1.10-1. & 1.10-2

(a) $S = \left(\frac{R^2}{v_0\theta}\right)^{1/3}(UVN)^{1/3} \Longrightarrow$ all satisfied

$\longrightarrow U = \left(\frac{v_0\theta}{R^2}\right)\frac{S^3}{VN} = U(S,V,N)$

Check
$$\begin{cases} \lambda S = S(\lambda U, \lambda V, \lambda N) \\ \left(\frac{\partial S}{\partial U}\right) \text{ or } \left(\frac{\partial U}{\partial S}\right) > 0 \\ T = \left(\frac{\partial U}{\partial S}\right) \xrightarrow{?} 0 \ (\text{when } S \to 0) \end{cases}$$

(b) $S = \left(\frac{R}{\theta^2}\right)^{1/3}\left(\frac{NU}{V}\right)^{2/3}$ (X)

$\lambda S \neq S(\lambda U, \lambda V, \lambda N)$

(c) $S = \left(\frac{R}{\theta}\right)^{1/2}\left(NU + \frac{R\theta V^2}{v_0^2}\right)^{1/2}$

$\lambda S = S(\lambda U, \lambda V, \lambda N)$

$\left(\frac{\partial S}{\partial U}\right)_{V,N} = \left(\frac{R}{\theta}\right)^{1/2}\frac{N}{2}\left(NU + \frac{R\theta V^2}{v_0^2}\right)^{-1/2} \sim \frac{N}{2}\cdot\frac{1}{S} > 0$

$\therefore \left(\frac{\partial U}{\partial S}\right) \to 0$ as $S \to 0$

$\therefore$ All the conditions (postulates) are satisfied

$U = \frac{1}{N}\left[\left(\frac{\theta}{R}\right)S^2 - \frac{R\theta V^2}{v_0^2}\right] = U(S,V,N)$

(d) $S = \left(\frac{R^2\theta}{v_0^3}\right)V^3/NU$ $\left(\to S \sim \frac{1}{U}\right)$

$\left(\frac{\partial S}{\partial U}\right) = -\frac{R^2\theta}{v_0^3}\frac{V^3}{NU^2} \sim -S^2 < 0$

$\left(\frac{\partial S}{\partial U}\right) < 0$ (X)

(e) $S = \left(\frac{R^3}{v_0\theta^2}\right)^{1/5}(N^2VU^2)^{1/5}$ $\left(S \sim U^{2/5}\right)$

$\lambda S = S(\lambda U, \lambda V, \lambda N)$

$\frac{\partial S}{\partial U} > 0$, $\left(\frac{\partial S}{\partial U}\right) \approx \frac{2}{5}U^{-\frac{3}{5}} \Rightarrow \left(\frac{\partial U}{\partial S}\right) \to 0$ as $S \to 0$

$\to U = \left(\frac{v_0\theta^2}{R^3}\right)S^5/N^2V$

(f) $S = NR\ln\left(\dfrac{UV}{N^2R\theta v_0}\right) \quad \left(\rightarrow U \sim e^{S/NR}\right)$

$\lambda S = S(\lambda U, \lambda V, \lambda N) \qquad \left(\dfrac{\partial S}{\partial U}\right)_{N,V} \sim \dfrac{1}{U} \sim e^{-S/NR} > 0$

but $\left(\dfrac{\partial U}{\partial S}\right) \sim e^{S/NR} \not\rightarrow 0$ as $S \rightarrow 0$

(X)

(g) $S = \left(\dfrac{R}{\theta}\right)^{1/2} (NU)^{1/2} e^{-V^2/2N^2v_0^2}$

$\lambda S = S(\lambda U, \lambda V, \lambda N)$

$\dfrac{\partial S}{\partial U} \sim \dfrac{1}{2} U^{-\frac{1}{2}} > 0 \qquad \left(\dfrac{\partial U}{\partial S}\right) \sim U^{\frac{1}{2}} \sim S \longrightarrow 0$ (as $S \rightarrow 0$)

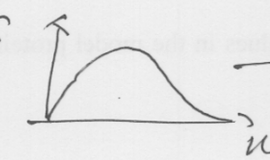
$\therefore$ All the postulates are fulfilled

$U = \dfrac{1}{N} S^2 \left(\dfrac{R}{\theta}\right)^{-1} \cdot e^{V^2/2N^2v_0^2}$

(h) $S = \left(\dfrac{R}{\theta}\right)^{1/2} (NU)^{1/2} e^{-\frac{UV}{NR\theta v_0}} \qquad$ (X)

$\lambda S \neq \lambda S(U, V, N)$

$(\partial S/\partial U)_{N,V} > 0 \rightarrow$ is not satisfied



$\rightarrow$ This is the shape of $S = S(U)$

S is not a monotonically increasing function of U

(i) $U = \left(\dfrac{v_0\theta}{R}\right) \dfrac{S^2}{V} \exp(S/NR)$

$\left(\dfrac{\partial U}{\partial S}\right) = \left(\dfrac{v_0\theta}{R}\right) \cdot \left(2S/V\, e^{S/NR} + \dfrac{S^2}{VNR} e^{S/NR}\right) > 0$ (for $\forall S > 0$)

$\longrightarrow 0$ (as $S \rightarrow 0$)

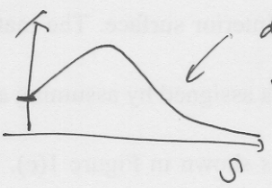
$U = U(S, V, N)$

(j) $U = \left(\frac{R\theta}{v_0}\right) NV \left(1 + \frac{S}{NR}\right) \exp(-S/NR)$

$\lambda U = U(\lambda S, \lambda V, \lambda N)$

$T = \left(\frac{\partial U}{\partial S}\right) > 0$

$\hookrightarrow$ not always satisfied



decreasing function of S

1.10-3. $S = \left(\frac{R^2}{v_0 \theta}\right)^{1/3} (NVU)^{1/3}$

A: $\boxed{\begin{array}{l} V_A = 9\times10^{-6}\, m^3 \\ N_A = 3 \text{ moles} \end{array}}$ B: $\boxed{\begin{array}{l} V_B = 4\times10^{-6}\, m^3 \\ N_B = 2 \text{ moles} \end{array}}$ $U_A + U_B = 80\,J = U_{tot}$

thermal contact through diathermal wall

$$S = S_A + S_B = \left(\frac{R^2}{v_0\theta}\right)^{1/3} \left[(N_A V_A U_A)^{1/3} + (N_B V_B U_B)^{1/3} \right]$$

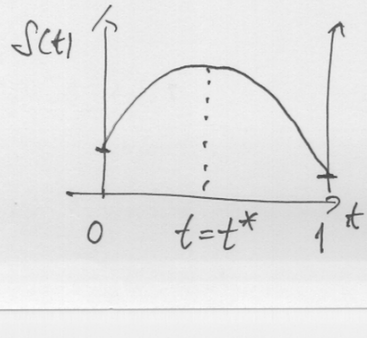
$$\left(t \equiv \frac{U_A}{U_{tot}}\right) \quad = \left(\frac{R^2}{v_0\theta}\right)^{1/3} \left[(N_A V_A U_{tot})^{1/3} t^{1/3} + (N_B V_B U_{tot})^{1/3} (1-t)^{1/3} \right]$$

$$0 = \frac{dS}{dt} = \left(\frac{R^2}{v_0\theta}\right)^{1/3} \left[\frac{1}{3}(N_A V_A U_{tot})^{1/3} t^{-2/3} - \frac{1}{3}(N_B V_B U_{tot})^{1/3} (1-t)^{-2/3} \right]$$

S becomes maximized (at $t = t^*$) (equilibrated)

$$\frac{1-t^*}{t^*} = \left(\frac{N_B V_B U_{tot}}{N_A V_A U_{tot}}\right)^{1/2} = \left(\frac{2}{3} \cdot \frac{4}{9}\right)^{1/2} = \sqrt{\frac{8}{27}}$$

$$\therefore\ t^* = \frac{U_A^*}{U_{tot}} = \frac{\sqrt{27}}{\sqrt{27} + \sqrt{8}}$$



$$\therefore \begin{cases} U_A^* = \dfrac{\sqrt{27}}{\sqrt{27}+\sqrt{8}} \times 80\,J \\[2ex] U_B^* = \dfrac{\sqrt{8}}{\sqrt{27}+\sqrt{8}} \times 80\,J \end{cases}$$

2.2-4

$$u = \left(\frac{\theta}{R}\right) s^2 - \left(\frac{R\theta}{v_0^2}\right) v^2$$

$$\Rightarrow \quad U = \left(\frac{\theta}{R}\right)\frac{S^2}{N} - \left(\frac{R\theta}{v_0^2}\right)\frac{V^2}{N}$$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} = -\left(\frac{\theta}{R}\right)\frac{S^2}{N^2} + \left(\frac{R\theta}{v_0^2}\right)\frac{V^2}{N^2}$$

$$= -\left\{\left(\frac{\theta}{R}\right) s^2 - \left(\frac{R\theta}{v_0^2}\right) v^2\right\} = -u$$

$$\therefore \quad \mu = -u$$

2.2-5

$$T = \left(\frac{\partial u}{\partial s}\right)_v = 2\left(\frac{\theta}{R}\right) s$$

$$p = -\left(\frac{\partial u}{\partial v}\right)_s = 2\left(\frac{R\theta}{v_0^2}\right) v$$

$$\therefore \quad u = \frac{1}{4}\left(\frac{R}{\theta}\right) T^2 - \frac{1}{4}\left(\frac{v_0^2}{R\theta}\right) p^2$$

Since $\mu = -u$ $\qquad \mu = -\frac{1}{4}\left(\frac{R}{\theta}\right) T^2 + \frac{1}{4}\left(\frac{v_0^2}{R\theta}\right) p^2$

2.2-6

$$u = \left(\frac{v_0\theta}{R}\right)\frac{s^2}{v} e^{s/R} \longrightarrow U = \left(\frac{v_0\theta}{R}\right)\frac{S^2}{V} e^{S/NR}$$

$$dU = TdS - pdV + \mu dN$$

$$T = \left(\frac{\partial U}{\partial S}\right)_{V,N} = \left(\frac{v_0\theta}{R}\right)\left\{\frac{2S}{V} + \frac{S^2}{NVR}\right\} e^{S/NR}$$

$$-p = \left(\frac{\partial U}{\partial V}\right)_{S,N} = -\frac{v_0\theta}{R}\frac{S^2}{V^2} e^{S/NR} \Rightarrow p = \frac{v_0\theta}{R}\frac{S^2}{V^2} e^{S/NR}$$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} = -\left(\frac{v_0\theta}{R}\right)\frac{S^2}{V}\frac{S}{RN^2} e^{S/NR}$$

$$= -\left(\frac{v_0\theta}{R}\right)\frac{S^3}{RN^2V} e^{S/NR}$$

2.2-7 $u = A v^{-2} \exp(S/R)$

initially at T_0 and P_0

expanded isentropically (constant S) until the pressure is halved $P = P_0/2$.

$$\begin{cases} T = \left(\frac{\partial u}{\partial S}\right)_v = A v^{-2}/R \cdot e^{S/R} \\ P = -\left(\frac{\partial u}{\partial v}\right)_S = 2 A v^{-3} e^{S/R} \end{cases}$$

$$T_0 = A v_i^{-2}/R \, e^{S/R}$$

$$P_0 = 2 A v_i^{-3} e^{S/R} \longrightarrow P_0/2 = 2 A v_f^{-3} e^{S/R}$$

$$\therefore \left(1/2\right) = \left(\frac{v_f}{v_i}\right)^{-3} \quad v_f = 2^{+\frac{1}{3}} v_i$$

$$\therefore \quad T_f = A v_f^{-2}/R \, e^{S/R} = A \, 2^{-\frac{2}{3}} v_i^{-2}/R \cdot e^{S/R}$$

$$= 2^{-\frac{2}{3}} T_0$$

$$\therefore T_f \simeq 0.63 \, T_0$$

2.3-5

a) $S/R = \frac{UV}{N} - \frac{N^3}{UV}$

$$\left(\frac{\partial S}{\partial U}\right)_{N,V} = \frac{1}{T} = \frac{V}{N}R + \frac{N^3}{U^2V}R \longrightarrow V\to\lambda V,\ U\to\lambda U,\ N\to\lambda N \longrightarrow$$

$$\frac{\lambda V}{\lambda N}R + \frac{(\lambda N)^3}{(\lambda U)^2\lambda V}R = \frac{1}{T}$$

$$\therefore\ \frac{1}{T} = \frac{1}{T}(\lambda U, \lambda V, \lambda N)$$

homogeneous 0th-order functions

$$\left(\frac{\partial S}{\partial V}\right) = +\frac{P}{T} = \frac{U}{N}R + \frac{N^3}{UV^2}R$$

$$\left(\frac{\partial S}{\partial N}\right) = -\frac{\mu}{T} = -\frac{UV}{N^2}R - \frac{3N^2}{UV}R$$

$$\therefore\ \frac{\mu}{T} = \frac{UV}{N^2}R + \frac{3N^2}{UV}R = \frac{\mu}{T}(\lambda U, \lambda V, \lambda N)$$

b) $1/T = \frac{V}{N}R + \frac{N^3}{U^2V}R > 0$ for all V, N, U

Therefore T is intrinsically positive

c)

$$① \quad \frac{1}{T} = vR + \frac{1}{u^2v}R$$

$$② \quad \frac{P}{T} = uR + \frac{1}{uv^2}R$$

from a)

$$① \times (u/v) \div ② \Rightarrow \frac{\frac{u}{v}\frac{1}{T}}{\frac{P}{T}} = 1 \quad \therefore\ P = \frac{u}{v}$$

from ① $\frac{R}{u^2v} = -vR + \frac{1}{T}$, $u = \left[\frac{R}{v}\left(-vR + \frac{1}{T}\right)^{-1}\right]^{1/2}$

$$\therefore\ P = \frac{1}{v}\left(\frac{R/v}{-vR + \frac{1}{T}}\right)^{1/2} = P(T, v)$$

d) from c) we know that $u = pv$

$$\therefore \; s/R = uv - \frac{1}{uv} = pv^2 - \frac{1}{pv^2}$$

constant.

$$pv^2 - \frac{1}{pv^2} = s/R$$

$$(pv^2)^4 - (s/R)\cdot(pv^2) - 1 = 0$$

$$pv^2 = \frac{s/R + \sqrt{(s/R)^2 + 4}}{2} = \text{constant}$$

$$\therefore \; pv^2 = \text{constant}.$$

(OR)

$du = -p\,dv \rightarrow$ this should be satisfied along adiabatic path.

$u = pv$

$$p\,dv + v\,dp = -p\,dv$$

$$v\,dp = -2p\,dv$$

$$\frac{1}{p}dp = -\frac{2}{v}dv$$

$$\ln p = -2\ln v + c$$

$$\therefore \; pv^2 = \text{constant}$$