

The abstract is as follows:

In this lecture series we will introduce Behrend constructible function on a scheme or a Deligne-Mumford stack. The Donaldson-Thomas type counting invariants will be defined by Behrend function on the Donaldson-Thomas moduli space. If the Donaldson-Thomas moduli space locally is the critical locus of a holomorphic function on a higher dimensional complex manifold, then the value of Behrend function at a point is given by the Euler characteristic of Milnor fibre of the holomorphic function at that point. We will explain how the Donaldson-Thomas type moduli space locally can be given by the critical locus of a holomorphic function. As applications, we will talk about the counting invariants in the derived category of quiver representations and cover the GW/DT/PT/NCDT-correspondence.

The outline is as follows:

- 1) Behrend functions and counting invariants.
- 2) Cyclic L-infinity algebras and transfer theorem.
- 3) Joyce-Song formulas for Behrend functions.
- 4) Applications to counting invariants for quiver representations.

References are:

- 1) K. Behrend, Donaldson-Thomas invariants via Microlocal geometry, arXiv:math/0507523.
- 2) M. Kontsevich and Y. Soibelman, Homological mirror symmetry and torus fibrations, arXiv:math/0011041.
- 3) M. Kontsevich and Y. Soibelman, Stability structures, motivic Donaldson-Thomas invariants and cluster transformations, arXiv:0811.2435.
- 4) J. Milnor, Singular points of complex hypersurfaces, Annals of Mathematics Studies 61 (Princeton University Press, 1968).
- 5) D. Joyce and Y. Song, A theory of generalized Donaldson-Thomas invariants, arXiv:0810.5645.
- 6) R. Pandharipande and R. thomas, Counting invariants via stable pairs in the derived category, arXiv:0707.2348.