MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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SS 2002
Sheet#10
June 25, 2002

Qualitative Theory of Ordinary Differential Equations

30. **Counterexamples to Hartman-Grobman if $\sigma(A) \cap i\mathbb{R} \neq \emptyset$.**
i) Show that $\dot{x} = x^2$ on $\mathbb{R}^1$ is not flow-equivalent to its linearization at 0.
ii) Show that $\dot{x} = f(x)$ with

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is not flow-equivalent to its linearization at 0.
iii) Find some function $f : \mathbb{R}^2 \to \mathbb{R}^2$ of the form

$$f(x) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x + o(x)$$

such that $\dot{x} = f(x)$ has a countable infinite number of periodic orbits. Conclude that $\dot{x} = f(x)$ is not flow-equivalent to its linearization at $(0,0)$.

(6)

31. Which condition must a polynomial $P(\mu) = \sum_{i=0}^{2} a_i \mu^i$ with $a_0, a_1, a_2 \in \mathbb{R}, a_2 \geq 0$ satisfy for that there exists some $\mu_0 \in \mathbb{R}$ such that the differential equation

$$\dot{x}_1 = P(\mu_0)\, x_1 + x_2 + \left(x_1^2 + x_2^2\right) x_1$$
$$\dot{x}_2 = P(\mu_0)\, x_2 - x_1 + \left(x_1^2 + x_2^2\right) x_2$$

has periodic solutions?

(4)

Due: Tuesday, July 2, 2002; 11:15 a.m.