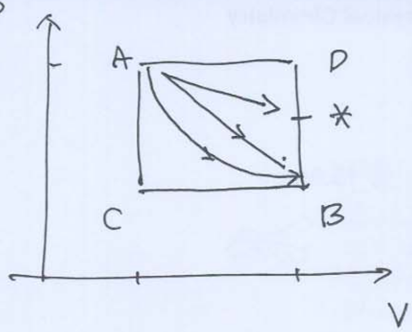
1.8-1



$U_* = \underbrace{(U_* - U_B)}_{①} + \underbrace{(U_B - U_A)}_{②}$ ($U_A = 0$) from example 1.

① $U_* - U_B = \frac{3}{2} V_B (P_* - P_B)$ (← use $\Delta U = \frac{3}{2} V \Delta p$)

② $U_B - U_A = -\int_{V_A}^{V_B} p\,dV$ (← choose adiabatic path ($Q=0$)) $pV^\gamma = C$

$$= -\frac{1}{1-\gamma} C \left(V_B^{1-\gamma} - V_A^{1-\gamma} \right)$$

$$= -\frac{1}{1-\gamma} (P_B V_B - P_A V_A) = \frac{3}{2} (P_B V_B - P_A V_A)$$

$\therefore U_* = \frac{3}{2}(P_* V_B - P_A V_A) = \frac{3}{2}(5 \cdot 10^4 \cdot 8 \times 10^{-3} - 10^5 \cdot 10^{-3})$ $[Pa \cdot m^{-3} = J]$

$$= \frac{9}{2} \times 10^2 J = 450 J.$$

1.8-2.

$$\Delta U_{(*)A} = U_{(*)} - U_A = Q + W = 450 J$$

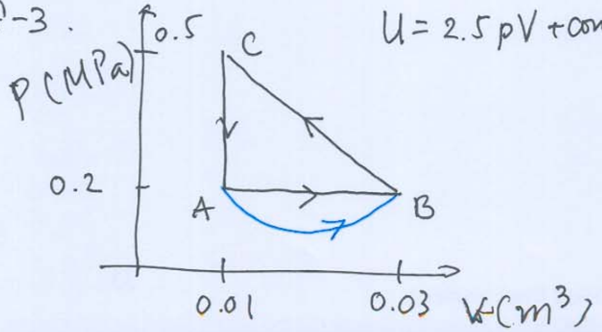
$$W = -\int_{A \to (*)} p\,dV = -\left\{ P_* (V_B - V_C) + \frac{1}{2}(P_D - P_*)(V_B - V_C) \right\}$$

$$= -\left\{ (5 \cdot 10^4 \cdot 7 \times 10^{-3}) + \frac{1}{2} 5 \cdot 10^4 \cdot 7 \times 10^{-3} \right\}$$

$$= -\frac{3}{2} \cdot 5 \cdot 7 \cdot 10 \; J = -525 J.$$

$\therefore Q = \Delta U - W = (450 + 525) J = 975 J$

1.8-3.



$U = 2.5 pV + \text{const} (A \to B)$

$U_{BA} = 2.5 (P_B V_B - P_A V_A) = 0.01 MJ = 10 kJ$

$W_{BA} = -0.2 (0.03 - 0.01) MJ$

$= -0.004 MJ = 4 kJ$

$\therefore Q_{BA} = 0.01 MJ + 0.004 MJ$

$= 0.014 MJ = 14 kJ$

(B → C)

$$U_{CB} = 2.5\,(P_C V_C - P_B V_B) = 2.5\,(0.5\cdot 0.01 - 0.2\cdot 0.03)\cdot 10^3\ \mathrm{kJ}$$
$$= 2.5\cdot(-0.001)\cdot 10^3\ \mathrm{kJ} = -2.5\ \mathrm{kJ}$$

$$W_{CB} = P_A(V_B - V_A) + \frac{1}{2}(P_C - P_A)(V_B - V_A)$$
$$= \frac{1}{2}(P_C + P_A)(V_B - V_A) = \frac{1}{2}\,0.7\cdot 0.02 = 0.007\ \mathrm{MJ} = 7\ \mathrm{kJ}$$

$$\therefore\ Q_{CB} = U_{CB} - W_{CB} = -9.5\ \mathrm{kJ}.$$

(C → A)

$$U_{AC} = 2.5\,(P_A V_A - P_C V_C) = 2.5\,(0.2\cdot 0.01 - 0.2\cdot 0.03)$$
$$= 2.5\,(-0.004) = -0.01\ \mathrm{MJ}$$
$$= -10\ \mathrm{kJ}$$

$$W_{AC} = 0 \quad (\because \Delta V = 0)$$

$$\therefore\ Q_{AC} = U_{AC} - \cancel{W_{AC}}^{\,0} = -10\ \mathrm{kJ}.$$

(Along the parabola)

$$\hookrightarrow\ p = 10^5 + 10^9 \times (V - 0.02)^2$$ [the unit should be "Pa"]

$$U_{BA} = 10\ \mathrm{kJ}$$ (state function path independent)

$$W_{BA} = -\int p\,dV = -\int_{V_A}^{V_B}\left[10^5 + 10^9 \times (V - 0.02)^2\right]dV$$

$$= -\left[10^5\ 0.02 + \frac{1}{3}\cdot 10^9\left\{\underbrace{(V_B - 0.02)^3}_{(0.01)^3} - \underbrace{(V_A - 0.02)^3}_{(-0.01)^3}\right\}\right]$$

$$= -\left(10^5\cdot 0.02 + \frac{2}{3}\cdot 10^9\ 10^{-6}\right)$$

$$= -\left(2\cdot 10^3 + \frac{2}{3}\cdot 10^3\right) = -\frac{8}{3}\cdot 10^3\ \mathrm{Pa\cdot m^3}$$

$$= -\frac{8}{3}\ \mathrm{kJ}.$$

$$Q_{BA} = \left(10 + \frac{8}{3}\right)\mathrm{kJ} = \frac{38}{3}\ \mathrm{kJ} \fallingdotseq 12.6\ \mathrm{kJ}.$$

1.8-5 Equation of adiabat for a system satisfying

$U = Ap^2V$ in (p, V) plane

$$dU = -pdV \rightarrow \quad 2ApVdp + Ap^2dV = -pdV$$

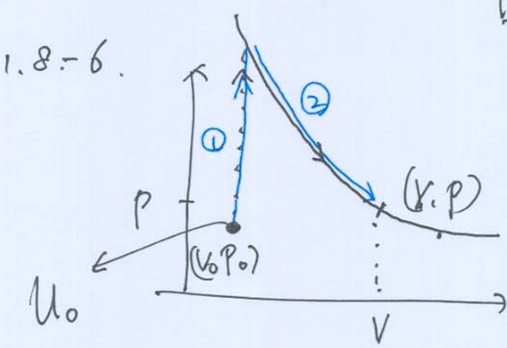
$$2AVdp + (Ap+1)\,dV = 0$$

$$\int \frac{2A}{Ap+1}\,dp + \int \frac{1}{V}\,dV = 0$$

$$2\log(Ap+1) + \log V = \text{const}.$$

$$\therefore \quad \boxed{(Ap+1)^2 V = \text{const}}$$

1.8-6.



Choose the thermodynamic path along ① & ② [the easiest path].

Along ① $\quad W = 0 \qquad \therefore \quad \Delta U_① = Q' = A(p' - p_0)$

Because (V_0, p') is on the $pV^\gamma = c \cdot$ surface

$$p' V_0^\gamma = pV^\gamma = c \qquad p' = p\left(\frac{V}{V_0}\right)^\gamma = pr^\gamma \quad \left(r \doteq \frac{V}{V_0}\right)$$

Along ② $\quad Q = 0$

$$\therefore \quad \Delta U_② = -\int_{V_0}^{V} p\,dV = -\frac{c}{1-\gamma}\left(V^{1-\gamma} - V_0^{1-\gamma}\right)$$

$$\cancel{= \frac{c}{\gamma-1} V_0^{1-\gamma}\left(r^{1-\gamma} - 1\right)}$$

$$\cancel{= \frac{p' V_0^\gamma V_0^{1-\gamma}}{\gamma-1}\left(r^{1-\gamma} - 1\right)}$$

$$= -\frac{cV^{1-\gamma}}{1-\gamma}\left(1 - \left(\frac{V_0}{V}\right)^{1-\gamma}\right) = \frac{pV}{\gamma-1}\left[1 - r^{\gamma-1}\right]$$

$$\therefore \quad \boxed{U = U_0 + A(pr^\gamma - p_0) + \frac{pV}{\gamma-1}\left(1 - r^{\gamma-1}\right)}$$