MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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SS 2002
Sheet#5
May 16, 2002

Qualitative Theory of Ordinary Differential Equations

15. Determine all $\alpha \in \mathbb{R}$ such that $(0,0) \in \mathbb{R}^2$ is stable for

$$\dot{x}_1 = -x_2 + \alpha \left(x_1^2 + x_2^2\right) x_1$$

$$\dot{x}_2 = x_1 + \alpha \left(x_1^2 + x_2^2\right) x_2.$$

(5)

16. Consider $\dot{x} = f(x)$ with $f \in C^{1-}(\mathbb{R}^n)$. Assume that $y \in \mathbb{R}^2$ is an asymptotically stable stationary point.

    Prove that each solution $x : (0, \infty) \to \mathbb{R}^n$ with $\omega(x) = \{y\}$ is asymptotically stable.

(6)

17. Consider the equation of the damped pendulum:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\alpha x_2 - \sin x_1 \ , \ \alpha > 0.$$

a) Show that the energy $E(x_1, x_2) = \frac{1}{2} x_2^2 - \cos x_1 + 1$ is a Lyapunov function.

b) Which maximal solutions are asymptotically stable, which maximal solutions are unstable?

(7)

Due: Thursday, May 23, 2002; 1915 a.m.