5.3-7. $H = AS^2 N^{-1} \log(P/P_0)$

From $dH = TdS + Vdp + \mu dN$

① $\left(\frac{\partial H}{\partial P}\right)_{S,N} = V = \frac{AS^2N^{-1}}{P}$, ② $\left(\frac{\partial H}{\partial S}\right)_{P,N} = T = 2ASN^{-1}\log(P/P_0)$

Eliminate P using ① & ②

$$T = 2ASN^{-1}\log\frac{AS^2N^{-1}}{P_0V}$$

Differentiate with T at constant $\cancel{P}$ V

$$1 = 2AN^{-1}\left(\frac{\partial S}{\partial T}\right)_V \underbrace{\log\frac{AS^2N^{-1}}{P_0V}}_{= \ln P/P_0} + 2ASN^{-1}\cdot 2\frac{1}{S}\left(\frac{\partial S}{\partial T}\right)_V$$

Since $C_v = \frac{T}{N}\left(\frac{\partial S}{\partial T}\right)_V$

$$C_v = \left[\frac{2A}{T}\log(P/P_0) + \frac{4A}{T}\right]^{-1}$$

5.3-12

$$(S-S_0)^4 = Avu^2$$

$$u = A^{-\frac{1}{2}}v^{-\frac{1}{2}}(S-S_0)^2$$

$$\Rightarrow \begin{cases} \left(\frac{\partial u}{\partial S}\right)_v = T = 2A^{-\frac{1}{2}}v^{-\frac{1}{2}}(S-S_0) \\ -\left(\frac{\partial u}{\partial v}\right)_S = P = \frac{1}{2}A^{-\frac{1}{2}}v^{-\frac{3}{2}}(S-S_0)^2 \end{cases} \rightarrow \frac{T^2}{P} = 8A^{-\frac{1}{2}}v^{\frac{1}{2}}$$

Legendre transform to $g(\equiv G/N)$

$$g = u - TS + Pv = \frac{1}{4}A^{\frac{1}{2}}v^{\frac{1}{2}}T^2 + \frac{1}{8}A^{\frac{1}{2}}v^{\frac{1}{2}}T^2 - \frac{1}{2}T^2A^{\frac{1}{2}}v^{\frac{1}{2}} - TS_0$$

$$= -\frac{A^{\frac{1}{2}}}{8}T^2v^{\frac{1}{2}} - TS_0 = -\frac{A}{64}T^4/P - TS_0$$

$$u = \frac{3}{2}pv, \quad p = AvT^4$$

$$\frac{1}{T} = \left(\frac{3}{2}\right)^{\frac{1}{4}} A^{\frac{1}{4}} u^{-\frac{1}{4}} v^{\frac{1}{2}}, \quad \frac{p}{T} = \frac{2}{3}\left(\frac{3}{2}\right)^{\frac{1}{4}} A^{\frac{1}{4}} u^{\frac{3}{4}} v^{-\frac{1}{2}}$$

S. eq $\rightarrow$

$$dS = \frac{1}{T}du + \frac{p}{T}dv = \left(\frac{3}{2}\right)^{\frac{1}{4}} A^{\frac{1}{4}} d\left(\frac{4}{3} u^{\frac{3}{4}} v^{\frac{1}{2}}\right)$$

$$\boxed{S = \left(\frac{3}{2}\right)^{\frac{1}{4}} A^{\frac{1}{4}} \cdot \frac{4}{3} u^{\frac{3}{4}} v^{\frac{1}{2}}} + \cancel{S_0} \rightarrow 0$$

$$u = u(S, v) = \left[\frac{3}{4}\left(\frac{3}{2}\right)^{-\frac{1}{4}}\right]^{\frac{4}{3}} A^{-\frac{1}{3}} S^{\frac{4}{3}} v^{-\frac{2}{3}}$$

$$T = \left(\frac{\partial u}{\partial S}\right)_v = \frac{4}{3} u \cdot S^{-1} \longrightarrow u = \frac{3}{4}TS$$

$$-p = \left(\frac{\partial u}{\partial v}\right)_S = -\frac{2}{3} u \cdot v^{-1} \longrightarrow u = \frac{3}{2}pv$$

$$f = f(T, v) = u - TS = +\frac{3}{4}TS - TS = -\frac{1}{4}TS = -\frac{1}{3}u = -\frac{1}{2}pv$$

Since $p = AvT^4 \longrightarrow pv = Av^2T^4$

$$\therefore f(T, v) = -\frac{1}{2}Av^2T^4$$

$$g = g(T, p) = u - TS + pv = \frac{1}{2}pv$$

Using $p = AvT^4$, we get $v = A^{-1}pT^{-4}$

$$\therefore g(T, p) = \frac{1}{2}p \cdot A^{-1}pT^{-4} = \frac{p^2}{2AT^4}$$

7.2.-3. $\alpha = \frac{1}{T}$ then $\left(\frac{\partial c_p}{\partial p}\right)_T = ?$

$$\left(\frac{\partial c_p}{\partial p}\right)_T = \frac{\partial}{\partial p}\left[T\left(\frac{\partial S}{\partial T}\right)_p\right]_T = T\cdot\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial p}\right)_T = \cancel{\oplus}\left[T\frac{\partial}{\partial T}\left(\frac{\partial V}{\partial T}\right)_p\right] =$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p \quad \text{(maxwell relation)}$$

$V\alpha = V/T$

$$-T\left[\frac{\partial}{\partial T}\frac{V}{T}\right] = -T\cdot\left[\frac{1}{T}\left(\frac{\partial V}{\partial T}\right)_p - V\frac{1}{T^2}\right]$$

$= 0$

$$\therefore \alpha = \frac{1}{T} \longrightarrow \left(\frac{\partial c_p}{\partial p}\right)_T = 0$$

7.3.-1.

$$dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp$$

$$TdS = T\left(\frac{\partial S}{\partial T}\right)_p dT + T\underbrace{\left(\frac{\partial S}{\partial p}\right)_T}_{= -\left(\frac{\partial V}{\partial T}\right)_p} dp$$

$$= Nc_p\,dT - VT\alpha\,dp$$

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial p}\right)_T dV \qquad = \left(\frac{\partial p}{\partial T}\right)_V = -\frac{\left(\frac{\partial V}{\partial T}\right)_p}{\left(\frac{\partial V}{\partial p}\right)_T} = \frac{\alpha}{k_T}$$

$$TdS = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV$$

$$= Nc_v\,dT + T\cdot\frac{\alpha}{k_T}\,dV$$

7.3.-2.

$$T\left(\frac{\partial S}{\partial T}\right)_V = Nc_p - VT\alpha\underbrace{\left(\frac{\partial p}{\partial T}\right)_V}_{= \alpha/k_T}$$

$$Nc_v = Nc_p - \frac{VT\alpha^2}{k_T} \longrightarrow \boxed{c_p - c_v = \frac{VT\alpha^2}{Nk_T}}$$

7.3-3.

$$\left(\frac{\partial H}{\partial V}\right)_{T,N} = T\left(\frac{\partial S}{\partial V}\right)_{T,N} + V\left(\frac{\partial P}{\partial V}\right)_{T,N}$$

$$= T\cdot\underbrace{\left(\frac{\partial P}{\partial T}\right)_V}_{=\frac{\alpha}{k_T}} - \frac{1}{k_T} = \left(\frac{T\alpha - 1}{k_T}\right)$$

7.4-6.

$$\left(\frac{\partial H}{\partial v}\right)_u = ?$$

$\hookrightarrow$ free expansion

$$\left(\frac{\partial H}{\partial v}\right)_u = T\left(\frac{\partial s}{\partial v}\right)_u + v\left(\frac{\partial P}{\partial v}\right)_u$$

$$= \cancel{T\cdot\frac{\left(\frac{\partial u}{\partial v}\right)_s = -P}{\left(\frac{\partial u}{\partial s}\right)_v = T} - v\cdot\frac{\left(\frac{\partial u}{\partial v}\right)_P}{\left(\frac{\partial u}{\partial P}\right)_v}}$$

$$= T\cdot\frac{P}{T} + v\cdot\left\{-\frac{\left(\frac{\partial u}{\partial v}\right)_P}{\left(\frac{\partial u}{\partial P}\right)_v}\right\}$$

$$= P + v\left\{-\frac{T\left(\frac{\partial s}{\partial v}\right)_P - P}{T\left(\frac{\partial s}{\partial P}\right)_v}\right\}$$

$$= P + v\cdot\left\{-\frac{T\,(\partial s/\partial T)_P/(\partial v/\partial T)_P - P}{T\cdot\frac{(\partial s/\partial T)_v}{(\partial P/\partial T)_v}}\right\}$$

$$= P + v\left\{-\frac{(c_P/v\alpha) - P}{c_v/(\alpha/k_T)}\right\} = P - \frac{c_P/\alpha - Pv}{c_v k_T/\alpha}$$

$$= P - \frac{c_P - Pv\alpha}{\left(c_P - \frac{Tv\alpha^2}{k_T}\right)k_T}$$

$$= P - \frac{c_P - Pv\alpha}{c_P k_T - Tv\alpha^2}$$

7.4-7 $$\left(\frac{\partial c_v}{\partial v}\right)_T = \frac{\partial}{\partial v}\left(T\left(\frac{\partial s}{\partial T}\right)_v\right)_T = T\left[\frac{\partial}{\partial T}\left(\frac{\partial s}{\partial v}\right)_T\right]_v$$

$$= T\cdot\frac{\partial}{\partial T}\left(\frac{\partial p}{\partial T}\right)_v = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v = T\cdot\frac{\partial}{\partial T}\left[\frac{R}{v-b}\right] = 0$$

vdW

7.4-8. $$\left(\frac{\partial c_p}{\partial p}\right)_T = \frac{\partial}{\partial p}\left[T\left(\frac{\partial s}{\partial T}\right)_p\right]_T$$

$$= T\left[\frac{\partial}{\partial T}\left(\frac{\partial s}{\partial p}\right)_T\right]_p$$

$$= T\cdot\frac{\partial}{\partial T}\left[\underbrace{-\left(\frac{\partial V}{\partial T}\right)_p}_{-V\alpha}\right]_p = -T\left(\frac{\partial}{\partial T}V\alpha\right)_p$$

$$= -T\alpha\underbrace{\left(\frac{\partial V}{\partial T}\right)_p}_{V\alpha} - TV\left(\frac{\partial \alpha}{\partial T}\right)_p = -Tv\left[\alpha^2 + \left(\frac{\partial \alpha}{\partial T}\right)_p\right]$$

for $p\left(v+\frac{A}{T^2}\right) = RT$

$$p\left[\overset{V\alpha}{\left(\frac{\partial v}{\partial T}\right)_p} - 2A\frac{1}{T^3}\right] = R.$$

$$\therefore \quad \alpha = \left(\frac{R}{p} + 2A/T^3\right)\frac{1}{v}$$

$$= \left(\frac{v}{T} + \frac{A}{T^3} + \frac{2A}{T^3}\right)\frac{1}{v}$$

$$= \frac{1}{T} + \frac{3A}{vT^3} \rightarrow \left(\frac{\partial \alpha}{\partial T}\right)_p = -\frac{1}{T^2} - \frac{9A}{T^4}\frac{1}{v} - \frac{2A}{T^3}\frac{1}{v^2}\left(\frac{\partial v}{\partial T}\right)_p$$

$$= -\frac{1}{T}\left[\frac{1}{T} + \frac{3A}{vT^3}\right] - \frac{6A}{vT^4} - \frac{3A}{vT^3}\alpha$$

$$p\left(v+\frac{A}{T^2}\right)=RT$$

$$v=\frac{RT}{p}-\frac{A}{T^2}$$

$$\left(\frac{\partial v}{\partial T}\right)_p=\frac{R}{p}+\frac{2A}{T^3}$$

$$\boxed{\alpha=\frac{R}{pv}+\frac{2A}{T^3}\frac{1}{v}}$$

$$\left(\frac{\partial \alpha}{\partial T}\right)_p=-\frac{R}{pv^2}\left(\frac{\partial v}{\partial T}\right)_p-\frac{6A}{T^4}\frac{1}{v}-\frac{2A}{v^2T^3}\left(\frac{\partial v}{\partial T}\right)_p$$

$$=-\frac{R}{pv}\alpha-\frac{6A}{T^4v}-\frac{2A}{vT^3}\alpha$$

$$=\underbrace{-\frac{1}{v}\left(\frac{R}{p}+\frac{2A}{T^3}\right)}_{\alpha}\alpha-\frac{6A}{T^4v}$$

$$\therefore\quad \alpha^2+\left(\frac{\partial \alpha}{\partial T}\right)_p=-\frac{6A}{T^4v}$$

$$\therefore\quad \left(\frac{\partial c_p}{\partial p}\right)_T=-Tv\left[\alpha^2+\left(\frac{\partial \alpha}{\partial T}\right)_p\right]=\frac{6A}{T^3}$$