Special topics in Phys. Chem. Mid-term exam (Fall 2008)

1. *Polymer models (30pt)*

A. Give a brief explanation on the following terminologies for polymer physics (To give a succinct explanation you may want to use some equations).

(a) Random flight polymer (or Freely Jointed Chain (FJC)) (4pt)

(b) Freely rotating polymer (4pt)

(c) Wormlike-chain (WLC) model (4pt)

(d) Self-avoiding walk (SAW) chain (4pt)

(e) persistence length (4pt)

B. Derive the force-extension relation for FJC of length $L(= Na)$ using statistical thermodynamics. (10pt)

$$\frac{\langle z \rangle}{L} = \coth\left(\frac{fa}{k_B T}\right) - \frac{k_B T}{fa} \tag{1}$$

2. *Polymer Physics using scaling argument (30pt)*

(a) In good solvent, polymers swell. The size of polymer (R_F) scales with the number of monomers (N) as

$$R_F \sim N^{\nu} \tag{2}$$

What is the Flory exponent of polymers in good solvent for d-dimension? Derive the value of ν in terms of d using Flory's argument. (15pt)

(b) For self-avoiding walk (SAW) chain, the extension (z) is scaled with an external force (f) as

$$z \sim f^{\alpha}. \tag{3}$$

Express the exponent α with Flory exponent ν using the scaling argument based on "blob picture". (15pt)

3. *Experimental Studies on Polymer Configurations (40pt)* (Hint:Use the results from Prob.2)

The figures below are from two FANTASTIC papers recently published in *Phys. Rev. Lett.* As each title suggests, configurations of double-stranded DNA are visualized at a single molecule level both in the bulk (3-dimension) (**A**) and on the surface (2-dimension) (**B**). The statistics

for the DNA configurations were collected from the ensemble of individual structures probed with electron microscope.

(a) Both end-to-end distance (R) in the bulk and radius of gyration for circular DNA on the surface (R_G) show cross-over in their scaling exponent at some length scale. Explain why this cross-over shows up. (10 pt)

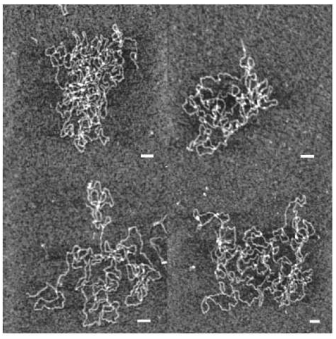
(b) The exponent $\nu_{3,2}$ either in $R \sim L^{\nu_3}$ (**A**) or $R_G \sim L^{\nu_2}$ (**B**) changes from one value to the other. First, estimate the values of these exponents from each graph (panels in the middle). Second, compare the estimates for exponents with theoretical scaling exponents (15 pt)

(c) The panel on the right in **A** plots the end-to-end distance distribution for DNA whose picture is shown on the panel on the left. The following equation is employed to fit the data

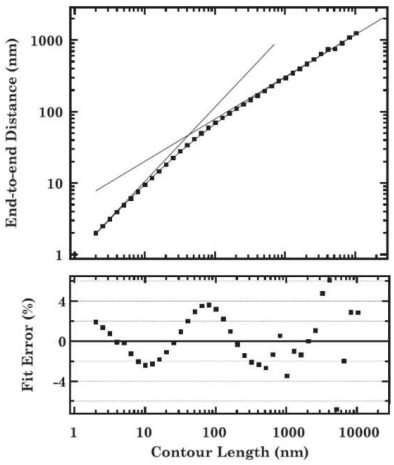
$$P(s) = as^g e^{-bs^\delta}, \tag{4}$$

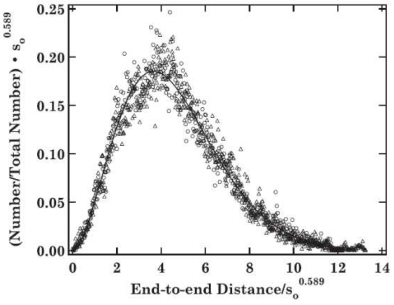
and the exponent δ is found as $\delta = 2.58 \pm 0.79$. Express δ in terms of ν_3 and justify the numerical value of δ from the fit above. (15pt)

A

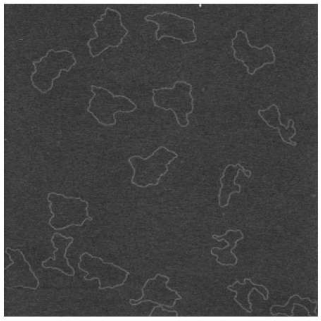


Phys. Rev. Lett. (2005) 95:158105
Scaling Exponents and
Probability Distributions of
DNA End-to-End Distance





B



Phys. Rev. Lett. (2008) 101:148103
Conformation of Circular DNA in Two Dimension