MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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SS 2002
Sheet#3
April 30, 2002

Qualitative Theory of Ordinary Differential Equations

8. Determine $\alpha(x)$ and $\omega(x)$ for an (arbitrary) solution of

$$\dot{x}_1 = -x_2 + \left(1 - x_1^2 - x_2^2\right) x_1$$

$$\dot{x}_2 = x_1 + \left(1 - x_1^2 - x_2^2\right) x_2.$$

(6)

9. Let $x : \mathbb{R} \to U$ be a solution of $\dot{x} = f(x)$. Assume $\omega(x) = \{y\}$, $y \in \overline{U}$.

   Show that $lim_{t\to\infty} x(t) = y$. (4)

10. Assume that $x : \mathbb{R} \to U$ solves $\dot{x} = f(x)$ and that $\omega(x)$ is bounded.

    Show that $\omega(x)$ is connected. (4)

11. Let $U \subset \mathbb{R}^n$ be an open subset, $g \in C^2(U; \mathbb{R})$ and $f : U \to \mathbb{R}^n$ be given by $f(x) := \operatorname{grad} g(x)$. Let $W := \{x \in U : f(x) = 0\}$.

    Prove: for any solution x of $\dot{x} = f(x)$ is

$$(\alpha(x) \cup \omega(x)) \cap U \subset W.$$

(5)

Due: Tuesday, May 7, 2002; 11:15 a.m.