Winter School in Algebraic Geometry and Applications

February 17th-20th, 2014 Konkuk University, Seoul, Korea

Exercise Sheet - Day 3

Exercise 1. Let $X = \mathbb{P}^3 \times \mathbb{P}^3 \times \mathbb{P}^4 \times \mathbb{P}^9 \times \mathbb{P}^{10} \times \mathbb{P}^{15}$. Compute first

i) the dimension of ambient space of the Segre embedding of X;

ii) the critical value for identifiability k_c.

Then, using results introduced in the lectures, compute the best known value k for which X is $k-$identifiable.

Exercise 2. Let X be the $2-$veronese embedding of $\mathbb{P}^n$. Show that X is $k-$defective for $2 \leq k \leq n$. Can you give an estimate of the $k-$defect ?

Exercise 3. A tensor $t \in \mathbb{C}^a \otimes \mathbb{C}^b \otimes \mathbb{C}^c$ defines three contractions

$(\mathbb{C}^a)^\vee \to \mathbb{C}^b \otimes \mathbb{C}^c$

$(\mathbb{C}^b)^\vee \to \mathbb{C}^a \otimes \mathbb{C}^c$

$(\mathbb{C}^c)^\vee \to \mathbb{C}^a \otimes \mathbb{C}^b$

and we call their ranks respectively $r_1(t)$, $r_2(t)$, $r_3(t)$.

- Prove that

$$\mathrm{rk}(t) = 1 \Longleftrightarrow \begin{cases} r_1(t) = 1 \\ r_2(t) = 1 \\ r_3(t) = 1 \end{cases}$$

- Prove that $(r_1, r_2, r_3) = (1, 2, 2)$ is admissible, while $(r_1, r_2, r_3) = (1, 1, 2)$ is not allowed.
- Prove that $r_i \leq \mathrm{brk}(t) \leq \mathrm{rk}(t) \leq r_j r_k \ \forall i, j, k$.
- Deduce that $r_1 \leq r_2 r_3$, $r_2 \leq r_1 r_3$, $r_3 \leq r_1 r_2$.