4th KIAS Workshop in Harmonic Analysis

December 26 – 27, 2019

Room 1503, Bldg #1, KIAS

PROGRAM

December 26

10:00–11:00 **Younghun Hong** (Chung-Ang University)
Quantum-classical correspondence between the Hartree and the Vlasov equations

11:10–12:10 **Juyoung Lee** (Seoul National University)
Singular spherical maximal operators on a class of two step nilpotent Lie groups

12:10–13:20 **Lunch**

13:20–14:20 **Jaehyeon Ryu** (Seoul National University)
L^p eigenfunction bounds for the Hermite operator

14:30–16:30 **Sanghyuk Lee** (Seoul National University)
Strichartz estimates for orthonormal families of initial data and weighted oscillatory integral estimates

16:30–17:00 **Break**

17:00–18:00 **Herim Ko** (Seoul National University)
An l^2 decoupling interpretation of efficient congruencing: The parabola

18:10–19:10 **Changhun Yang** (KIAS)
On the Korteweg-de Vries limit for the Fermi-Pasta-Ulam system

December 27

10:00–11:00 **Jungjin Lee** (UNIST)
Global Kato smoothing estimates via local ones for dispersive equations

11:10–12:10 **Joonil Kim** (Yonsei University)
Oscillatory integrals and Resolution of Singularities

12:10–13:20 **Lunch**

13:20–14:20 **Sewook Oh** (Seoul National University)

Oscillatory Integrals and Regularity of Dispersive Equations

14:30–15:30 **Seokchang Hong** (Seoul National University)
Well-Posedness in a critical space of Chern-Simons-Dirac System in the Lorenz gauge

15:30–16:00 **Break**

16:00–17:00 **Gyeongha Hwang** (Yeungnam University)
Probabilistic well-posedness of the mass-critical NLS with radial data below L^2

17:10–18:10 **Ihyeok Seo** (Sungkyunkwan University)
On Morawetz estimates with time-dependent weights for the Klein-Gordon equation

ABSTRACTS

(Aphabetically ordered by last names)

Well-Posedness in a critical space of Chern-Simons-Dirac System in the Lorenz gauge

Seokchang Hong

Seoul National University, seokchangh11@snu.ac.kr

In this paper, we consider the Cauchy problem of local well-posedness of the Chern-Simons-Dirac system in the Lorenz gauge for $B^{\frac{1}{4}}_{2,1}$ initial data. We improve the low regularity well-posedness, by using the localization of space-time Fourier side and bilinear estimates. Then we show the Dirac spinor flow of Chern-Simons-Dirac system is not C^2 at the origin in H^s if $s < \frac{1}{4}$. From this point of view, the space $B^{\frac{1}{4}}_{2,1}$ can be regarded as a critical space for the local well-posedness. This is joint work with Yonggeun Cho.

Quantum-classical correspondence between the Hartree and the Vlasov equations

Younghun Hong

Chung-Ang University, yhhong@cau.ac.kr

Both the Hartree and the Vlasov equations describe mean-field dynamics of collision-less interacting particles, but the former is for quantum particles and the latter is for classical particles. In this talk, we compare the two models in the semi-classical regime in various aspects, including (1) variational problems (for free energy minimizers), (2) linear evolutions (in the form of Strichartz estimates for orthogonal functions and those for transport equations), and (3) nonlinear evolutions (for moment propagation). This is a survey talk.

Probabilistic well-posedness of the mass-critical NLS with radial data below L^2

Gyeongha Hwang

Yeungnam University, Gyeongha.Hwang@gmail.com

In this paper, we consider the Cauchy problem of the mass-critical nonlinear Schrödinger equation (NLS) with radial data below L^2. We prove almost sure local well-posedness along with small data global existence and scattering. Furthermore, we also derive conditional almost sure global well-posedness of the defocusing NLS under the assumption of a probabilistic *a priori* energy bound. The main ingredient is to establish the probabilistic radial Strichartz estimates.

Oscillatory integrals and Resolution of Singularities

Joonil Kim
Yonsei University, jikim7030@yonsei.ac.kr

We consider a class of oscillatory integrals with polynomial phase functions P over global domains $D \subset \mathbb{R}^2$. We first obtain the resolution of singularity for P over the global domain D. Next investigate the two main issues (i) whether it converges or not and (ii) how fast it decays. We establish a criterion on (P, D) to determine its convergence, and find the oscillation indices $\rho_{\text{osc}} = \rho_{\text{osc}}(P, D)$ when it converges. They are described in terms of a generalized notion of Newton polyhedra associated with (P, D). Finally, we discuss about its applications.

An l^2 decoupling interpretation of efficient congruencing: The parabola

Herim Ko
Seoul National University, kohr@snu.ac.kr

We review the work of Zane Kun Li which provide a different proof of l2 decoupling for the parabola. The proof uses bilinear reduction and iteration argument inspired by efficient congruencing.

Global Kato smoothing estimates via local ones for dispersive equations

Jungjin Lee
UNIST, jungjinlee@unist.ac.kr

We will consider some Kato type smoothing estimates. The sharp regularity rages of the global Kato smoothing estimates are already known, but the case of the local Kato smoothing estimates are not. In spacetime $\mathbb{R}\times\mathbb{R}$, Sun, Trélat, Zhang and Zhong have shown that the known local Kato smoothing estimates are also sharp by taking an example. We recently resolved this problem in higher dimensions by showing that local Kato smoothing estimates are essentially equivalent to the global Kato smoothing estimates. This is obtained by using some basic techniques developed for the Fourier restriction conjecture such as wave-packet decompositions, induction-on-scale arguments and epsilon removal lemma. In this talk, we will present our results and ideas in detail.

Singular spherical maximal operators on a class of two step nilpotent Lie groups

Juyoung Lee
Seoul National University, ljy219@snu.ac.kr

Let $H^n \cong \mathbf{R}^{2n} \ltimes \mathbf{R}$ be the Heisenberg group and let μ_t be the normalized surface measure for the sphere of radius t in $\mathbf{R}^{2n}$. Consider the maximal function defined by $Mf = \sup_{t>0} |f * \mu_t|$. We are interested in the boundedness property of this function on L^p-spaces. For $n \geq 2$, this maximal operator is bounded on L^p when $p > \frac{2n}{2n-1}$. This result can be generalized to a more general class of surfaces and to groups satisfying a nondegeneracy condition. This talk is based on the paper written by D. Müller and A. Seeger.

Strichartz estimates for orthonormal families of initial data and weighted oscillatory integral estimates

Sanghyuk Lee
Seoul National University, shklee@snu.ac.kr

This talk concerns the Strichartz estimates for orthonormal families of initial data in the case of the wave, Klein–Gordon and fractional Schrödinger equations. Our results extend those of Frank–Sabin in the case of the wave and Klein–Gordon equations, and generalize work of Frank–Lewin–Lieb–Seiringer and Frank–Sabin for the Schrödinger equation. Due to a certain technical barrier, except for the classical Schrödinger equation, the Strichartz estimates for orthonormal families of initial data have not previously been established up to the sharp summability exponents in the full range of admissible pairs. We obtain the optimal estimates in various notable cases and improve the previous results. The main novelty of this paper is the use of estimates for weighted oscillatory integrals which we combine with an approach due to Frank and Sabin. This strategy also leads us to proving new estimates for weighted oscillatory integrals with optimal decay exponents which we believe to be of wider independent interest. This is joint work with Neal Bez and Shohei Nakamura.

Oscillatory Integrals and Regularity of Dispersive Equations

Sewook Oh
Seoul National University, dhtpdnr0220@snu.ac.kr

Oscillatory integrals have close relationship with the local and global smoothing properties of dispersive equations. To see regularity of dispersive equation, we will consider oscillatory integrals with some damping factor. Regarding this, I will review old results based on Kenig, Ponce, Vega's paper: Oscillatory Integrals and Regularity of Dispersive Equations.

L^p eigenfunction bounds for the Hermite operator

Jaehyeon Ryu
Seoul National University, miro21670@snu.ac.kr

We introduce the Hermite operator H, which is defined by $H = -\Delta + x^2$. As an operator acting on $L^2(\mathbb{R}^d)$, this has some interesting properties. It has distinct positive eigenvalues, and corresponding eigenfunctions which form a basis in $L^2(\mathbb{R}^d)$. Then we direct our attention to the eigenfunction $(-\Delta + x^2)\phi_\lambda = \lambda^2\phi_\lambda$. These are concentrated inside the ball $\{|x| \leq \lambda\}$, and have exponential Airy type decay beyond this boundary.

We consider the problem of obtaining L^p bounds of eigenfunction for H. This question has been considerable interest in the context of Riesz summability for the Hermite operator in the work of Thangavelu and Karadzhov. In this talk, we present the work of Koch and Tataru, which strengthen the results of aforementioned two authors and gives a complete picture.

References

[1] H. Koch and D. Tataru, *L^p eigenfunction bounds for the Hermite operator*, Duke Math. J. **128** (2005), 369-392. 4, 27

On Morawetz estimates with time-dependent weights for the Klein-Gordon equation

Ihyeok Seo
Sungkyunkwan University, ihseo@skku.edu

We obtain some new Morawetz estimates for the Klein-Gordon flow of the form

$$\big\| |\nabla|^\sigma e^{it\sqrt{1-\Delta}} f \big\|_{L^2_{x,t}(|(x,t)|^{-\alpha})} \lesssim \|f\|_{H^s}$$

where $\sigma, s \geq 0$ and $\alpha > 0$. The conventional approaches to Morawetz estimates with $|x|^{-\alpha}$ are no longer available in the case of time-dependent weights $|(x,t)|^{-\alpha}$. Here we instead apply the Littlewood-Paley theory with Muckenhoupt A_2 weights to frequency localized estimates thereof that are obtained by making use of the bilinear interpolation between their bilinear form estimates which needs to carefully analyze some relevant oscillatory integrals according to the different scaling of $\sqrt{1-\Delta}$ for low and high frequencies. This is joint work with Jungkwon Kim, Hyeongjin Lee and Jihyeon Seok.

On the Korteweg-de Vries limit for the Fermi-Pasta-Ulam system

Changhun Yang
KIAS, maticionych@kias.re.kr

The Fermi-Pasta-Ulam system (FPU) is a simple nonlinear dynamical lattice model describing a one-dimensional chain of vibrating strings with nearest neighbor interactions. This model was introduced by Fermi, Pasta and Ulam in 1955. It was anticipated at that time that chaotic nonlinear interactions would lead to thermalization. Surprisingly however, numerical simulations showed the opposite behavior it exhibited quasi-periodic motions. This phenomena is known as the FPU paradox. This puzzle has been solved by Zabusky and Kruskal by discovering a formal convergence of FPU to the Kortewegde Vries equation. Later, the convergence has been rigorously justified. We revisit this convergence problem, and show how to put it into the dispersive PDE framework. This talk is based on joint work with Younghun Hong and Chulkwang Kwak.