MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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SS 2002
Sheet#4
May 7, 2002

Qualitative Theory of Ordinary Differential Equations

12. Let $x : \mathbb{R} \to \mathbb{R}^2$ be a solution of $\dot{x} = f(x)$. Assume that $\omega(x)$ is a non-constant periodic orbit. Prove: if $x(0) \notin \omega(x)$, then $\alpha(x) \cap \omega(x) = \emptyset$. (3)

13. Let $x : \mathbb{R} \to \mathbb{R}^2$ be a *bounded* solution of $\dot{x} = f(x)$. Prove that there is $y \in \mathbb{R}^2$ with $f(y) = 0$.

    Hint: Use Zorn's lemma and the fact that a decreasing family of compact sets has nonempty intersection (Cantor's theorem). (5)

14. Consider the system of equations

$$\dot{u} = v - G(u)$$

$$\dot{v} = -u$$

where $G \in C^1(\mathbb{R}, \mathbb{R})$ satisfies the following conditions:
(a) G is odd: $G(-u) = -G(u)$ for all $u \in \mathbb{R}$,
(b) $lim_{u\to\infty} G(u) = \infty, lim_{u\to-\infty} G(u) = -\infty$,
(c) there is a $\beta > 0$ such that $G(\beta) = 0$,
(d) $G(u)$ is monotonely increasing for $\mid u \mid > \beta$.

Prove that there exists a non-constant periodic orbit. (8)

Due: Tuesday, May 14, 2002; 11:15 a.m.