1. (a) $S(\lambda U, \lambda V, \lambda N) = \lambda N S_0 + \lambda N R \log \dfrac{(\lambda U)^{3/2}(\lambda V)}{(\lambda N)^{5/2}} = \lambda S(U,V,N)$; S is extensive

$\dfrac{\partial S}{\partial U} = \dfrac{3}{2} NR/U > 0$ (for $\forall U > 0$) $\therefore$ S is monotonically increasing function of U

However $\left(\dfrac{\partial U}{\partial S}\right) = \dfrac{U}{\frac{3}{2}NR} = \dfrac{1}{\frac{3}{2}NR} e^{(S-NS_0)/NR \cdot \frac{2}{3}} \not\longrightarrow 0$

as $S \to 0$

Therefore, the Nernst theorem is not satisfied.

Ideal gas equation is only valid when $T \gg 0$

(b) $\left(\dfrac{\partial S}{\partial U}\right)_{V,N} = \dfrac{1}{T} = \dfrac{3}{2}NR/U \longrightarrow T = \dfrac{U}{\frac{3}{2}NR} = \dfrac{2U}{3NR}$

$\left(\dfrac{\partial S}{\partial V}\right)_{U,N} = \dfrac{P}{T} = \dfrac{NR}{V} \longrightarrow P = \dfrac{NR}{V} \cdot T = \dfrac{2}{3}\dfrac{U}{V}$ $\quad P = \dfrac{2U}{3V}$

(c) $W = -\displaystyle\int_{V_0}^{2V_0} p\,dV = -\int_{V_0}^{2V_0} \frac{RT_0}{V} dV = -RT_0 \ln 2$

$\Delta U = 0$ (isothermal, $U = \frac{3}{2}RT \longrightarrow \Delta U = \frac{3}{2}R\Delta T = 0$)

$Q = -W = RT_0 \ln 2$

(d) $W = -\displaystyle\int_{V_0}^{2V_0} p\,dV = -p\overbrace{\Delta V}^{(2V_0 - V_0)} = -pV_0$

$\Delta U = \frac{3}{2}R\Delta T = \frac{3}{2}P\Delta V = \frac{3}{2}PV_0$

$Q = \Delta U - W = \frac{5}{2}PV_0$

(e) From equation (1), $\left(U^{3/2}V/N^{5/2}\right) = \text{const}$

$\Longleftrightarrow \quad T^{3/2}V = \text{const}$

$\Longleftrightarrow$

$\quad PV^{5/3} = \text{const} = P_0V_0^{5/3} = c$

$\therefore W = -\displaystyle\int_{V_0}^{2V_0} p\,dV = -\int_{V_0}^{2V_0} cV^{-5/3} dV = \frac{3}{2} P_0V_0^{5/3}\left[(2V_0)^{-2/3} - V_0^{-2/3}\right]$

$= \frac{3}{2}P_0V_0\left(2^{-\frac{2}{3}} - 1\right)$

$Q = 0$

$\therefore \Delta U = W = \frac{3}{2}\left(2^{-2/3} - 1\right)P_0V_0$

2. (a) $$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$[a] = \left(N/m^2\right)\cdot\left(m^3/mol\right)^2$; correction accounting for intermolecular attraction

$[b] = m^3/mol$; correction accounting for excluded volume

(b)
$$\frac{pv}{RT} = \frac{1}{1-b/v} - \frac{a}{v}\frac{1}{RT}$$
$$\simeq 1 + \frac{b}{v} + \frac{b^2}{v^2} - \frac{a}{v}\frac{1}{RT} \cdots$$
$$= 1 + \frac{1}{v}\left(b - \frac{a}{RT}\right) + \frac{b^2}{v^2} + \cdots$$

when $\underline{T = T_B = \frac{a}{bR}}$ $\quad \frac{pv}{RT} \simeq 1$

(c)
$$\frac{P}{T} = \frac{R}{v-b} - \frac{a}{v^2}\frac{1}{T} \qquad \text{From } ds = \frac{1}{T}du + \frac{P}{T}dv$$

$$\frac{\partial}{\partial v}\left(\frac{1}{T}\right)_u = \frac{\partial}{\partial u}\left(\frac{P}{T}\right)_v = \frac{\partial}{\partial u}\left(\frac{R}{v-b} - \frac{a}{v^2}\frac{1}{T}\right)_v = -\frac{a}{v^2}\frac{\partial}{\partial u}\left(\frac{1}{T}\right)_v$$

$$\frac{\partial}{\partial(1/v)} = \frac{a}{1}\frac{\partial}{\partial u}\left(\frac{1}{T}\right)_u \Longleftrightarrow \frac{\partial}{\partial(a/v)}\left(\frac{1}{T}\right)_u = \frac{\partial}{\partial u}\left(\frac{1}{T}\right)_v$$

$$\therefore \quad \frac{1}{T} = \frac{cR}{u + a/v}$$

$$\therefore \quad ds = \frac{cR}{u + a/v}du + \left(\frac{R}{v-b} - \frac{a}{v^2}\frac{cR}{u + a/v}\right)dv$$

$$\underline{S = Ns_0 + NR\ln\left(u + \frac{a}{v}\right)^c (v-b)} \quad \text{where } c = 3/2 \text{ for monatomic gas}$$

(d) For isentropic condition

$$\left(u + a/v\right)^c (v-b) = \text{const.}$$

$$T^c(v-b) = \text{const} \Longrightarrow \left(p + a/v^2\right)(v-b)^{\frac{c+1}{c}} = \text{const}$$

$$\Longrightarrow \left(p + a/v^2\right)(v-b)^{5/3} = \text{const}$$