KIAS WORKSHOP ON PARTIAL DIFFERENTIAL EQUATION AND HARMONIC ANALYSIS

Time: 13:30 - 17:00 on December 13 (Tuesday) - 14 (Wednesday)

Place: Seminar Room 7323 at KIAS

December 13 (Tuesday)

Talk 1 (13:30-14:10) Sun-Sig Byun (Seoul National University), *Global L^p Estimates for Weak Solutions of Parabolic Nonlinear Equations*

Abstract: We will study in this talk a global L^p estimate for $2 \leq p < \infty$ for the weak solution of a Dirichlet problem for a divergence structure nonlinear parabolic partial differential equation that is not of variational form. We will ask what are optimal conditions to place both on the nonlinearity and on the boundary for the $W^{1,p}$ regularity theory to this large class of parabolic problems.

Talk 2 (14:20-15:00) Kyungwoo Song (Kyunghee University), *Two dimensional transonic flows through a nozzle for the steady full Euler equations*

Abstract: We establish the existence and uniqueness of transonic shocks in the steady flows through a two-dimensional nozzle with varing cross-sections. The flow is governed by the steady full Euler equations. We show that the solutions behind the shock front remain subsonic in a downstream region and the shock front is smooth. The problem is approached by a one-phase free boundary problem where the shock front is a free boundary. The steady full Euler equations are decomposed into elliptic equations and a system of transport equations for the free boundary problem.

Talk 3 (15:30-16:10) Chan woo Yang (Korea University), *Multiparameter singular integrals and maximal operators along flat surfaces*

Abstract: We study multiple Hilbert transforms and their maximal functions along surfaces $(t_1, \cdots, t_{n-1}, \Gamma(t_1, \cdots, t_{n-1}))$ with $t_i \geq 0$. The L^p boundedness of the strong maximal functions is obtained under only convexity condition. Multiple Hilbert transforms is also proved to be bounded in L^p if Γ is extended to $\mathbb{R}^{n-1}$ as an even function in each variable. If Γ is

extended to $\mathbb{R}^{n-1}$ as an odd function in one variable and even function in other variables, we establish L^p boundedness with additional comparability conditions.

Talk 4 (16:20-17:00) Namkwon Kim (Seoul National University), *Self-dual and nonself-dual Chern-Simons vortices*

Abstract: We review Chern-Simons vortex type solutions and present recent results on the self-dual and nonself-dual Chern-Simons vortices. In particular, existence and asymptotic behavior of the vortices are dealt.

December 14 (Wednesday)

Talk 5 (13:30-14:10) Jihoon Lee (Sungkyunkwan University), *On the regularity to the Navier-Stokes equations related with the direction of the vorticity*

Abstract: We are interested in the regularity problem for suitable Navier-Stokes equations

$$\text{(1)} \qquad \begin{cases} \dfrac{\partial v}{\partial t} + (v \cdot \nabla)v - \Delta v = -\nabla p & \text{in } \ Q_T := \Omega \times (0,T), \\ \text{div } v = 0 \end{cases}$$

where Ω is a domain in $\mathbf{R}^3$, v is the flow velocity, p is the scalar pressure, and $\nu > 0$ is the viscosity constant. We are especially concerned with the initial boundary value problem on a bounded and smooth domain, and therefore, we require together with (1) initial and boundary conditions:

$$\text{(2)} \qquad \begin{cases} v(x,0) = v_0(x), & x \in \Omega \\ v(x,t) = 0, & x \in \partial\Omega, \ \ 0 < t < T. \end{cases}$$

We study the interior regularity criteria for the vorticity of "suitable" weak solutions to the Navier-Stokes equations. We prove that if two components of a vorticiy is in $L^{q,p}_{\text{loc}}$ with $3/p+2/q \le 2$ and $3/2 < p \le \infty$ at an interior point, then solution is bounded near the point. We also show that if the direction of vorticity is locally in some Triebel-Lizorkin spaces in a neighborhood of an interior point, then solution is bounded near the point. This is the joint work with Professor Dongho Chae and Professor Kyungkeun Kang.

Talk 6 (14:20-15:00) Ya Ryong Heo (POSTECH), *Some remarks on multi-parameter maximal operators*

Abstract: Let μ be a compactly supported finite Borel measure in $\mathbf{R}^n \times \mathbf{R}^m$ and $E, F \subset (0,\infty)$. Then we consider the maximal operator given by

$$T^*_{E,F} f(x,y) = \sup_{(t,s)\in E\times F} |T_{t,s} f(x,y)|$$
$$= \sup_{(t,s)\in E\times F} \Big| \int_{\mathbf{R}^n\times\mathbf{R}^m} f(x-t\tilde{x}, y-s\tilde{y}) d\mu(\tilde{x},\tilde{y})\Big|.$$

We study the L^p-boundedness properties of $T^*_{E,F}$ as a function of E and F.
Definition. If $E \subset [1,2]$, we define Minkowski dimension of E, given by

$$d(E) = \limsup_{\delta\to 0} \frac{\log N(E,\delta)}{\log \frac{1}{\delta}},$$

where $N(E,\delta)$ represents the minimum number of intervals of length δ needed to cover E. For a general set E in $(0,\infty)$, we define $d(E)$ as

$$d(E) = \sup_{l\in\mathbf{Z}} d(2^{-l}(E\cap[2^l, 2^{l+1}]).$$

Theorem. If μ is a compactly supported finite Borel measure in $\mathbf{R}^n \times \mathbf{R}^m$ with $|\widehat{\mu}(\xi,\eta)| \lesssim |\xi|^{-a}|\eta|^{-b}$, $a, b > 1/2$ then $T^*_{E,F}$ is a L^p-bounded operator for $p > \max\Big(1+\frac{d(E)}{2a}, 1+\frac{d(F)}{2b}\Big)$.
Theorem. Let μ be a compactly supported finite Borel measure in $\mathbf{R}^n\times\mathbf{R}^m$ such that

$$|\widehat{\mu}(\xi,\eta)| \lesssim (|\xi|+|\eta|)^{-a} \text{ for some } 2a > d(E)+d(F).$$

Then $T^*_{E,F}$ is a L^p-bounded operator for $p > 1 + \frac{d(E)+d(F)}{2a}$.

Talk 7 (15:30-16:10) Hyungjin Huh (Seoul National University), *Local and global solutions of the Chern-Simons-Higgs system*

Abstract: We study low regularity local solutions of the Chern-Simons-Higgs equations. Under the Coulomb gauge condition they are formulated in the hyperbolic equation coupled with elliptic equation. The Lorentz gauge condition makes them hyperbolic equations with the null form. The div-curl decomposition is used in the condition of temporal gauge in which a global existence of smooth solution will be showed.