Korea: Lecture 4
Stirring by Microswimmers

Julia Yeomans
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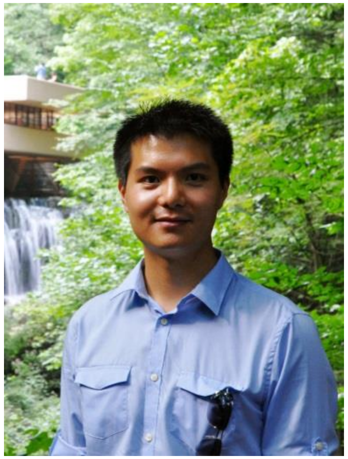


[Fluid Mixing by Curved Trajectories of Microswimmers]
Dmitri O. Pushkin and Julia M. Yeomans
Phys. Rev. Lett. **111**, 188101 (2013)

Fluid transport by individual microswimmers
Journal of Fluid Mechanics **726** (2013) 5-25
DO Pushkin, JM Yeomans, H Shum

Henry Shum
University of Pittsburgh

Jorn Dunkel
University of Cambridge / MIT

Funding: ERC Advanced Grant

Introduction

Loops

What’s wrong with loops

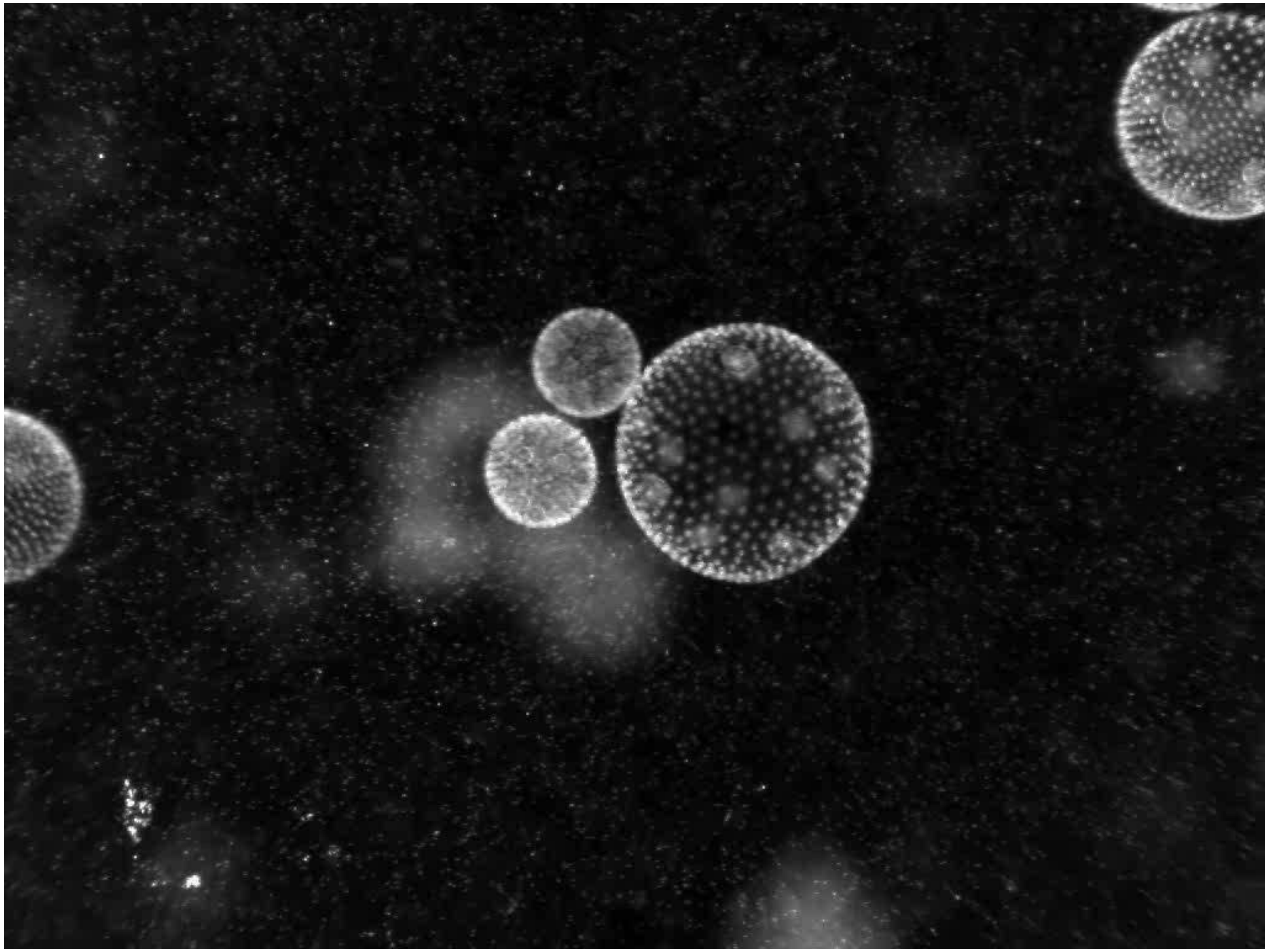
Entrainment

Random reorientations

Stirring by microswimmers

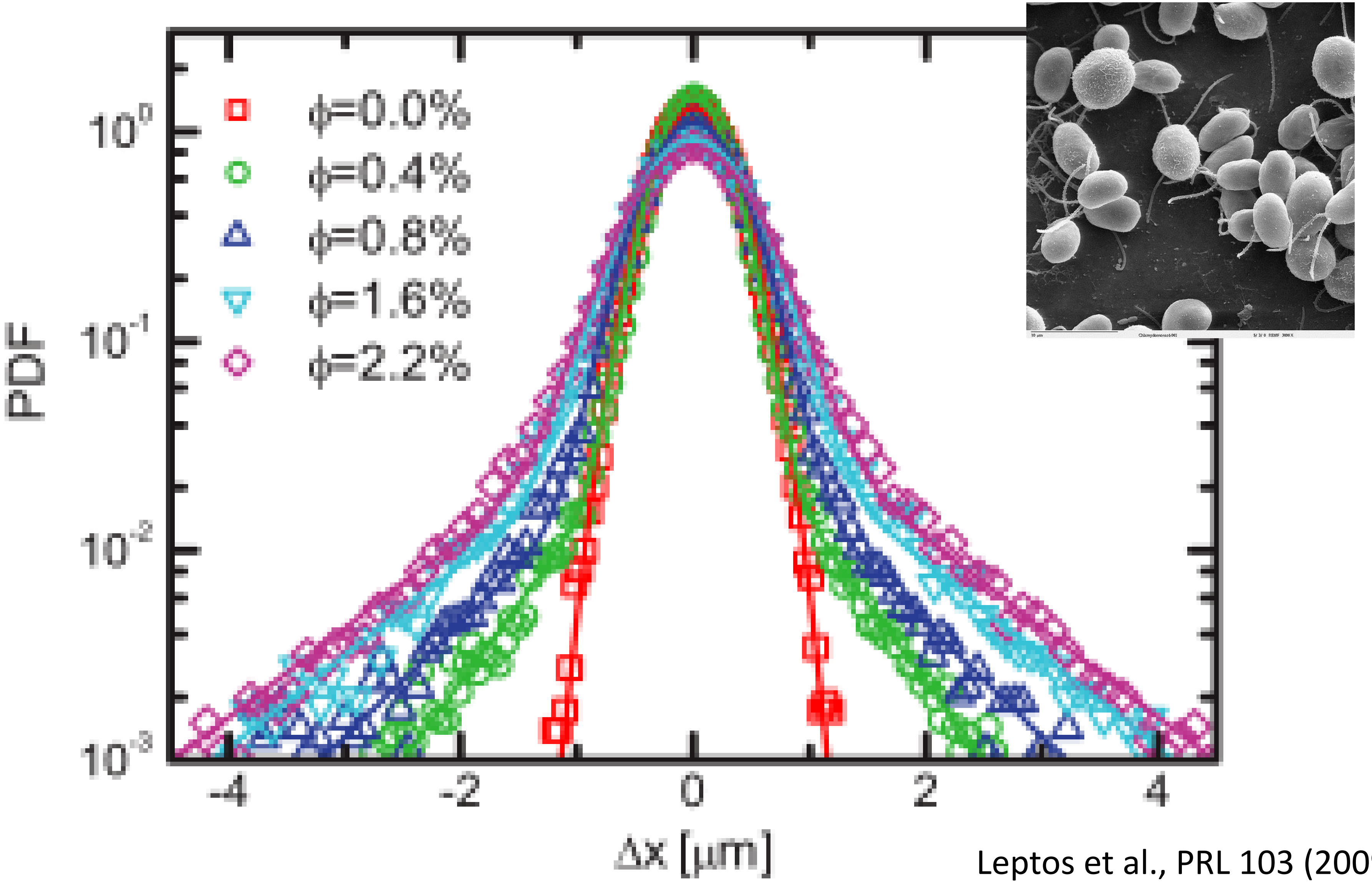
(How) do microswimmers stir the fluid they swim in?

Why do microswimmers stir the fluid they swim in?



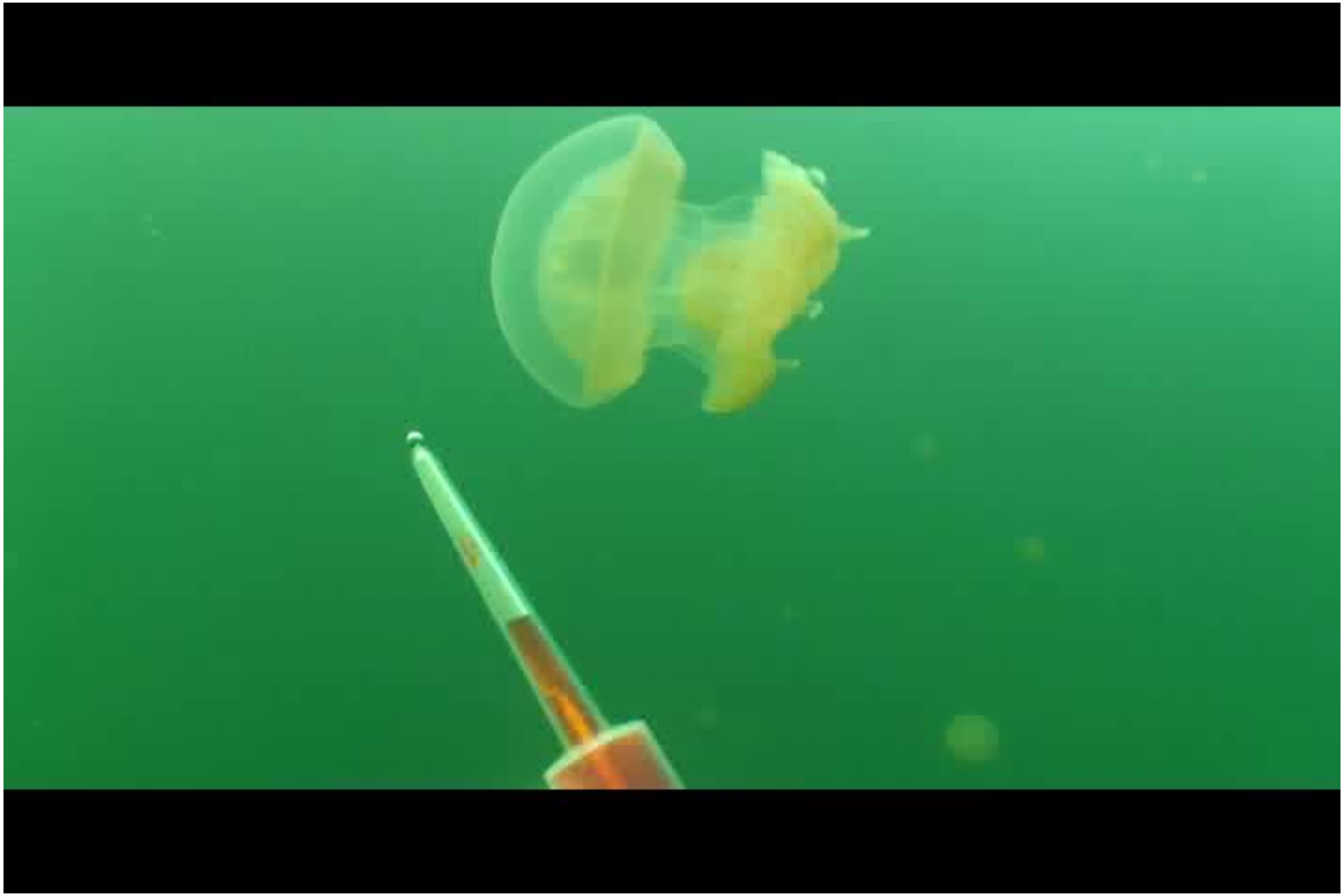
Volvox Goldstein group, Cambridge

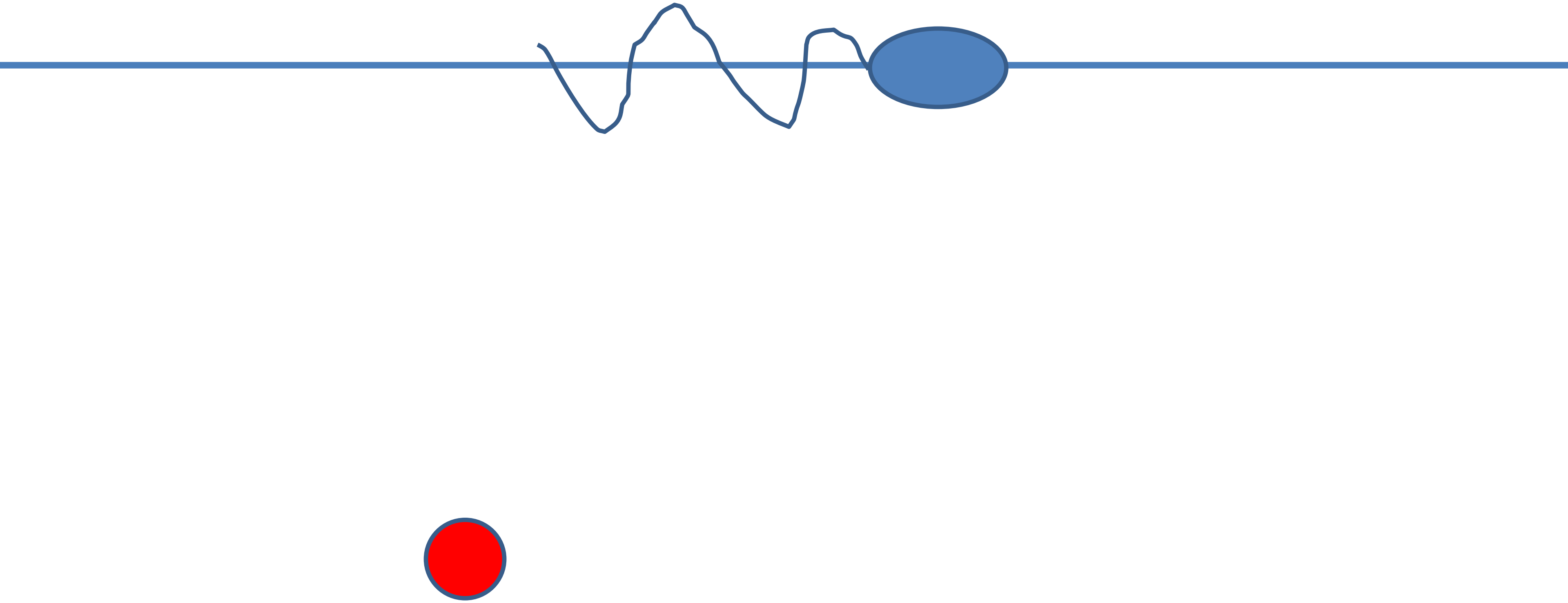
Swimmers enhance diffusion

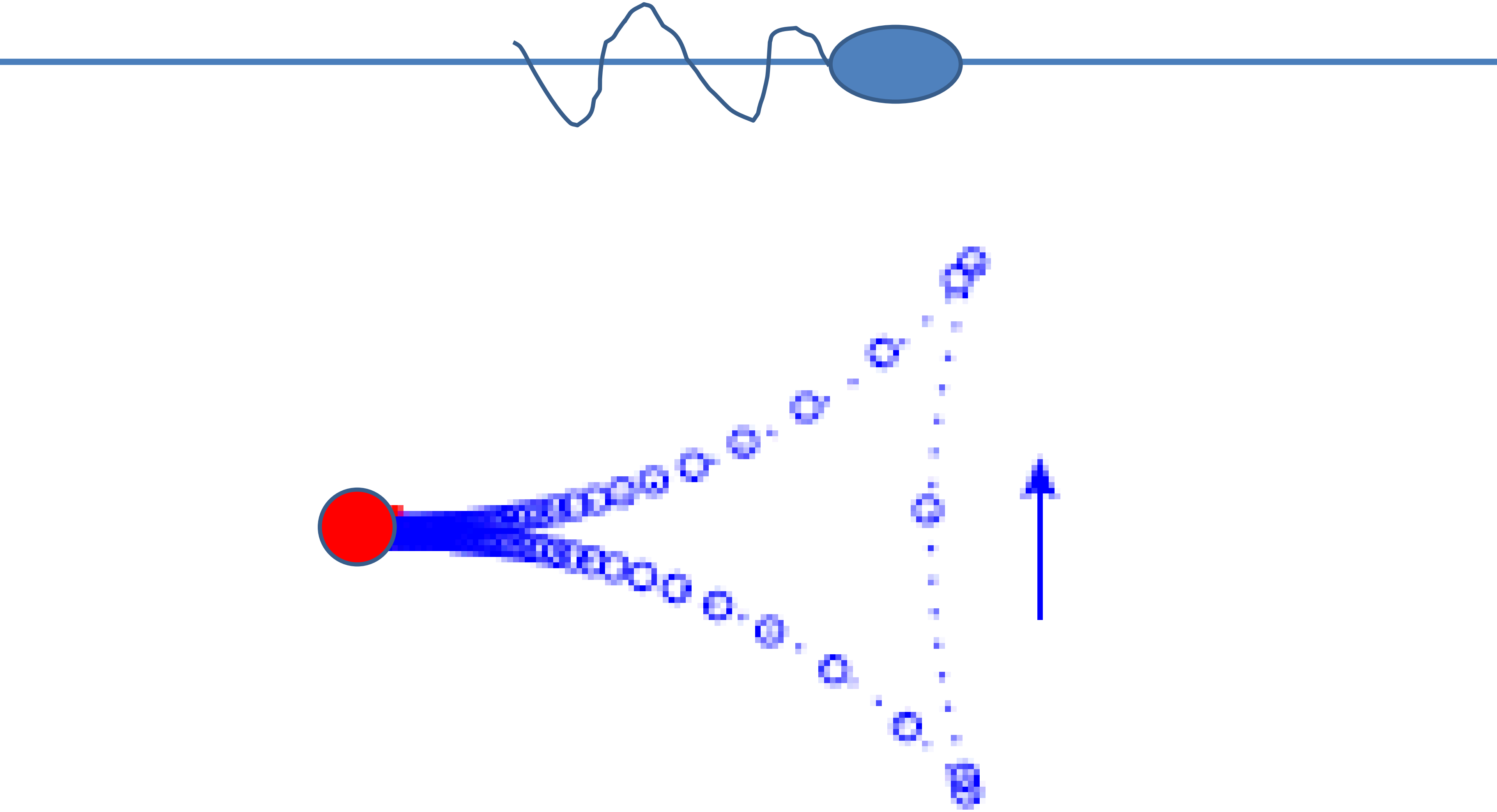


Leptos et al., PRL 103 (2009)

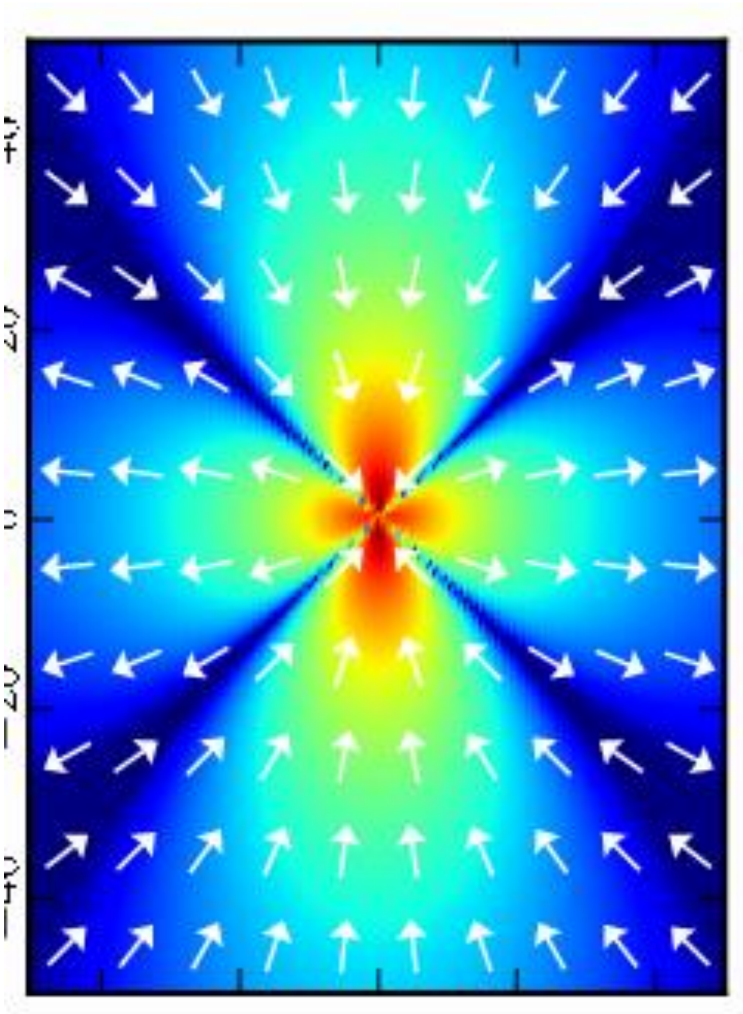
Do small swimmers mix the ocean? K. Katija, J.O. Dabiri, G. Subramanian, A.M. Leshansky, L.M. Pismen, A.W. Visser





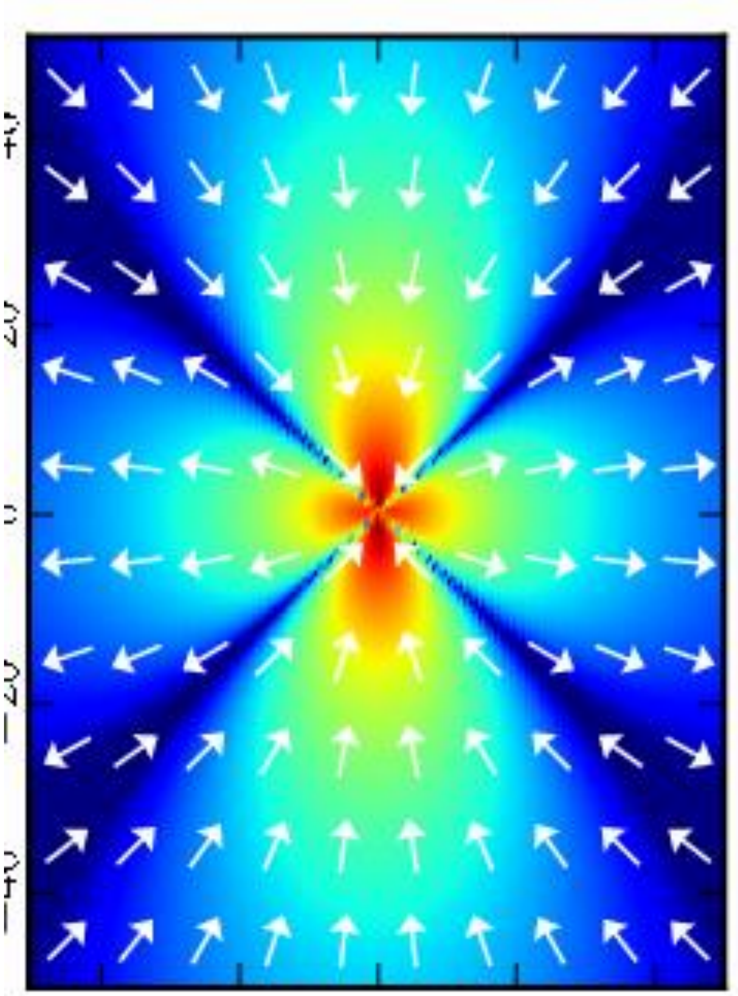


Multipole flow fields



Dipole flow field

Multipole flow fields



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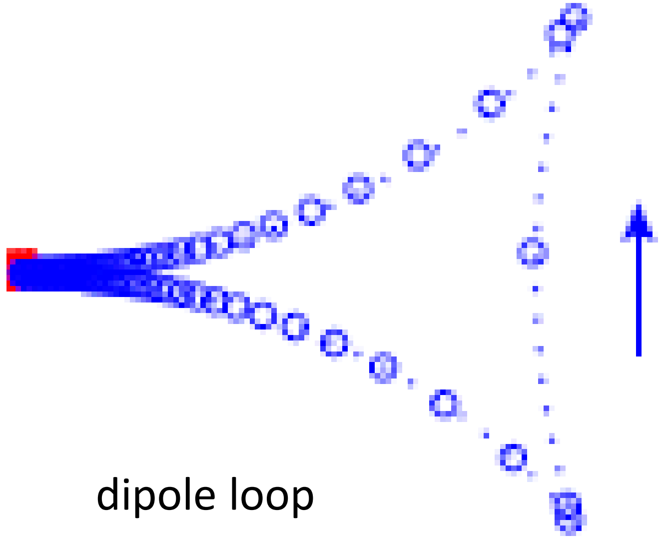
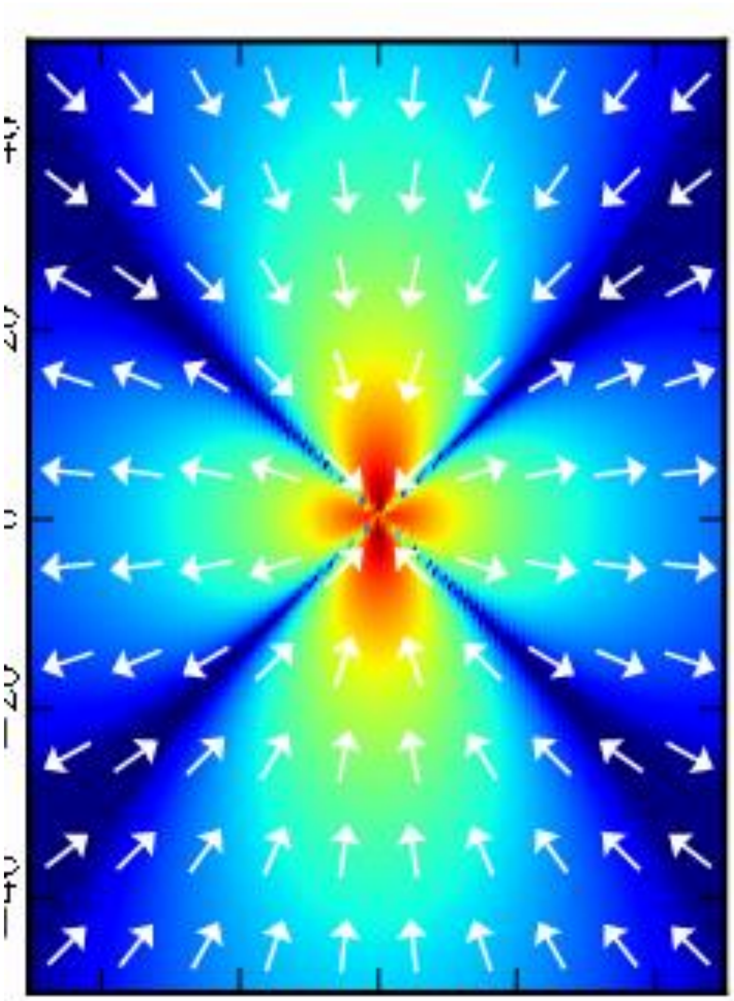
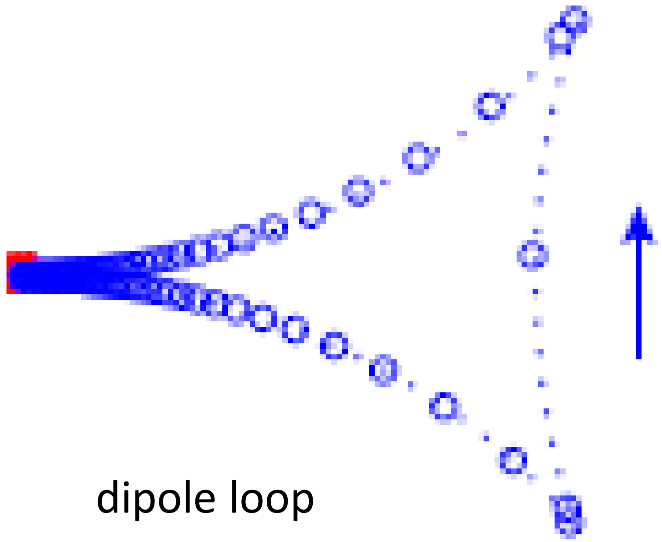


dipole loop

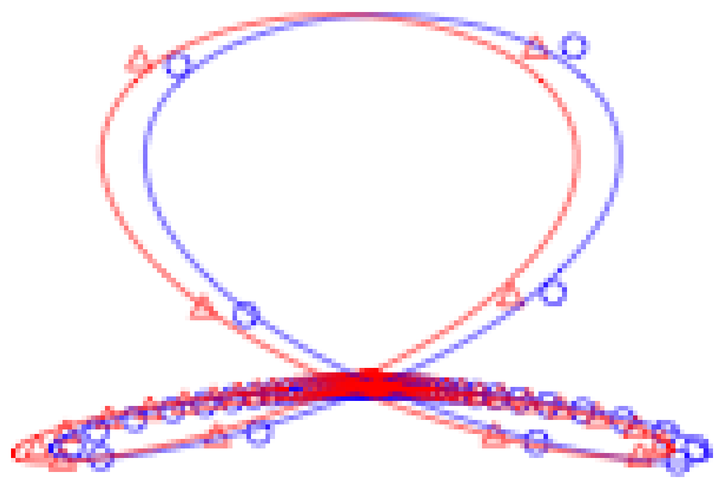
Multipole flow fields



Dipole flow field



dipole loop



quadrupole loop

?? enhanced diffusion and loops ??

Stokeslet:

$$\mathbf{u}^{\mathrm{S}}(\mathbf{r},\mathbf{k}) = \mathbf{k}\cdot\mathbf{J}, \quad \mathbf{J} = \frac{\mathbf{I}}{r} + \frac{\mathbf{r}\mathbf{r}}{r^3}$$

Dipole term:

$$\mathbf{u}^{\mathrm{D}}(\mathbf{r},\mathbf{k}) = -\kappa(\mathbf{k}\cdot\nabla)\,\mathbf{u}^{\mathrm{S}}(\mathbf{r},\mathbf{k})$$

Quadrupole term:

$$\mathbf{u}^{\mathrm{Q}}(\mathbf{r},\mathbf{k}) = -\frac{1}{2}\left(Q_{\parallel}(\mathbf{k}\cdot\nabla)^2 + Q_{\perp}\nabla_{\perp}^2\right)\mathbf{u}^{\mathrm{S}}(\mathbf{r},\mathbf{k})$$

$$Q_{\perp} = -\frac{1}{2}\int_S f_z \rho^2 dS$$

Can write the swimmer flow field as a derivative:

$$\mathbf{u}(\mathbf{r},\mathbf{k}) = (\mathbf{k}\cdot\nabla)\,\mathbf{U}_0(\mathbf{r},\mathbf{k}) + O(r^{-4})$$

$$\frac{d\mathbf{r}_T}{dt} = \mathbf{u}\left(\mathbf{r},\mathbf{k}\right) = \left(\mathbf{k}\cdot\nabla\right)\mathbf{U}_0\left(\mathbf{r},\mathbf{k}\right)$$

Lagrangian derivative at the position of the tracer $\mathbf{r}_T$:

$$\frac{d\mathbf{U}_0}{dt} = \left(\mathbf{V}\cdot\nabla\right)\mathbf{U}_0 - \left(\frac{d\mathbf{r}_T}{dt}\cdot\nabla\right)\mathbf{U}_0, \quad \mathbf{V} = V\mathbf{k}$$

Eulerian derivative:

$$\frac{d\mathbf{U}_0}{dt} \approx V\left(\mathbf{k}\cdot\nabla\right)\mathbf{U}_0$$

tracer velocity:

$$\frac{1}{V}\frac{d\mathbf{U}_0}{dt} \approx \frac{d\mathbf{r}_T}{dt}$$

total tracer displacement:

$$\Delta\mathbf{r}_T = \int_{-\infty}^{+\infty}\frac{d\mathbf{r}_T}{dt}dt = -\frac{\kappa}{V}\left(\mathbf{U}_0(+\infty) - \mathbf{U}_0(-\infty)\right) = \mathbf{0}$$

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entrainment

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infinite swimmer path

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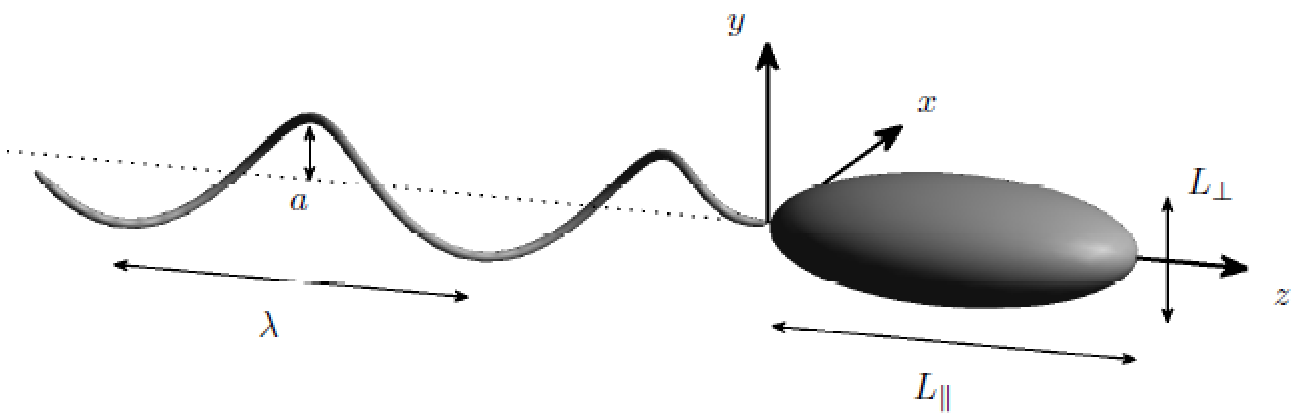
Rhodobacter sphaeroides



Boundary element simulations

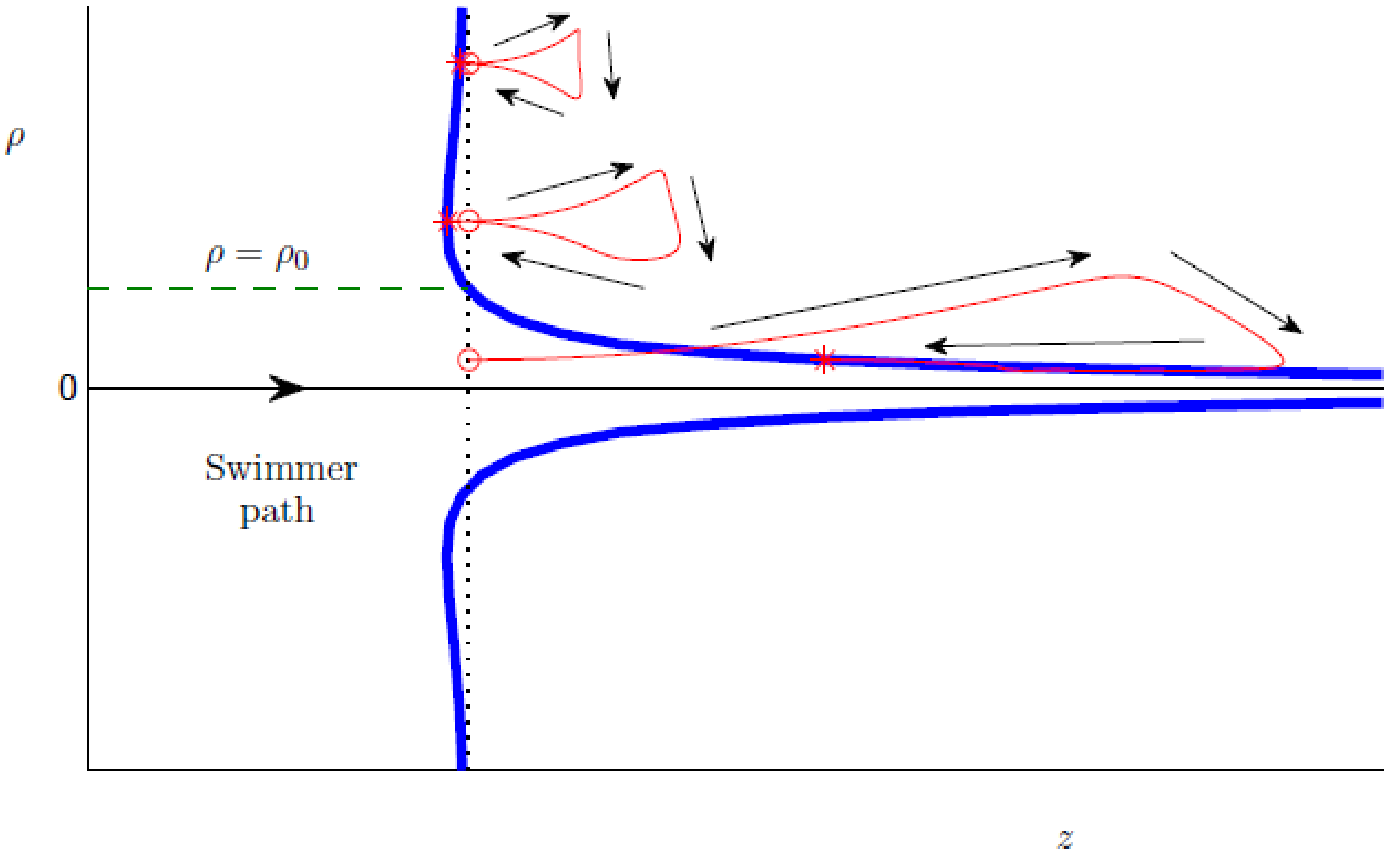
Solve Stokes equations, no slip on swimmer surface, swimmer force and torque free

Swimmer radius 1; swimmer velocity 1; ~ 10 rotations of tail to advance one body length

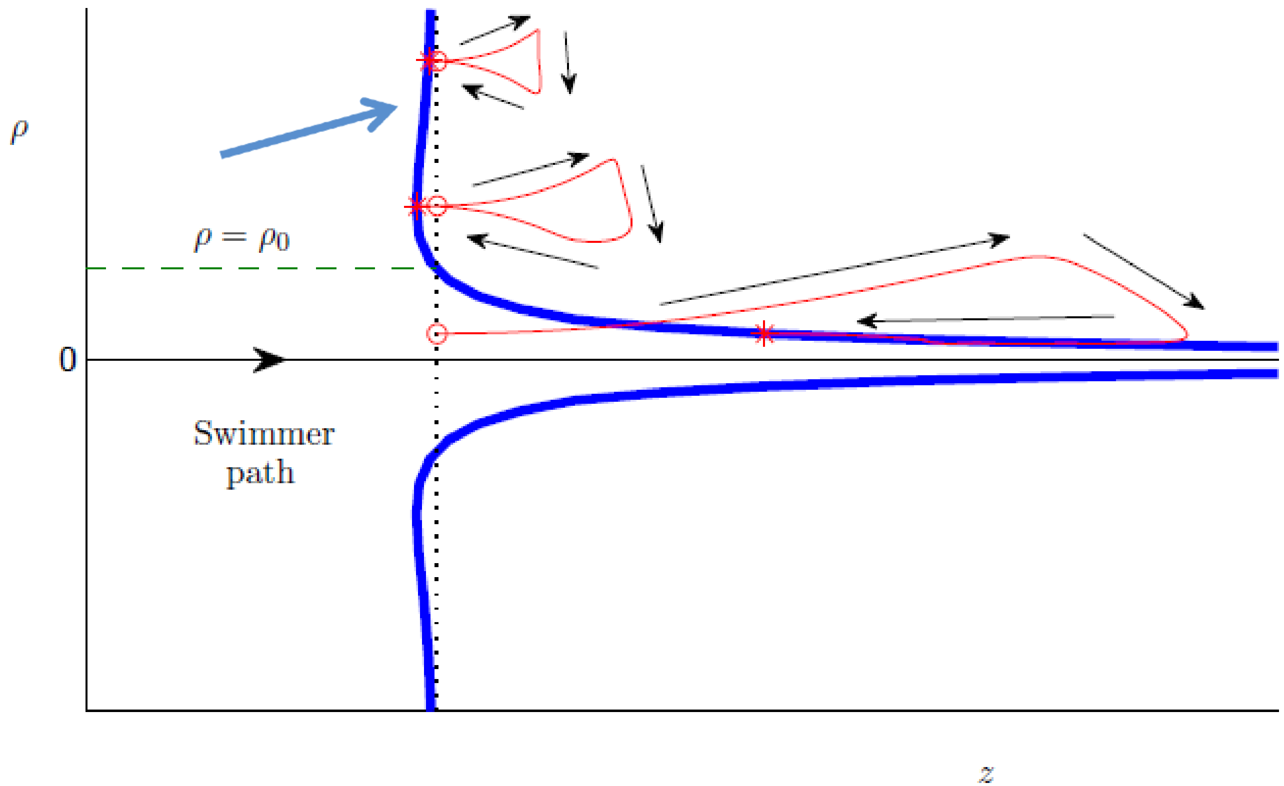
Net tracer displacement along z – deviations from the z-direction very small

Swimmer moves from z= -1000 to z= +1000, and extrapolate to infinite swimmer path

Entrainment



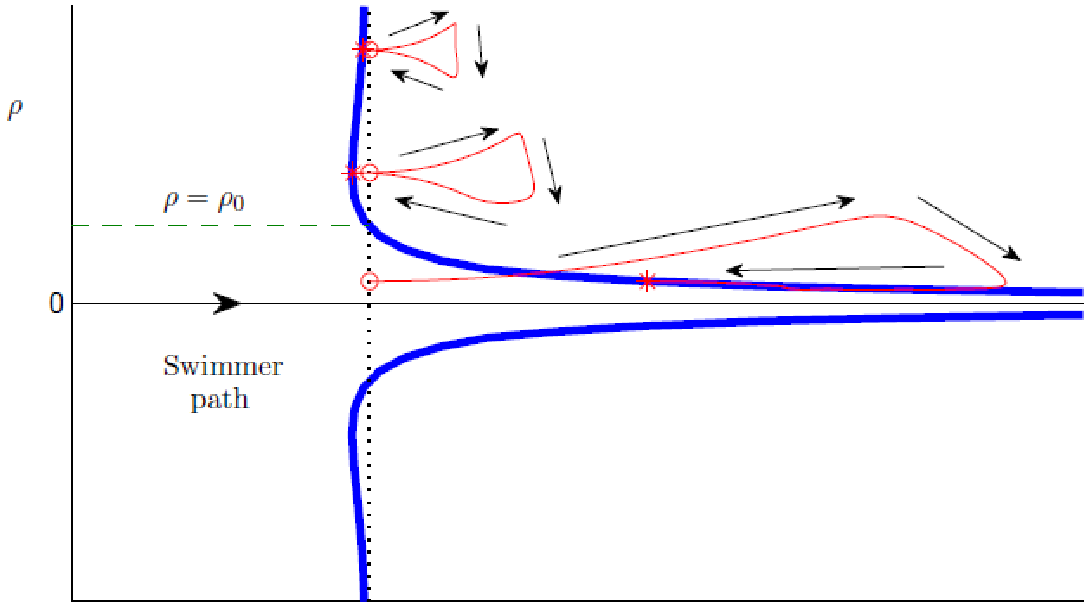
Entrainment



Far field entrainment:

$$\Delta = -C_1 \frac{\kappa^2}{V^2} \frac{1}{\rho^3} + C_2 \frac{\kappa Q_\perp}{V^2} \frac{1}{\rho^4} + O(\rho^{-5})$$

Darwin drift



Darwin
Benjamin
Eames
Belcher
Hunt
Gobby
Dalziel
Leshansky
Pismen

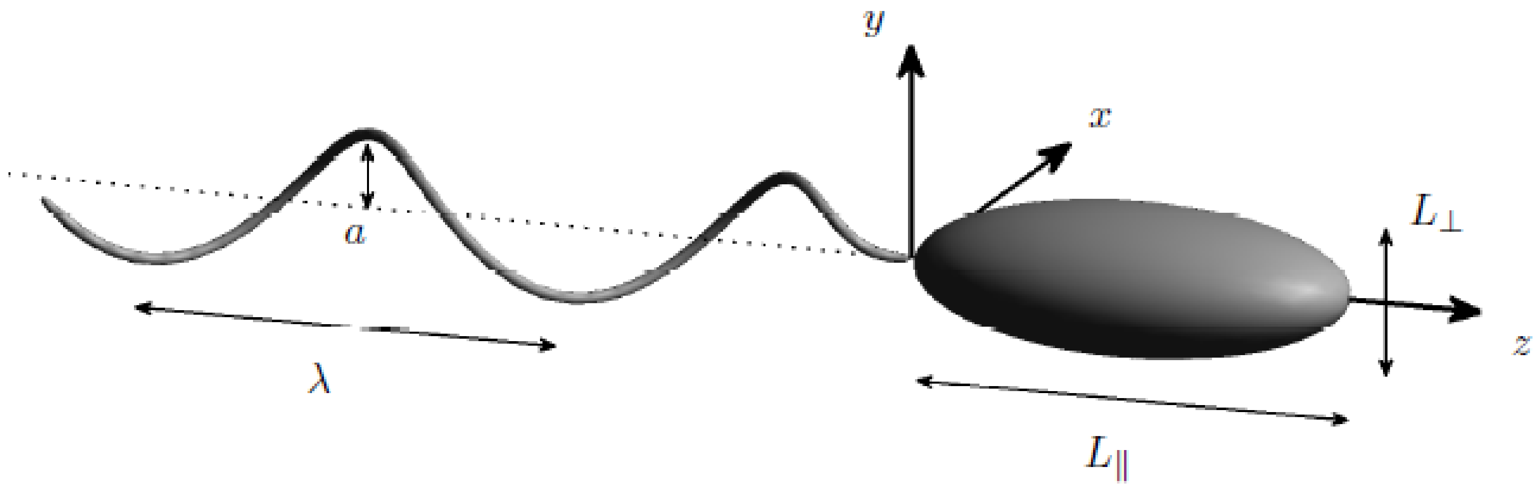
Total fluid volume moved by swimmer

Darwin drift:

$$v_D = \frac{4\pi Q_\perp}{V} - v_*,$$

$$v_* = v_s + v_{wake}$$

$$Q_\perp = -\frac{1}{2}\int_S f_z \rho^2 dS$$

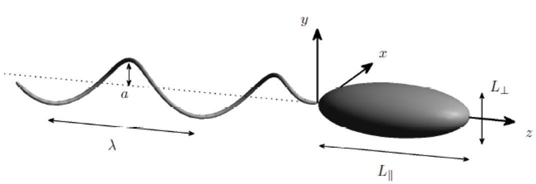


Base parameters: $L_\|/L_\perp = 2$, $\lambda = 2$, $L = 10$, $a = \lambda/2\pi$.

Comparison of analytic and numerical results for the Darwin drift

TABLE 1. Base parameters: $L_{\|}/L_{\perp} = 2$, $\lambda = 2$, $L = 10$, $a = \lambda/2\pi$.

Shape	$Q_{\perp}/V$	v_D (from equation)	v_D (from simulations)
Base	-0.15	-6.10	-6.11
$L_{\|}/L_{\perp} = 0.5$	-0.68	-12.74	-12.78
$L_{\|}/L_{\perp} = 3.5$	-0.08	-5.24	-5.33
$L = 5$	-0.17	-6.36	-6.30
$L = 15$	-0.14	-5.98	-6.03
$\lambda = 0.5$	-0.20	-6.76	-6.78
$\lambda = 3.5$	-0.04	-4.71	-4.68
$\lambda = 8, L = 20$	0.58	3.04	3.07



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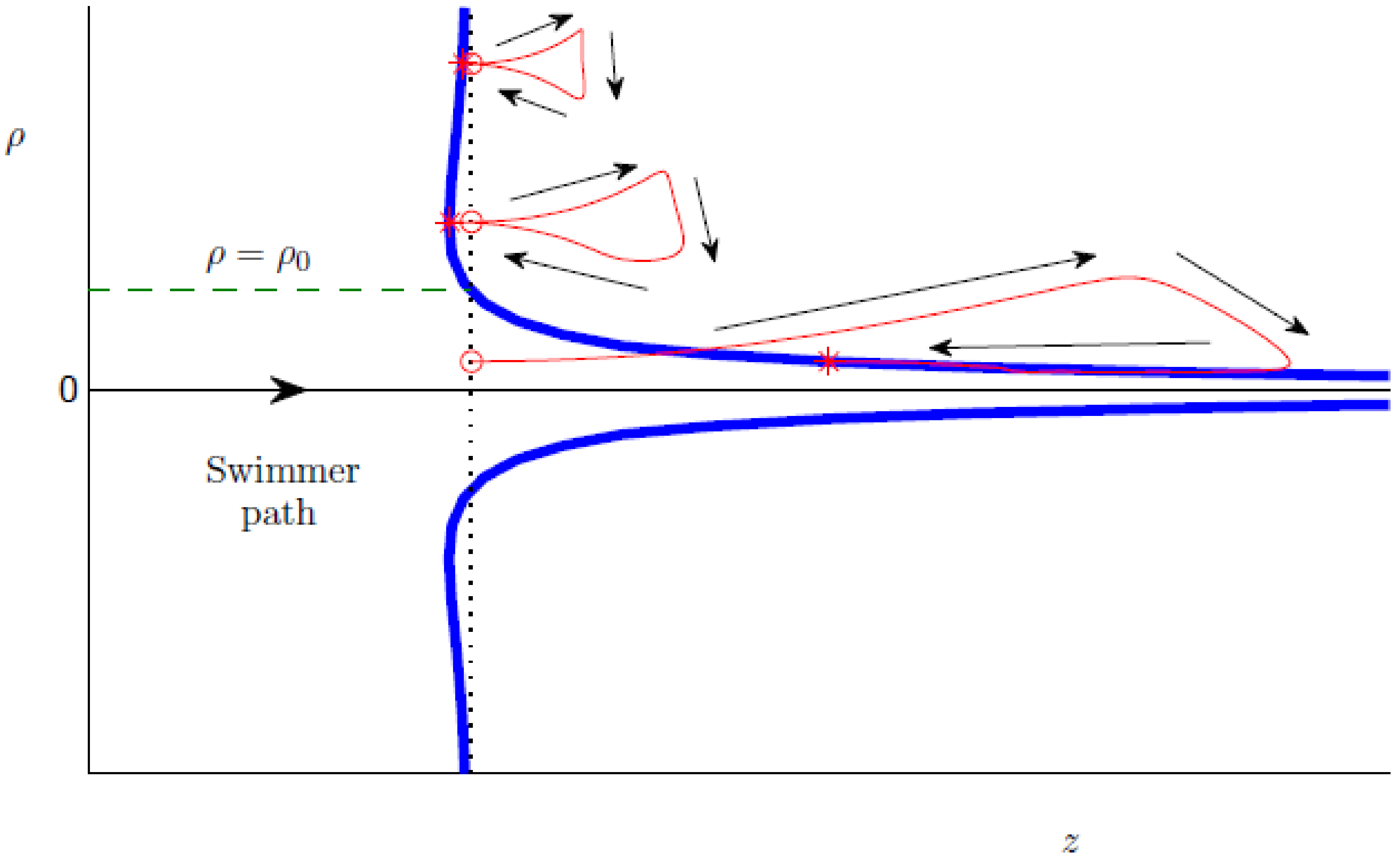

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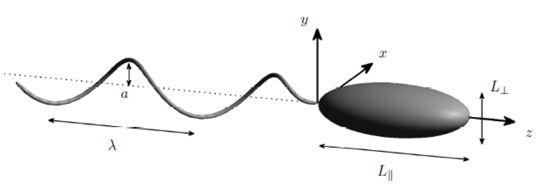

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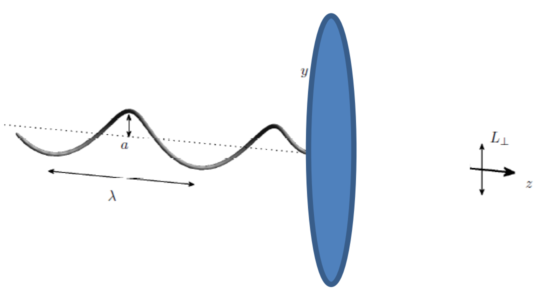

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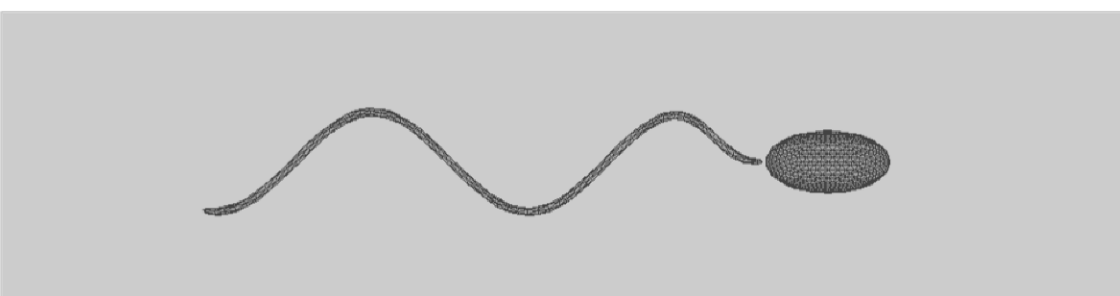
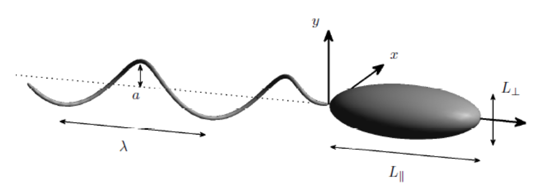
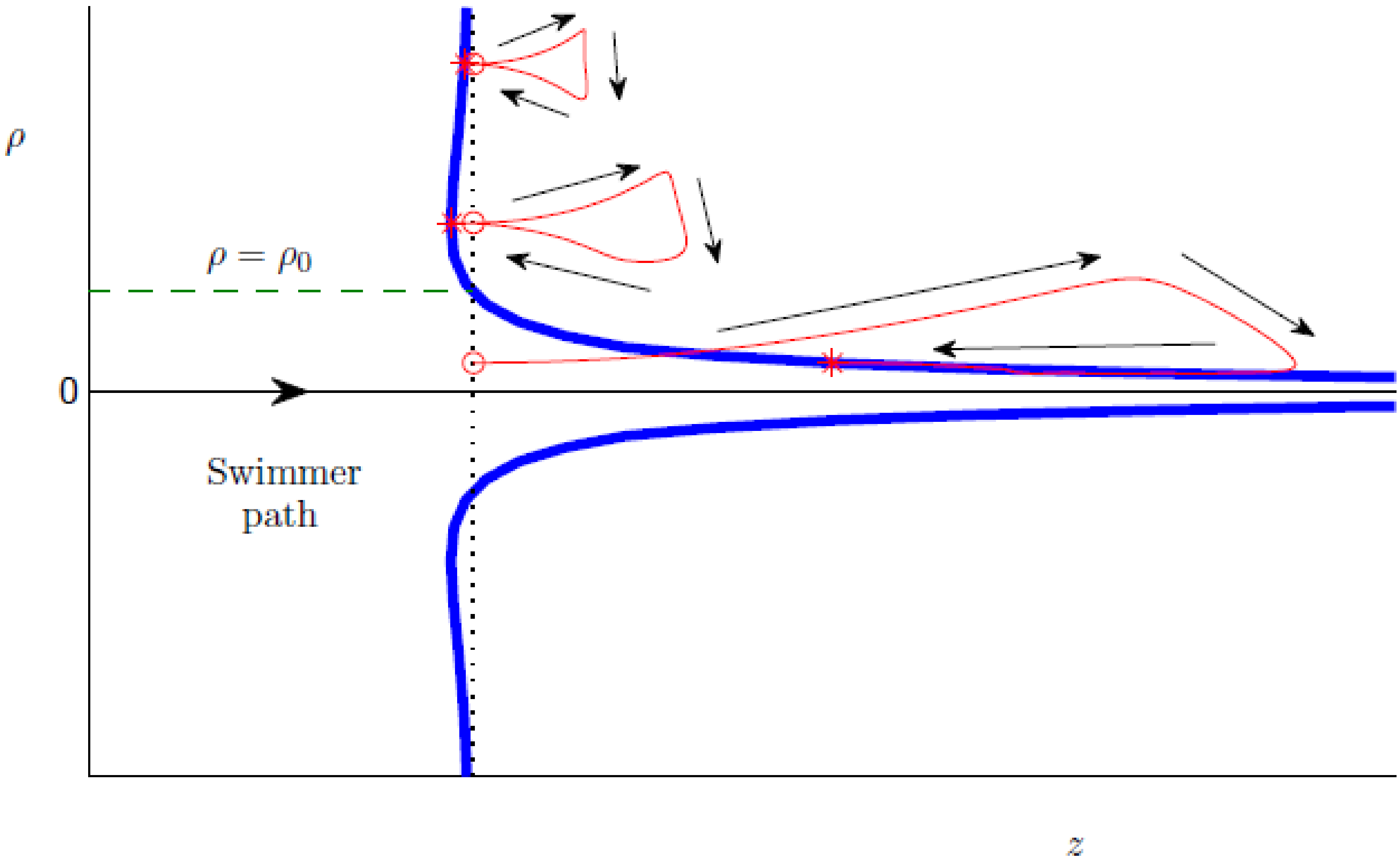




Entrainment



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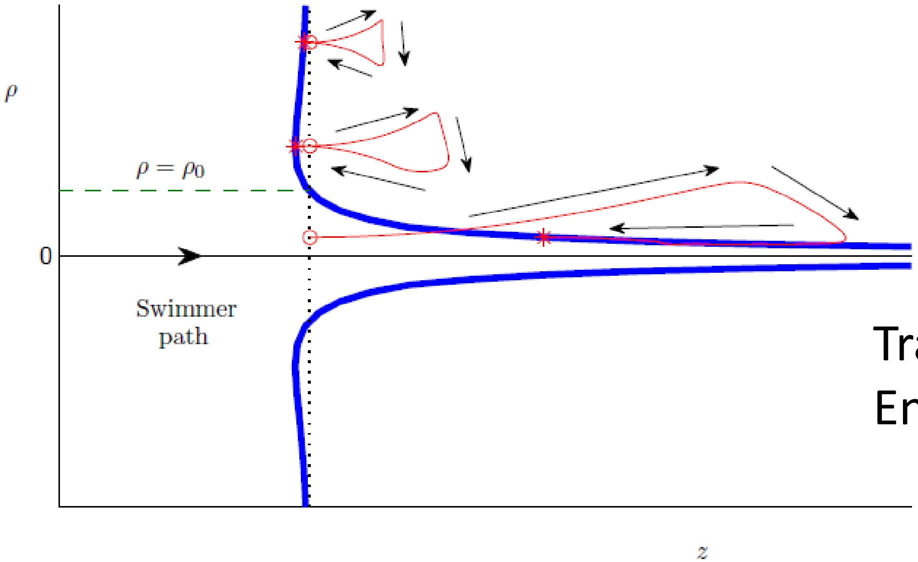


Tracer moves in loops far from swimmer

Entrainment close to the swimmer

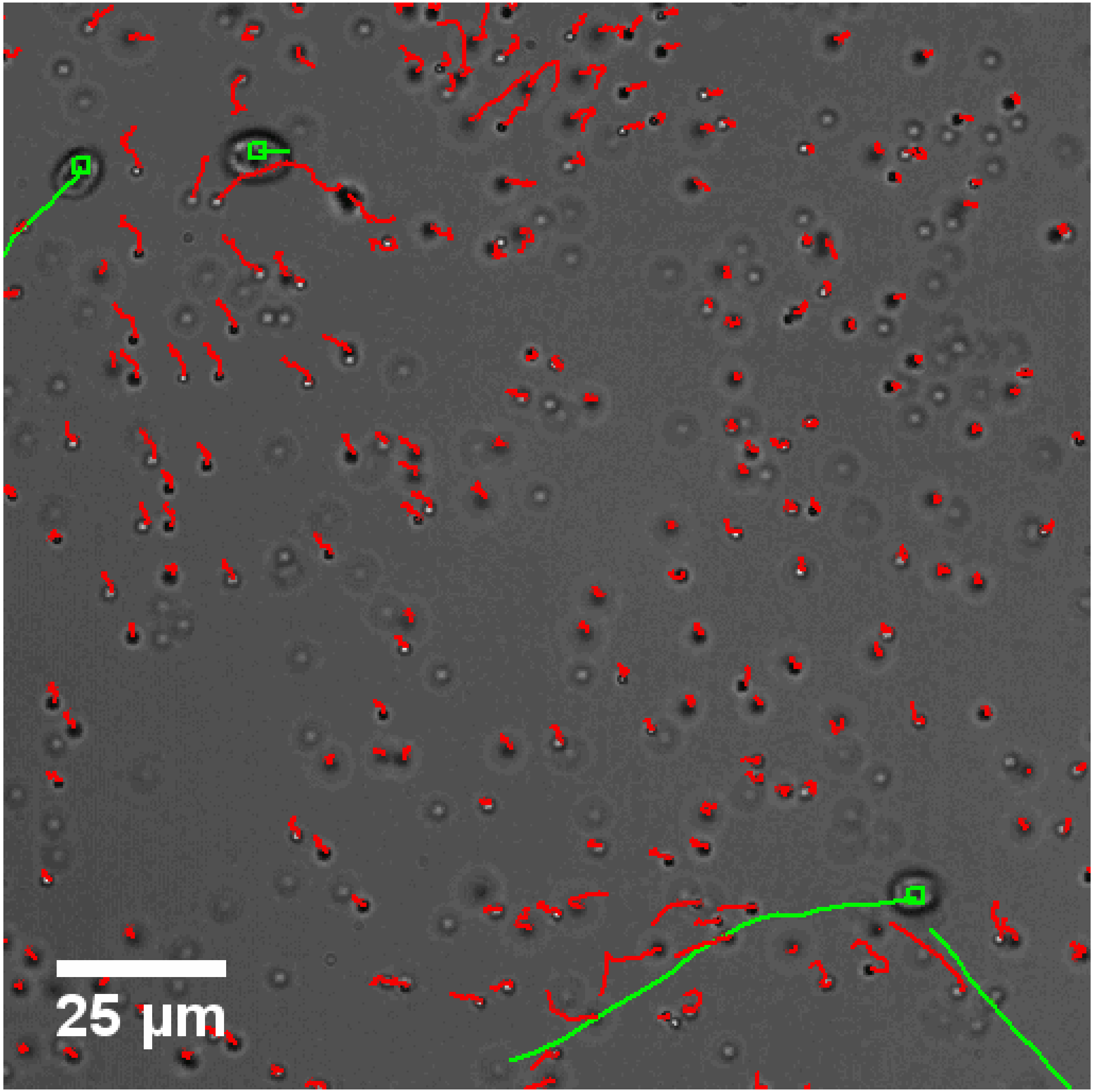
Volume of fluid moved by the swimmer:

Darwin drift:

$$v_D = \frac{4\pi Q_\perp}{V} - v_*,$$

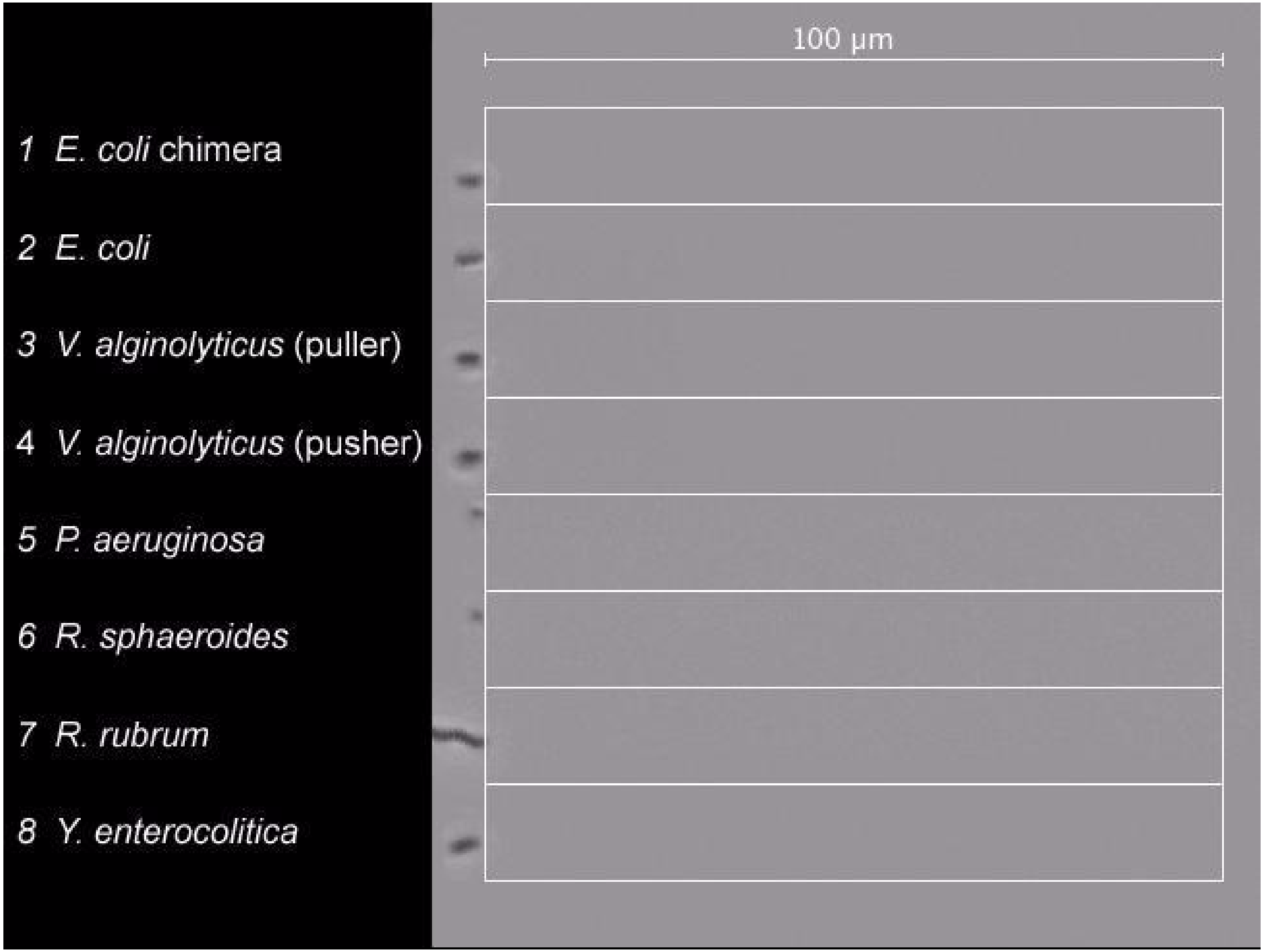
$$v_* = v_s + v_{wake}$$

$$D_{entr} \approx \frac{1}{6} n\, V\, a\, \frac{4\pi}{3} a^3$$



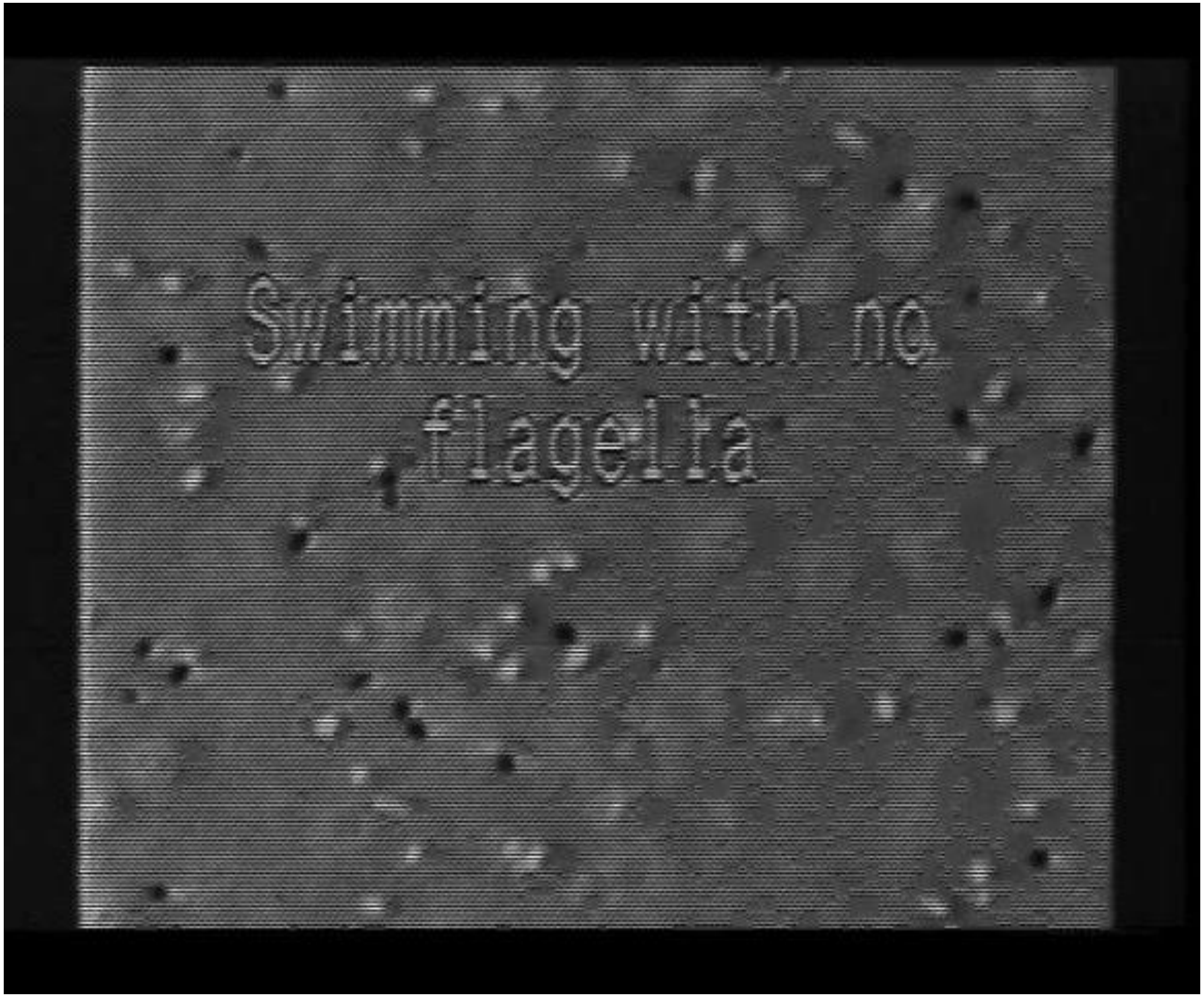
Guasto website

!!!!! Bacterial Olympics: 100 micrometres !!!!



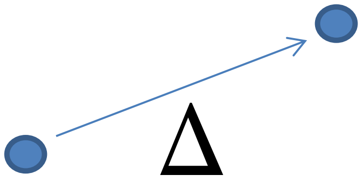
Judith Armitage,
Oxford biophysics

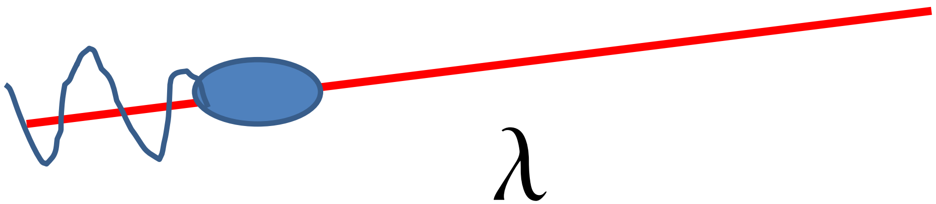
Random reorientations



http://mcb.harvard.
edu/Faculty/Berg.html

Random reorientations





$$\langle \Delta^2 \rangle = (Vol\, n_s)\, \frac{1}{Vol} \int d^d\mathbf{r_i}\, \Delta^2(\mathbf{r_i})$$

$$D = \langle \Delta^2 \rangle / \,(2\, d\, t) = \frac{1}{2d} \frac{n\, V}{\lambda} \int d^d\mathbf{r_i}\, \Delta^2(\mathbf{r_i})$$

Lin, Thiffeault, Childress JFM (2011)

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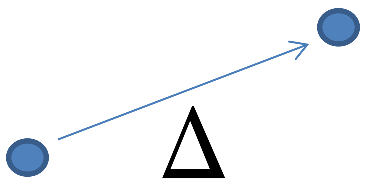
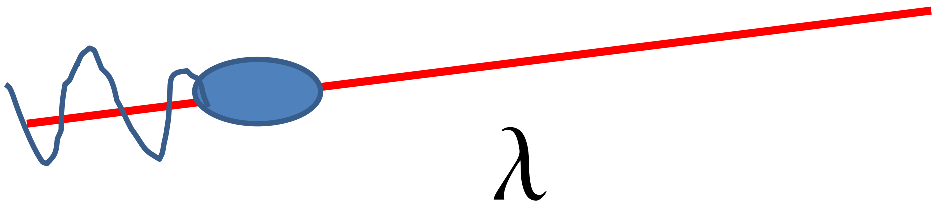
infinite swimmer path

Random reorientations





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Lin, Thiffeault, Childress JFM (2011)

Random reorientations

$$D_{rr} = n\lambda^d \cdot \frac{V}{\lambda} \cdot \left(\frac{\kappa_m}{V\lambda}\right)^2 \cdot I_{d,m}\left(\frac{a}{\lambda}\right)$$

number of swimmers within λ of tracer

number of independent path segments

displacement per path segment

term from lower limit of rescaled integral

Random reorientations

$d > d_*(m) = 2(m-1)$

Regular (eg dipolar swimmer, m=2, d=3)

$$D_{rr} \sim \tilde{\kappa}_m^2 \, n \, V \, \lambda^{d-2m+1} \, a^{2m}$$

- Independent of swimmer run length for dipolar swimmers in 3D
  Lin, Thiffeault, Childress JFM (2011)
- Distribution of tracer run lengths converges to a Gaussian

Random reorientations

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$d < d_*(m) = 2(m-1)$

Singular (eg quadrupolar swimmer, m=3, d=3)

$$D_{rr} \sim \tilde{\kappa}_m^2 \, n \, V \, \lambda^{-1} \, a^{d+2},$$

- Distribution of tracer run lengths is a truncated Levy distribution

Entrainment	Random reorientations

Regular (eg dipolar swimmer, m=2, d=3)

$$\frac{D_{rr}}{D_{entr}} \approx \frac{\tilde{\kappa}^2 \; n \; V \;\; a^4}{n \; V \;\; a^4} \sim \tilde{\kappa}^2$$

Singular (eg quadrupolar swimmer, m=3, d=3)

$$\frac{D_{rr}}{D_{entr}} \approx \frac{\tilde{\kappa}^2 \; n \; V \;\; \lambda^{-1} \;\; a^5}{n \; V \;\; a^4} \sim \tilde{\kappa}^2 \frac{a}{\lambda}$$

Random reorientations

For 3D dipolar swimmers

$$D_{rr} = \frac{4\pi}{3}\kappa_m^2 n\, V\, a^4$$

independent of swimmer run length.

In the regular case (eg 3D dipolar swimmers)
diffusion due to random reorientations is dominant
distribution of lengths of tracer paths Gaussian

In the singular case (eg 3D quadrupolar swimmers)
diffusion due to entrainment is dominant:
tracer paths lengths form a truncated Levy distribution

Open questions

Correlated re-orientations

Denser swimmer suspensions

Surfaces and confinement

Links between stirring, swimmer structure and stroke and biological fitness

Dmitri (Mitya) Pushkin
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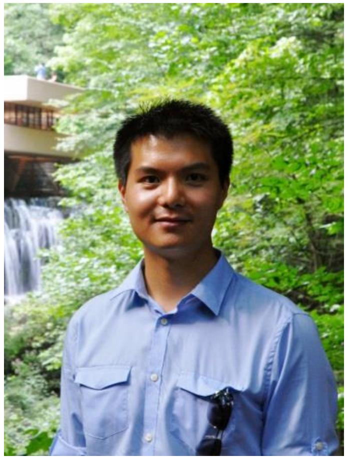

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