$ and $(x, y)(0) \in A_1$. Show:
a) There is $t_1 \in I \cap]0, \infty[$ with $(x, y)(]0, t_1[) \subset B_1$ and $(x, y)(t_1) \in A_2$.
b) There is $t_2 \in I \cap]t_1, \infty[$ with $(x, y)(]t_1, t_2[) \subset B_2$ and $(x, y)(t_2) \in A_3$.
c) (x, y) turns clockwise around $(0, 0)$ infinitely many times.

(6)

23. Let $\phi = (\phi_1, \phi_2) : \Omega \subset \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}^2$ be the flow of the van der Pol's equation. Let $\alpha : \mathbb{R}^+ \to \mathbb{R}^+, \alpha(y_0) := -\phi_2(t_2, (0, y_0))$ with $t_2 \in \mathbb{R}^+$ be the minimal $t_2 > 0$ with the property from exc.22b. Show:
a) t_2 depends continuously on y_0. (Hint: implicit function theorem.)
b) α is continuous.
c) $\phi(., (0, y_0))$ periodic $\Longleftrightarrow \alpha(y_0) = y_0$.
d) There is a unique $y_1 \in \mathbb{R}^+$ with $\phi_2(t_1, (0, y_1)) = 0$, where $t_1 \in \mathbb{R}^+$ is the minimal $t_1 > 0$ with the property from exc.22a.

(10)

Due: Tuesday, June 11, 2002; 11:15 a.m.