1 Title: Holomorphic Lagrangian fibrations on hyperkahler manifolds

Abstract: The only fibration that can exist on a K3 surface is an elliptic fibration, i.e., with generic fibre an elliptic curve and base $\mathbf{P}^1$. Holomorphic symplectic manifolds (compact hyperkahler manifolds) are higher-dimensional analogues of K3 surfaces. Matushita found very strong restrictions on the kind of holomorphic fibration such a manifold can admit: namely, the fibres should be complex tori which are Lagrangian with respect to the holomorphic symplectic structure. This is the higher-dimensional generalization of an elliptic K3 surface. In this talk I will review Matsushita's result and describe some examples.

Reference: math.AG/0404362

2 Title: Existence of Lagrangian fibrations

Abstract: A K3 surface admits an elliptic fibration if and only if it contains a divisor of square zero. There is an analogous condition which conjecturally ensures the existence of a Lagrangian fibration on a holomorphic symplectic manifold. Hilbert schemes of points on K3 surfaces are a family of examples of holomorphic symplectic manifolds. I will prove the existence of Lagrangian fibrations on these spaces in accordance with the above conjecture.

Reference: math.AG/0509224

3 Title: Dual fibrations

Abstract: The relative Jacobian arises naturally in the study of elliptic surfaces. In higher dimensions, given a fibration by abelian varieties it is natural to look at the relative Picard scheme. This can be thought of as the dual fibration. I will describe how Mukai's derived equivalence for an abelian variety and its dual can, in some instances, be extended to families of abelian varieties. This gives derived equivalences between Lagrangian fibrations and their duals, which perhaps can be regarded as a manifestation of homological mirror symmetry for holomorphic SYZ fibrations.

References: math.AG/0404365, math.AG/0509222

4 Title: Classification of Lagrangian fibrations

Abstract: Markushevich proved the following result about four-dimensional Lagrangian fibrations. Let $X \to B$ be a fibration by Jacobians of genus two curves over a complex surface B, with total space X a holomorphic symplectic four-fold. Then the genus two curves form a complete linear system ($B \cong \mathbf{P}^2$) of curves in a hyperelliptic K3 surface S, and X is a deformation of the Hilbert scheme of two points on S. Using ideas of Hurtubise on foliations of hypersurfaces in X, I will describe a higher-dimensional generalization of Markushevich's result for fibrations by Jacobians of genus n curves.

Reference: in preparation