

From Riemannian geometry to topology via Witten deformation (WHS theory)

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Witten-Helffer-Sjstrand theory is an addition to Morse theory and Hodge-de-Rham theory for Riemannian manifolds and considerably improve on them by injecting some spectral theory of elliptic operators (but rather elementary aspects). It is based on Witten deformation of the de-Rham complex into Morse complex associated with a Morse function and a Riemannian metric (equivalently a cell structure on the manifold). It can serve to prove results about comparison /equality of numerical invariants of compact manifolds defined analytically , i.e. using Riemannian metric, or combinatorially, i.e.using triangulations.

Lecture 1. De-Rham complex and additional structures. Laplace Beltrami operators and elementary spectral properties; Morse complex and integration theory.

Lecture 2. Witten deformation, and Witten-Helffer-Sjstrand theory. Asymptotic Hodge-de-Rham theory.

Lecture 3. Generalizations, applications ; -replacing a Morse function by a Bott function , or by a closed one form . -complex WHS theory A few applications.

References:

1. Dan Burghelea: A short course on Witten Helffer-Sjstrand theory, arXiv:math/0101063
2. D.Burghelea and S. Haller: Complex valued Ray-Singer torsion II, arXiv:math/0610875
3. E.Witten: Supper symmetry and Morse theory, J.of Diff. Geom.17, (1982) 661-692