MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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SS 2002
Sheet#2
April 23, 2002

Qualitative Theory of Ordinary Differential Equations

5. Consider the system of ordinary differential equations

$$\dot{x_1} = x_2$$
$$\dot{x_2} = -x_1\left(1 - x_1^2\right).$$

Compute its equilibria, the respective linearisations in the equilibria, and draw the phase portraits in neighborhoods of the equilibria.

Let $0 <\mid a \mid< 1$. Show that the solutions $(x_1(t), x_2(t))$ with $(x_1(0), x_2(0)) = (a, 0)$ are periodic. (Hint: use symmetries of the ODE.)

Sketch the complete phase portrait. (6)

6. Find some $f : \mathbb{R}^4 \to \mathbb{R}^4$ such that there exists a solution $x : \mathbb{R} \to \mathbb{R}^4$ of $\dot{x} = f(x)$ with the following property: the orbit $x(\mathbb{R})$ is a dense subset of

$$T := \left\{(z_1, z_2) \in \mathbb{R}^2 \times \mathbb{R}^2 = \mathbb{R}^4 :\mid z_1 \mid=\mid z_2 \mid= 1\right\}.$$

(T is called the 2-torus.) The denseness has to be proved! (5)

7. Let I be an interval with $0 \in I$ and $\dot{I} \neq \emptyset$, and let $U \subset \mathbb{R}^m$ be an open subset of $\mathbb{R}^m$. Prove the following statement for $L = \mathbb{R}^n$ and $L = \mathrm{End}(\mathbb{R}^n)$:

for any $A \in C^0(I \times U; L)$ and any $x_0 \in L$, there exists a **continuous** mapping $\Phi \in C^0(I \times U; L)$ such that, for any $u \in U$, $x(.) := \Phi(., u)$ is the unique solution of

$$\dot{x} = A(t, u)\, x$$
$$x(0) = x_0.$$

(7)

Due: Tuesday, April 30, 2002; 11:15 a.m.