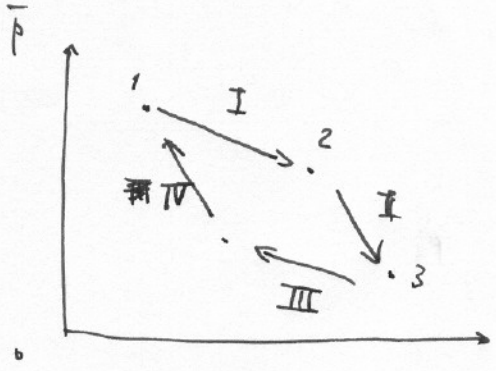
※ An example of quasi-static process



ideal gas in volume V.

(N molecules)

$$\left\{\begin{array}{l} \bar{p}V = NkT \\ \bar{E} = \frac{3}{2}NkT \end{array}\right.$$

($\bar{E}$ will be discussed in equipartition theorem)

process I : isothermal expansion

$V_1 \to V_2$ at constant temperature T_h

isotherm $\Rightarrow$ $\frac{V_2}{V_1} = \frac{P_1}{P_2}$,

$$\Delta\bar{E}_{12} = E_2 - E_1 = \Delta Q_{12} - \Delta W_{12} = 0$$

$$\Rightarrow \Delta Q_{12} = \Delta W_{12} = \int \bar{p}\,dV = \int_{V_1}^{V_2} \frac{NkT_h}{V}\,dV = NkT_h \ln(V_2/V_1)$$

process II : Adiabatic expansion

$V_2 \to V_3$., $T_h \to T_c$

adiabatic $\Rightarrow$ $đQ = 0 = d\bar{E} + đW$

$$\underbrace{d\bar{E} = \frac{3}{2}k\cancel{N}dT \;,\; đW = pdV}_{} = \frac{NkT}{V}dV$$

$$\Rightarrow \frac{3}{2}dT = \frac{T}{V}dV \Rightarrow \left(\frac{V_3}{V_2}\right) = \left(\frac{T_h}{T_c}\right)^{3/2}$$

process III : isothermal . $(V_3 \to V_4)$ at temperature T_c

$$V_4/V_3 = P_3/P_4 . \quad \Delta Q_{34} = +\Delta W_{34} = NkT_c \ln\frac{V_4}{V_3}$$

process IV : adiabatic $(V_4 \to V_1)$, $(T_c \to T_h)$

$$\left(\frac{V_1}{V_4}\right) = \left(\frac{T_c}{T_h}\right)^{3/2} .$$

Using the above relations between $(V_1 . V_2 . V_3 . V_4 , T_c . T_h)$,

One can show $\frac{\Delta Q_{12}}{T_h} + \frac{\Delta Q_{34}}{T_c} = 0 \Rightarrow \boxed{\oint \frac{đQ}{T} = 0 .}$

$\Leftrightarrow \frac{đQ}{T}$ ~ exact differential in quasi-static processes.