MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
H. Steinlein

SS 2002
Sheet#1
April 16, 2002

Qualitative Theory of Ordinary Differential Equations

1. Let $U \subset \mathbb{R}^n$ be open and $f \in C^{1-}(U;\mathbb{R}^k)$ $(k, n \in \mathbb{N})$. Let $K \subset U$ be compact. Show that $f|K$ is Lipschitz continuous. (4)

2. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) := 1 + x^2$. Determine explicitly the flow $\varphi : D(\varphi) \subset \mathbb{R}^2 \to \mathbb{R}$ and draw its domain $D(\varphi)$. (4)

3. Let
$$\Omega := \{(t,x) \in \mathbb{R}^2 \quad | \quad x > 0 \Rightarrow x > t\}$$
and $\varphi : \Omega \to \mathbb{R}$,
$$\varphi(t,x) := x - t.$$
Show that Ω is not open, but all other conditions for a flow are fulfilled.
Show that φ can be extended in exactly one way to a flow $\overline{\varphi}$. (6)

4. Let $U \subset \mathbb{R}^n$ be open and $f \in C^{1-}(U;\mathbb{R}^n)$. Let $x : \mathbb{R} \to U$ be a solution of $x' = f(x)$. Show that if x is not injective, then x is constant or periodic. (4)

Due: Tuesday, April 23, 2002; 11:00 a.m.