MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
H. Steinlein

SS 2002
Sheet#6
May 28, 2002

Qualitative Theory of Ordinary Differential Equations

18. Consider the equations

$$\dot{x} = x(\alpha - \epsilon x - \beta y)$$
$$\dot{y} = y(-\gamma + \delta x - \kappa y)$$

with $\alpha, \beta, \gamma, \delta > 0$ and $\epsilon, \kappa \geq 0$.

Prove:
a) If $\epsilon \in \left]0, \frac{\alpha\delta}{\gamma}\right[$, then the open quadrant $]0,\infty[^2$ contains exactly one equilibrium (x_0, y_0).
b) If moreover $\epsilon, \kappa > 0$, then for any solution with $(x(0), y(0)) \in\ ]0,\infty[^2$ we have $\lim_{t\to\infty}(x(t), y(t)) = (x_0, y_0)$.
Hint: Consider $F(x,y) = \delta(x - x_0 \ln x) + \beta(y - y_0 \ln y)$. (6)

19. Let $f \in C^1(U, \mathbb{R}^n)$, $U \subset \mathbb{R}^n$ open. Show that the hyperbolic points of $\dot{x} = f(x)$ are isolated. (3)

20. We consider the van der Pol's equation

$$\dot{x} = y - x^3 + x$$
$$\dot{y} = -x.$$

Linearize the van der Pol's equation at its equilibrium, compute the eigenvalues of this linearization and sketch the phase portrait. (4)

21. Compute and draw the vector field of the van der Pol's equation on the set

$$\left\{\left(x, x^3 - x\right) : x \in \mathbb{R}\right\} \cup \{(0, y) : y \in \mathbb{R}\} \subset \mathbb{R}^2.$$

(3)

Due: Tuesday, June 4, 2002; 11:15 a.m.