2.6-3.

$N^{(1)} = 2$, $N^{(2)} = 3$

$$\frac{1}{T^{(1)}} = \frac{3}{2} R \frac{N^{(1)}}{U^{(1)}} , \quad \frac{1}{T^{(2)}} = \frac{5}{2} R \frac{N^{(2)}}{U^{(2)}}$$

$$U^{(1)} + U^{(2)} = 2.5 \times 10^3 \, J$$

$\Rightarrow$ At equil

$$\frac{1}{T^{(1)}} = \frac{1}{T^{(2)}}$$

$$\therefore \frac{\frac{3}{2} R N^{(1)}}{U^{(1)}} = \frac{\frac{5}{2} R N^{(2)}}{U^{(2)}}$$

$$\Rightarrow U^{(1)} : U^{(2)} = 2 : 5$$

Therefore $U^{(1)} = \frac{2}{7} \cdot 2.5 \times 10^3 \, J = 714.3 \, J$

$$U^{(2)} = (2.5 \times 10^3 - 714.3) \, J = 1785.7 \, J$$

2.6-4

Initially $T_i^{(1)} = 250 \, K$, $T_i^{(2)} = 350 \, K$ $\Longrightarrow T_f = ?$

$$U_i^{(1)} = \frac{3}{2} R N^{(1)} T_i^{(1)}$$
$$U_i^{(2)} = \frac{5}{2} R N^{(2)} T_i^{(2)}$$

$$\Rightarrow \quad \frac{3}{2} R N^{(1)} T_f = U_f^{(1)}$$
$$\frac{5}{2} R N^{(2)} T_f = U_f^{(2)}$$

$\hookrightarrow$ reach an identical temp.

$U_f^{(1)} + U_f^{(2)} = U_i^{(1)} + U_i^{(2)}$: energy conservation

$$\therefore \quad T_f = \frac{3/2 \cdot \cancel{R} \cdot N^{(1)} T_i^{(1)} + 5/2 \, \cancel{R} \cdot N^{(2)} T_i^{(2)}}{3/2 \, \cancel{R} \cdot N^{(1)} + 5/2 \cdot \cancel{R} \cdot N^{(2)}} \approx 321 \, K .$$

$$U_f^{(1)} = \frac{3}{2} R \cdot N^{(1)} T_f = \frac{3}{2} R \cdot 2 \cdot 321 \cong 8 \, kJ$$

$$U_f^{(2)} = \frac{5}{2} R \cdot N^{(2)} T_f = \frac{5}{2} R \cdot 3 \cdot 321 \cong 20 \, kJ$$

2.7-2.

Energy conservation & equilibrium condition ($T_f^{(1)} = T_f^{(2)}$)

$$U_i^{(1)} + U_i^{(2)} = \frac{3}{2}R\cdot 0.5\cdot 200 + \frac{5}{2}R\cdot 0.75\cdot 300 \qquad \overset{\shortparallel}{T_f}$$

$$= \frac{3}{2}R\cdot 0.5\, T_f + \frac{5}{2}R\cdot 0.75\cdot T_f$$

$$T_f = \frac{3/2\cdot R\cdot 0.5\cdot 200 + 5/2\, R\cdot 0.75\cdot 300}{3/2\cdot R\cdot 0.5 + 5/2\cdot R\cdot 0.75} \cong 271\,K.$$

Volume conservation & equilibrium condition ($P^{(1)}/T_1 = P^{(2)}/T_2$) $\qquad \overset{\shortparallel}{P_f}$

$$V_i^{(1)} + V_i^{(2)} = 20\,l = \underbrace{\frac{\overset{0.5}{N^{(1)}}\, R\, T_f}{P_f}}_{V_f^{(1)} = 8\,l} + \underbrace{\frac{\overset{0.75}{N^{(2)}}\, R\cdot T_f}{P_f}}_{V_f^{(2)} = 12\,l}$$

$$\therefore\ P_f/T_f = (0.5 + 0.75)R/20\,l$$

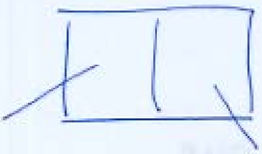
$$\therefore\ P_f = \frac{1.25\cdot 8.314\ J/\overset{mol}{mol}\cdot K}{20\cdot 10^{-3}\ m^3} \times 271\,K$$

$$= 1.405\times 10^5\ J/m^3 = 1.405\times 10^5\ Pa$$

$$U_f^{(1)} = \frac{3}{2}R\cdot 0.5\cdot 271 = 1689\,J \approx 1700\,J$$

$$U_f^{(2)} = \frac{5}{2}R\cdot 0.75\cdot 271 = 4224\,J \approx 4200\,J$$

2.8-1



$N_1^{(1)} = 0.5$

$N_2^{(1)} = 0.75$

$T^{(1)} = 300$

$N_1^{(2)} = 1$

$N_2^{(2)} = 0.5$

$T^{(2)} = 250$

$$S = NA + NR \ln \frac{U^{3/2} V}{N^{5/2}} - N_1 R \ln N_1/N - N_2 R \ln N_2/N$$

$$= NA + NR \ln \frac{U^{3/2} V}{N^{3/2}} - N_1 R \ln N_1 - N_2 R \ln N_2$$

$$S = S^{(1)} + S^{(2)} = N^{(1)} A + N^{(1)} R \ln \left[\left(U^{(1)}\right)^{3/2} V^{(1)} / \left(N^{(1)}\right)^{3/2} \right] - N_1^{(1)} R \ln N_1^{(1)} - N_2^{(1)} R \ln N_2^{(1)}$$

$$+ N^{(2)} A + N^{(2)} R \ln \left[\left(U^{(2)}\right)^{3/2} V / \left(N^{(2)}\right)^{3/2} \right] - N_1^{(2)} R \ln N_1^{(2)} - N_2^{(2)} R \ln N_2^{(2)}$$

At equil ... $\frac{1}{T^{(1)}} = \frac{1}{T^{(2)}}$, $\frac{\mu_1^{(1)}}{T^{(1)}} = \frac{\mu_2^{(2)}}{T^{(2)}}$

energy & number conservation are satisfied.

$$\frac{1}{T^{(i)}} = \left(\frac{\partial S}{\partial U^{(i)}}\right) \ldots = \frac{3}{2} N_f^{(i)} R / U_f^{(i)}$$

$$-\frac{\mu_1^{(i)}}{T^{(i)}} = \left(\frac{\partial S}{\partial N_1^{(i)}}\right) = A + R \ln \left(\frac{U^{(i)}}{N^{(i)}}\right)^{3/2} V^{(i)} - \frac{3}{2} R$$

$$- R \ln N_1^{(i)} - R$$

$$\frac{1}{T^{(1)}} = \frac{1}{T^{(2)}} \Rightarrow \frac{N_f^{(1)}}{U_f^{(1)}} = \frac{N_f^{(2)}}{U_f^{(2)}}$$

$$\frac{\mu_1^{(1)}}{T^{(1)}} = \frac{\mu_1^{(2)}}{T^{(2)}} \Rightarrow R \ln \left(\frac{U_f^{(1)}}{N_f^{(1)}}\right)^{3/2} V^{(1)} - R \ln N_{1f}^{(1)} = R \ln \left(\frac{U_f^{(2)}}{N_f^{(2)}}\right)^{3/2} V^{(2)} - R \ln N_{1f}^{(2)}$$

$$\Longrightarrow N_{1f}^{(1)} = N_{1f}^{(2)} \quad \left(\because U_f^{(1)} / N_f^{(1)} = U_f^{(2)} / N_f^{(2)}\right)$$

$$U_i^{(1)} + U_i^{(2)} = U_f^{(1)} + U_f^{(2)}$$

$$N_{1i}^{(1)} + N_{1i}^{(2)} = 1.5 = N_{1f}^{(1)} + N_{1f}^{(2)}$$

$$\therefore \begin{cases} U_f^{(1)} + U_f^{(2)} = \frac{3}{2} \overset{1.25}{N_i^{(1)}} R\, 300 + \frac{3}{2} \overset{1.5}{N_i^{(2)}} R \cdot 250 \\[2ex] \dfrac{N_f^{(1)}}{U_f^{(1)}} = \dfrac{N_f^{(2)}}{U_f^{(2)}} \Longrightarrow \dfrac{N_{1f}^{(1)} + \overset{0.75}{N_{2f}^{(1)}}}{U_f^{(1)}} = \dfrac{N_{1f}^{(2)} + \overset{0.5}{N_{2f}^{(2)}}}{U_f^{(2)}} \\[2ex] N_{1f}^{(1)} = N_{2f}^{(2)} \\[1ex] N_{1f}^{(1)} + N_{1f}^{(2)} = 1.5 \end{cases}$$

4 unknowns & 4 equations.

$$N_{1f}^{(1)} = N_{1f}^{(2)} = 0.75 \Rightarrow N_f^{(1)} = N_{1f}^{(1)} + N_{2f}^{(1)} = 1.5$$

$$N_f^{(2)} = N_{1f}^{(2)} + N_{2f}^{(2)} = 1.25$$

$$\cancel{U_f^{(1)} =}$$

$$U_i^{(1)} + U_i^{(2)} = \left(\frac{3}{2} N_i^{(1)} R \cdot 300 + \frac{3}{2} N_i^{(2)} \cdot R \cdot 250 \right) = U_f^{(1)} + U_f^{(2)}$$

$$= \left(\frac{3}{2} N_f^{(1)} R \cdot T_f + \frac{3}{2} N_f^{(2)} \cdot R \cdot T_f \right)$$

$$T_f = \frac{\overset{1.25}{N_i^{(1)}} R \cdot 300 + \overset{1.5}{N_i^{(2)}} \cdot R \cdot 250}{\underbrace{N_f^{(1)} + N_f^{(2)}}_{= 2.75}} = 272.7 \text{ K}$$

$$\frac{P_f^{(i)}}{T_f^{(i)}} = \frac{\partial S}{\partial V^{(i)}} = \frac{3}{2} N_f^{(i)} R \Big/ V^{(i)}$$

$$\Rightarrow P^{(i)} = \cancel{3}\, N_f^{(i)} R \cdot 272.7 \text{ K} \Big/ 5\,l$$

$$\therefore \begin{cases} P^{(1)} = \cancel{3} \cdot 8.314 \cdot 1.5 \cdot 272.7 \Big/ 5 \cdot 10^{-3} \text{ Pa} = 0.68 \text{ MPa} \\[2ex] P^{(2)} = \cancel{3} \cdot 8.314 \cdot 1.25 \cdot 272.7 \Big/ 5 \cdot 10^{-3} \text{ Pa} \simeq 0.57 \text{ MPa} \end{cases}$$