# Industrious Number Theory 3

November 10 – 13, 2010 at Seminar Room 1503 Korea Institute for Advanced Study

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# Time Table

	10日(水)	11日(木)	12日(金)	13日(土)
9:30-10:30	Park	Han	Hattori	FD
11:00-12:00	Tsushima	Abe	Lee	FD
2:00-3:00	Jeon	Mieda	Yoshida	EX
3:30-4:30	Arai	Bender	Jang	EX
5:00-6:00			Hiranouchi	EX

FD := Free Discussions

EX := Excursion

# Program

### 11月10日(水)

9:30–10:30 PARK Jeehoon (POSTECH)  $U_p$ -eigenvalue of eigenvariety for definite quaternion algebras and Teitelbaum-type L-invariant

- 11:00–12:00 TSUSHIMA Takahiro (Keio Univ.) On the stable reduction of  $X_0(p^4)$
- 12:00–14:00 Lunch
- 14:00–15:00 JEON Daeyeol (Kongju National Univ.) Bielliptic modular curves
- 15:30–16:30 ARAI Keisuke (Tokyo Denki Univ.) Algebraic points on Shimura curves of  $\Gamma_0(p)$ -type

#### 11月11日(木)

- 9:30–10:30 HAN Lin (Inha Univ.) Generation of coefficient fields of Galois representations by a single trace of Frobenius
- 11:00–12:00 ABE Tomoyuki (Univ. of Tokyo) The theory of arithmetic *D*-modules and characteristic cycles
- 12:00–14:00 Lunch
- 14:00–15:00 MIEDA Yoichi (Kyushu Univ.) Lefschetz trace formula for adic spaces and applications
- 15:30–16:30 BENDER Andreas (POSTECH) Quantitative aspects of the Goldbach conjecture in the function field case

# 11月12日(金)

- 9:30–10:30 HATTORI Shin (Kyushu Univ.) Ramification correspondence of finite flat group schemes of equal and mixed characteristics
- 11:00–12:00 LEE Jungjo (Yonsei Univ.) On a ramification filtration computation
- 12:00–14:00 Lunch
- 14:00–15:00 YOSHIDA Manabu (Kyushu Univ.) Fontaine's property  $(P_m)$  at the maximal ramification break
- 15:30–16:30 JANG Junmyeong (KIAS) The ordinarity of an isotrivial elliptic fibration
- 17:00–18:00 HIRANOUCHI Toshiro (Hiroshima Univ.) On the image of the cycle map for the product of elliptic curves over a p-adic field

# 11月13日(土)

- 午前 Free discussions
- 午後 Excursion

# Abstracts

#### ABE Tomoyuki (Univ. of Tokyo)

**Title:** The theory of arithmetic  $\mathscr{D}$ -modules and characteristic cycles **Abstract:** In this talk, I want to introduce the characteristic cycles of arithmetic  $\mathscr{D}$ -modules due to P. Berthelot, and see how they can be used. After a brief review of the theory of Berthelot, we will see the relation between characteristic cycles and Swan conductors. Then I will introduce the ring of microdifferential operators for the further analysis on characteristic cycles. We will conclude this talk by pointing out the future application of the theory.

#### ARAI Keisuke (Tokyo Denki Univ.)

**Title:** Algebraic points on Shimura curves of  $\Gamma_0(p)$ -type

**Abstract:** We classify the characters associated to algebraic points on Shimura curves of  $\Gamma_0(p)$ -type, and show that there are few points over quadratic fields on such Shimura curves.

#### **BENDER Andreas** (POSTECH)

**Title:** Quantitative aspects of the Goldbach conjecture in the function field case

**Abstract:** Under some restrictive assumptions it is possible to prove a statement analogous to the Goldbach conjecture for polynomials in one variable over a finite field of odd characteristic. I shall discuss heuristic expectations of how many pairs of primes sum up to a given polynomial and how to prove this in certain cases.

#### HAN Lin (Inha Univ.)

**Title:** Generation of coefficient fields of Galois representations by a single trace of Frobenius

**Abstract:** The original question starts from the case of Galois representation attached to classical newforms. Galois representations coming from newforms generally have a type of images as large as possible. Then we are interested in whether the coefficient fields of these representations can be generically generated by just a single trace of Frobenius element. We generalize this result to the case of Galois representation where Eichler-Shimura type theory is established.

#### HATTORI Shin (Kyushu Univ.)

**Title:** Ramification correspondence of finite flat group schemes of equal and mixed characteristics

Abstract: Let K be a complete discrete valuation field of mixed characteritic (0, p) with perfect residue field k. It is well-known that we have an equivalence of categories between the category of finite flat group schemes over  $\mathcal{O}_K$  killed by p and a category of finite flat group schemes over k[[u]] killed by their Verschiebungs which induces the field-of-norm functor on the generic fiber. In this talk, we show that the ramification subgroups of two corresponding finite flat group schemes of equal and mixed characteristics via this equivalence are isomorphic to each other.

#### HIRANOUCHI Toshiro (Hiroshima Univ.)

**Title:** On the image of the cycle map for the product of elliptic curves over a p-adic field

**Abstract:** We study the Chow group of 0-cycles (modulo  $p^n$ ) on the product of elliptic curves over a *p*-adic field. In particular, we obtain the structure of the image of the Albanese kernel for such abelian variety by the cycle class map. The key ingredient is the structure theorem of the graded quotients associated with a certain isogeny of formal groups. (joint work with Seiji Hirayama)

#### **JANG Junmyeong** (KIAS)

Title: The ordinarity of an isotrivial elliptic fibration

**Abstract:** In this talk, we will see a criterion of the ordinarity for isotrivial elliptic surfaces over a field of positive characterisitc in terms of the generic fiber and a suitable covering of the base. Using this, we will prove the ordinary reduction theorem for some isotrivial elliptic surfaces defined over a number field.

#### JEON Daeyeol (Kongju National Univ.)

Title: Bielliptic modular curves

**Abstract:** A smooth, projective curve X with genus  $g(X) \ge 2$  is called *bielliptic* if it admits a map from X to an elliptic curve of degree 2. In this talk, we present a brief review of the classification of bielliptic modular curves and its applications.

#### LEE Jungjo (Yonsei Univ.)

Title: On a ramification filtration computation

**Abstract:** For a finite extension  $F|\mathbb{Q}_p$ , we describe completely the structure of the filtered  $\mathbb{F}_p[G]$ -module  $K^{\times}/K^{\times p}$  where  $K = F(\sqrt[p-1]{F^{\times}})$ , the maximal abelian extension of F of exponent dividing p-1, and  $G = \operatorname{Gal}(K|F)$ . We also determine the conjugation action of  $\operatorname{Gal}(K|F)$  on  $\operatorname{Gal}(M|K)$  where M is the maximal abelian extension of exponent p of the local field K.

## MIEDA Yoichi (Kyushu Univ.)

Title: Lefschetz trace formula for adic spaces and applications

Abstract: In this talk, I will give a Lefschetz trace formula for quasicompact adic spaces over an algebraically closed non-archimedean field, and explain how it can be used to calculate the  $\ell$ -adic cohomology of some Rapoport-Zink spaces. If time permits, I also would like to explain another Lefschetz trace formula for open adic curves, which is related to the ramification theory for certain valuation rings of higher rank.

#### PARK Jeehoon (POSTECH)

**Title:**  $U_p$ -eigenvalue of eigenvariety for definite quaternion algebras and Teitelbaum-type *L*-invariant.

**Abstract:** We give a precise relationship between the Teitelbaum type L-invariant for Hilbert modular forms and the derivative of the  $U_p$ -eigenvalue of a p-adic family of automorphic forms on a definite quaternion algebra over a totally real field. This is a joint work with Chung Pang Mok and Masataka Chida.

#### TSUSHIMA Takahiro (Keio Univ.)

**Title:** On the stable reduction of  $X_0(p^4)$ 

Abstract: By the Deligne-Mumford theorem, a curve C over a local field K has a unique stable model over some finite extension K'/K, as long as the genus of C is greater than one. If defining equations of a curve are given, it is not always easy to compute the stable model of the curve concretely. In our talk, we concentrate on the stable model of a modular curve. The stable models of  $X_0(p)$ ,  $X_1(p)$  and X(p) are classically known by works of Igusa and Deligne-Rapoport. B. Edixhoven calculated the stable model of  $X_0(p^2)$  in 1990. In 2006, Coleman-McMurdy found the stable model of  $X_0(p^3)$  on the basis of the rigid geometry. They use the Gross-Hopkins theory to calculate irreducible components appearing in the stable reduction of  $X_0(p^3)$ . We will introduce a more elementary method to calculate the defining equations of

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the irreducible components in the model of  $X_0(p^3)$ . Starting with Kronecker's polynomial, we will find components by only using blow-up. By this method, we also find the stable reduction of  $X_0(p^4)$ . Deligne-Lusztig curves for  $SL_2(\mathbb{F}_p)$  appear in the stable model of  $X_0(p^4)$ . In our talk, we will explain how to compute the irreducible components appearing in this model of  $X_0(p^4)$ .

## YOSHIDA Manabu (Kyushu Univ.)

**Title:** Fontaine's property  $(P_m)$  at the maximal ramification break **Abstract:** Let K be a complete discrete valuation field with perfect residue field. Consider a certain condition called Fontaine's property  $(P_m)$  for a finite Galois extension L/K and real numbers m. Fontaine and the speaker characterized the maximal ramification break of L/Kby the truth of  $(P_m)$ . In this talk, we completely determine the truth of  $(P_m)$  at the maximal ramification break by using the local class field theory of Serre and Hazewinkel. (joint work with Takashi Suzuki)