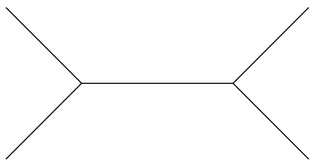
## Winter School in Algebraic Geometry and Applications

**February 17th-20th, 2014 Konkuk University, Seoul, Korea**

## Exercise Sheet - Day 2

**Exercise 1.** Suppose ve are considering a Markov model on the tree



The model has 2 states $\{0,1\}$ and moreover we require that it is symmetric, that is, each transition matrix has the form

$$\begin{pmatrix} 1-z_e & z_e \\ z_e & 1-z_e \end{pmatrix}$$

where $z_e$ can varies for each edge.

We have the following observed patterns:

| | |
|---|---|
| Species A | 00010000100011110101101001110110110010101110 |
| Species B | 01100001011000010011010001111011110001101110 |
| Species C | 00101111000011101100101110000100001110011101 |
| Species D | 10100100111000101110100110110001011001010101 |

i) Establish, using informative invariants, which species are homologues ? (two species are homologues if, at the end of a process of phylogenetic inference, their associated leaves form a cherry)

ii) Write down the joint distribution tensor of the Markov models associated to different labeling and then compute their invariant ideals.

**Exercise 2.**

- (i) Find the closest rank one matrix, according to the Frobenius norm, to

$$A = \begin{pmatrix} -3\sqrt{3}+2 & -3\sqrt{3}-2 \\ 2\sqrt{3}+3 & -2\sqrt{3}+3 \end{pmatrix}$$

It is useful to compute that $A \cdot A^t = \begin{pmatrix} 62 & -10\sqrt{3} \\ -10\sqrt{3} & 42 \end{pmatrix}$, with eigenvalues 72, 32 and eigenvectors respectively $(\sqrt{3}, -1)$ and $(1, \sqrt{3})$

$A^t \cdot A = \begin{pmatrix} 52 & 20 \\ 20 & 52 \end{pmatrix}$, with eigenvalues 72, 32 and eigenvectors respectively $(1, 1)$ and $(1, -1)$.

Find all the critical points of the euclidean distance from $A$ to the variety of rank $\leq 1$ matrices and the tangent space at these points.

- (ii) Find the closest orthogonal matrix, according to the Frobenius norm, to $A$. Find all the critical points of the euclidean distance from $A$ to the variety of orthogonal matrices and the tangent space at these points.