(5) Connection to Statistical Mechanics

ensemble $\longrightarrow$ canonical ensemble
grand —.

probability $\longrightarrow$ $P_r = \frac{1}{Z} e^{-\beta E_r}$

statistics $\longrightarrow$ eg. energy
$\bar{E} = \sum P_r E_r$, $\overline{\Delta E^2}$, ...

generating fn $\longrightarrow$ partition function.

(ex) $G(\lambda) = \sum_n W_N(n) e^{i n \lambda}$ $\qquad Z = \sum_r e^{-\beta E_r}$

Multinormial distribution (skip at this moment. reappear in course 3 or 4)

n_j : # of particles occupying the jth level.
$j \in [1 . r]$

$$\sum_j n_j = N$$

$$W_N(\{n_j\}) = \frac{N!}{n_1! \cdots n_r!} p_1^{n_1} \cdots p_r^{n_r}$$

$$\sum_{j=1}^{r} p_j = 1$$

$$\underbrace{k \ln W_N}_{\propto \text{ free energy}} = \underbrace{\sum n_j \log p_j}_{\propto \text{ energy}} + \underbrace{[\log N! - \sum \log n_j!]}_{\propto \text{ entropy}}$$

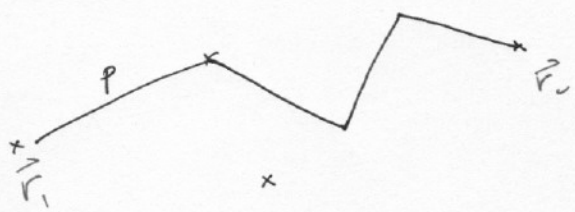
Minimize free energy $\Rightarrow$ $\tilde{n}_j = p_j N = \langle n_j \rangle$ (use stirly formula)

$\longleftrightarrow$ ~~ergodic hypothesis~~ : system goes to
the most probable
configuration
(equilibrium)

(6) Others.

· quantum effect in diffusion : Weak localization

$P(r_1 \to r_2)$



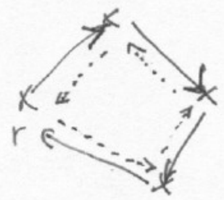
$$A_p = |A_p| \exp i S_p$$

$$S_p = \int_p \left(k - \frac{e}{\hbar} A(r)\right) dr - E dt$$

classical : $P(r_1 \to r_2) = \sum_p A_p A_p^*$

quantum : $P(r_1 \to r_2) = \left(\sum_p A_p\right)\left(\sum_{p'} A_{p'}^*\right)$

$$= P(\bar{r}_1 - \bar{r}_2)_{class} + \sum_{p \neq p'} A_p A_{p'}^*$$



$$P(r \to r)_{quantum} = 2 \langle P(r, r) \rangle_{classical}$$

(interference between a closed path and its time-reversal path.)

· Random matrix : Ensemble of matrix. ~ random, but keeping the symmetries of the system

(e.g. Hermitian for random Hamiltonian matrix, time-reversal symmetry, etc..)

· Counting statistics $\langle I \rangle$, $\langle \delta I \delta I \rangle$, ...

(photon counting statistics
electron counting statistics