Large $\tan\beta$ SUSY QCD corrections to $B \to X_s\gamma$

Youichi Yamada (Tohoku Univ.)

F. Borzumati, C. Greub, YY,
hep-ph/0305063; to appear soon

- $b \to (s\gamma, sg)$ decay in the MSSM:
  $H^{\pm}$ contribution

- Large $\mathcal{O}(\alpha_s \tan\beta)$ corrections
  by squark-gluino subloops

- Comparison of the "nondecoupling approx." by effective 2HD lagrangian and exact two-loop calculation

Inclusive radiative decay $B \to X_s\gamma$

Branching ratio BR($B \to X_s\gamma$):
well described by short-distant parton decays
$b \to s\gamma$ (and $b \to sg$)

Important process to probe the "beyond SM" physics

* small uncertainty from hadronic corr.

* loop-generated in SM:
  $\Rightarrow$ new physics contributes at the same order

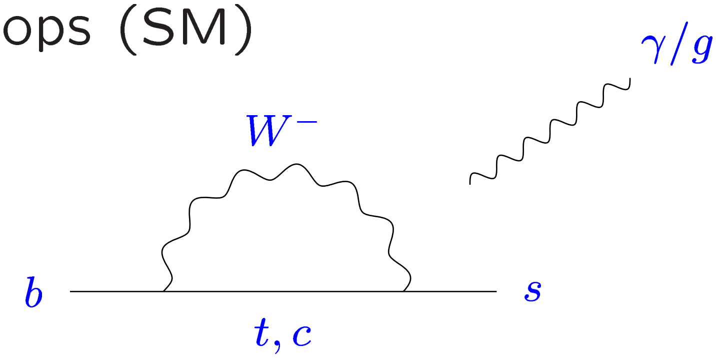
* BR(exp)$\simeq$ BR(SM, NLO)
  $\to$ constraints on new physics

We analyze the decays $b \to (s\gamma, sg)$ in the MSSM (minimal supersymmetric standard model) with large $\tan\beta$ (ratio of VEVs of two Higgs doublets).

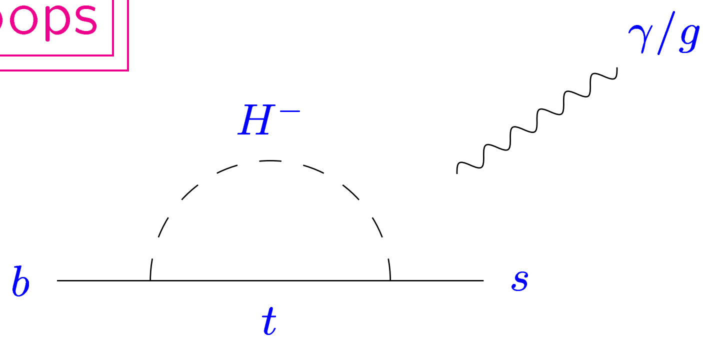
$b \to (s\gamma, sg)$ in the MSSM

one-loop contributions:

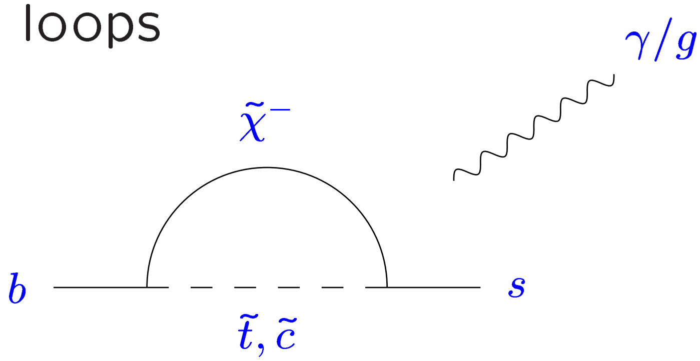
$W^\pm$ loops (SM)



$H^\pm$ loops



$(\tilde{\chi}^\pm, \tilde{t})$ loops

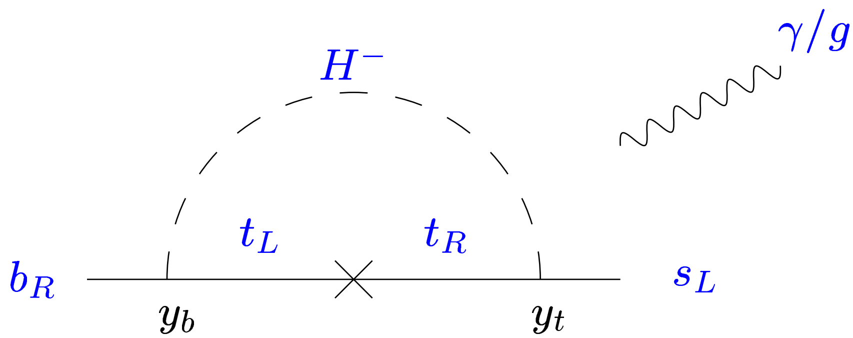


$\mathcal{O}(h_q^2)$: comparable to $W^\pm$ loops

$(\tilde{g}, \tilde{b})$ loops: usually smaller than others

H^+ contribution to $b \to s\gamma$ for large $\tan\beta$

Dominant one-loop diagram



$y_b = \frac{g_2 m_b}{\sqrt{2} m_W} \tan\beta$, $y_t = \frac{g_2 m_t}{\sqrt{2} m_W} \cot\beta$
$\tan\beta \equiv \langle H_U \rangle / \langle H_D \rangle$
y_b (y_t) is enhanced (suppressed) for $\tan\beta \gg 1$

However, the H^+ couplings receive $\mathcal{O}(\alpha_s \tan\beta)$ corrections from squark-gluino loops, which can be comparable to the one-loop contributions.

$\mathcal{O}(\alpha_s \tan\beta)$ corrections to H^+-quark couplings

(1) Correction from counterterm δm_b

Hempfling , Hall et al., Carena et al.

At tree-level, $d_{iR} = (d, s, b)_R$ couple to only H_D, one of two Higgs doublets (constraint by SUSY)

$$\mathcal{L}(\text{int}) \supset -h_b \bar{b}_R q_L H_D$$

$m_b = h_b v_D/\sqrt{2}$: suppressed by $\tan\beta = v_U/v_D \gg 1$

Squark-gluino loops induce the SUSY-breaking effective coupling $h_b \Delta_b \bar{b}_R q_L H_U$.

$$\begin{aligned} m_b(\text{running, SM}) &= \frac{h_b \bar{v}}{\sqrt{2}} \cos\beta [1 + \Delta_b \tan\beta] \\ &= m_b(\text{running, MSSM}) + \delta m_b \\ &\Downarrow \end{aligned}$$

$$\begin{aligned} y_b(H^+ \bar{t}_L b_R)^{\text{eff}} &= V_{tb} h_b \sin\beta (1 - \Delta_b \cot\beta) \\ &\to V_{tb} \frac{\sqrt{2} m_b(\text{SM})}{\bar{v}} \tan\beta \frac{1}{1 + \Delta_b \tan\beta} \end{aligned}$$

$|\Delta_b \tan\beta| \sim 1$ is possible despite $\Delta_b = \mathcal{O}(\alpha_s)$:

very large corr. for $\tan\beta \gg 1$.

δm_b also contribute to $\tilde{\chi} q \tilde{q}$ couplings.

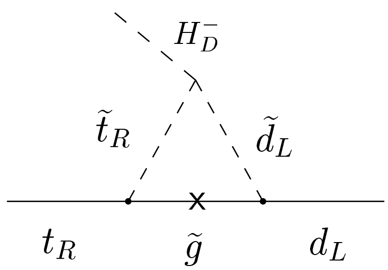
(2) 1PI correction to $H^- \bar{s}_L t_R$ coupling:

Carena et al., Babu-Kolda, D'Ambrosio et al., . . .

$$H^+ = \sin\beta H_D^+ + \cos\beta H_U^+ \sim H_D^+ (\text{for } \tan\beta \gg 1)$$

At tree-level, $u_{iR} = (u, c, t)_R$ only couple to H_U.
$H^- \bar{s}_L t_R$ coupling $V_{ts} h_t \cos\beta = V_{ts} \frac{\sqrt{2}}{\bar{v}} m_t \cot\beta$ is suppressed.

Squark-gluino loops induce the effective $\bar{t}_R q_L H_D$ couplings.



$\sim h_t \Delta_t$

$$\begin{aligned} y_t (H^+ \bar{s}_L t_R)^{\text{eff}} &= V_{ts} h_t \cos\beta (1 - \Delta_t \tan\beta) \\ &\to V_{ts} \frac{\sqrt{2} m_t}{\bar{v}} \cot\beta (1 - \Delta_t \tan\beta) \end{aligned}$$

very large corr. for $\tan\beta \gg 1$, relative to $\tan\beta$-suppressed tree-level coupling

$\Delta_{b,t} = \mathcal{O}(\alpha_s \mu m_{\tilde{g}} / M_{\tilde{q}}^2) = \mathcal{O}(\alpha_s M_{\text{SUSY}}^0)$:

Non-decoupling in large M_{SUSY} limit

Two-loop $\mathcal{O}(\alpha_s \tan\beta)$ corrections to the H^+-mediated $b \to (s\gamma, sg)$ decays

(1) δm_b corr. to $y_b(\bar{t}_L H^+ b_R)$

(2) 1PI vertex corr. to $y_t(\bar{s}_L H^- t_R)$

"Nondecoupling" approximation:

(Degrassi et al., Carena et al., D'Ambrosio et al., Buras et al., . . .)

(a) integrate out squarks and gluino to obtain effective 2HD lagrangian,
(b) calculate one-loop (t, H^+) diagrams in this effective theory

⋆ Theoretically justified approximation when the momenta of $(t, H^\pm)$ are sufficiently smaller than M_{SUSY}.

⋆ Analytically simple results

Validity of this approximation

(1) δm_b corr.

Given at momentum m_b: no problem

resummation of $(\alpha_s \tan\beta)^n$ corr. is possible

by using effective lagrangian

(2) 1PI vertex corr.

Loop momentum $\sim m_{H^+}$ may contribute.

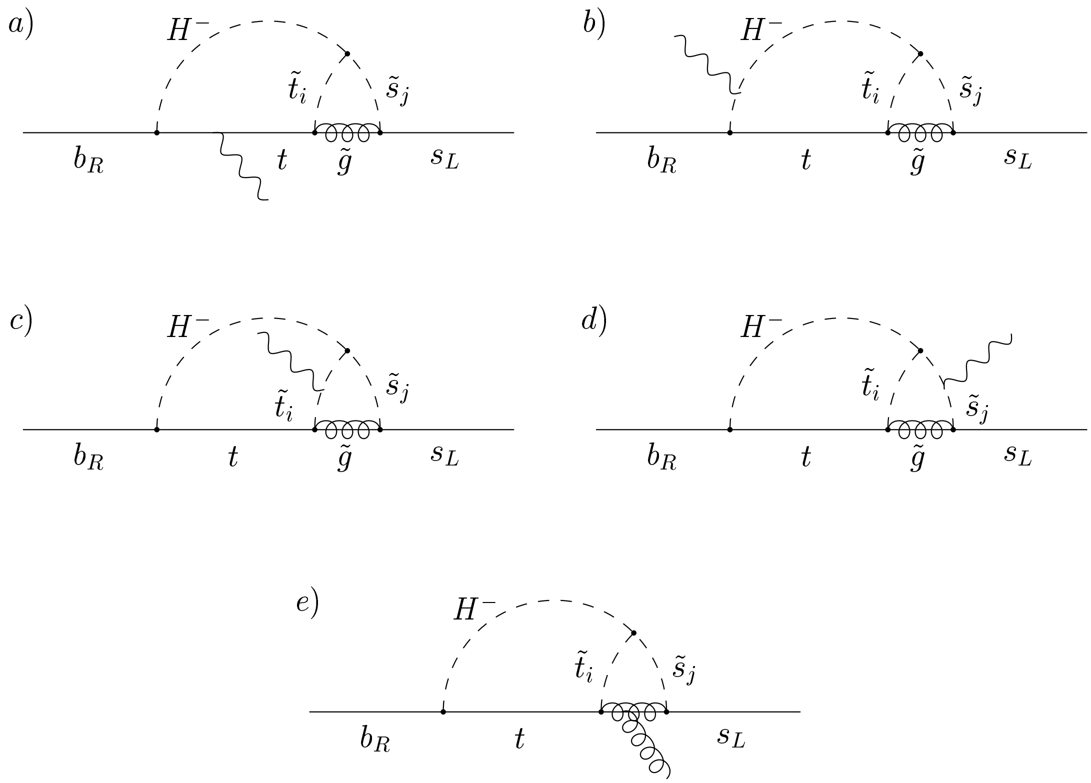
not theoretically justified if $m_{H^+} \geq M_{\mathrm{SUSY}}$

Significant deviation from the approximation is expected if $m_{H+} \geq M_{\mathrm{SUSY}}$

$$\Downarrow$$

We perform an exact evaluation of the two-loop diagrams, to study the deviation.

$\mathcal{O}(\alpha_s \tan\beta)$ corrections to the $H^+\bar{s}t$ coupling: Full two-loop diagrams



Contributions absent in nondecoupling approx.:

- finite momenta $k_{t,H,s}$ for SUSY subloop
- γ/g emission from SUSY particles (fig. c-e)
- chirality flip on $\tilde{t}_i$, instead of on t
  (effective $H^-\bar{s}_L t_L$ couplings)

Effective hamiltonian for $b \to s\gamma$ at $\mu \sim \mu_{weak}$

$$
\begin{aligned}
H_{\text{eff}} &\supset -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(C_7(\mu)\mathcal{O}_7(\mu) + C_8(\mu)\mathcal{O}_8(\mu) \right) \\
\mathcal{O}_7(\mu) &= \frac{e}{16\pi^2} m_b(\mu) \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu} (\text{photon}) \\
\mathcal{O}_8(\mu) &= \frac{g_s}{16\pi^2} m_b(\mu) \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu} (\text{gluon})
\end{aligned}
$$

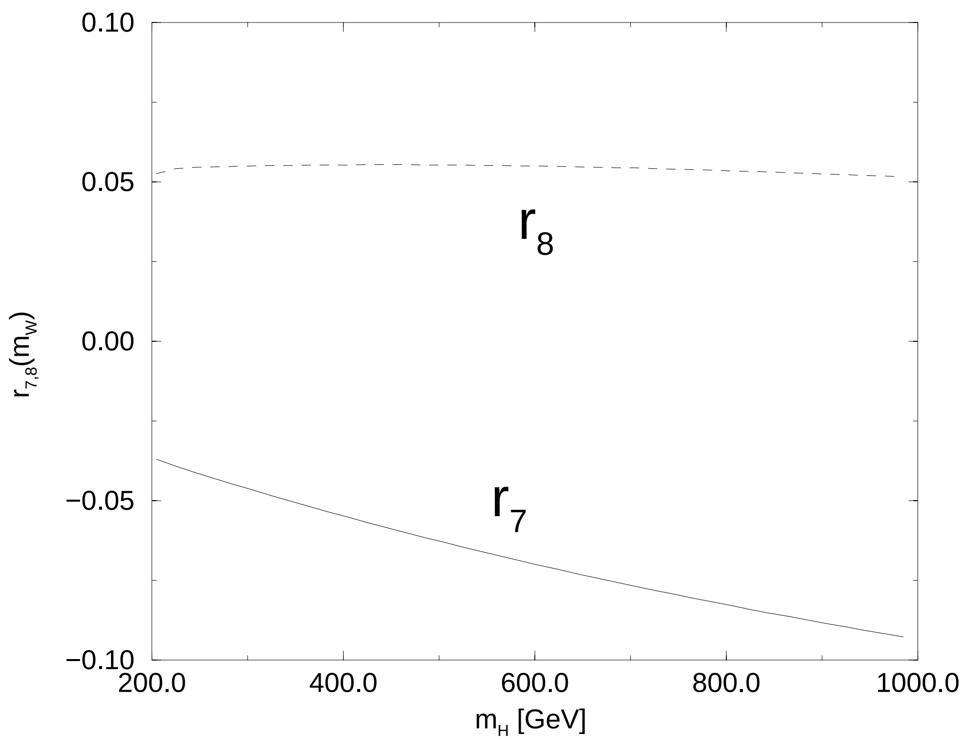
We calculate the $H^{\pm}$ loop contributions to $C_{7,8}(\mu_W)$ and evaluate the "goodness" of the nondecoupling approximation.

$$
r_i(\mu_W) \equiv \frac{C_i^H(\mu_W)|_{\text{approx}} - C_i^H(\mu_W)|_{\text{exact}}}{C_i^H(\mu_W)|_{\text{exact}}} \quad (i = 7, 8)
$$

(δm_b contributions factored out)

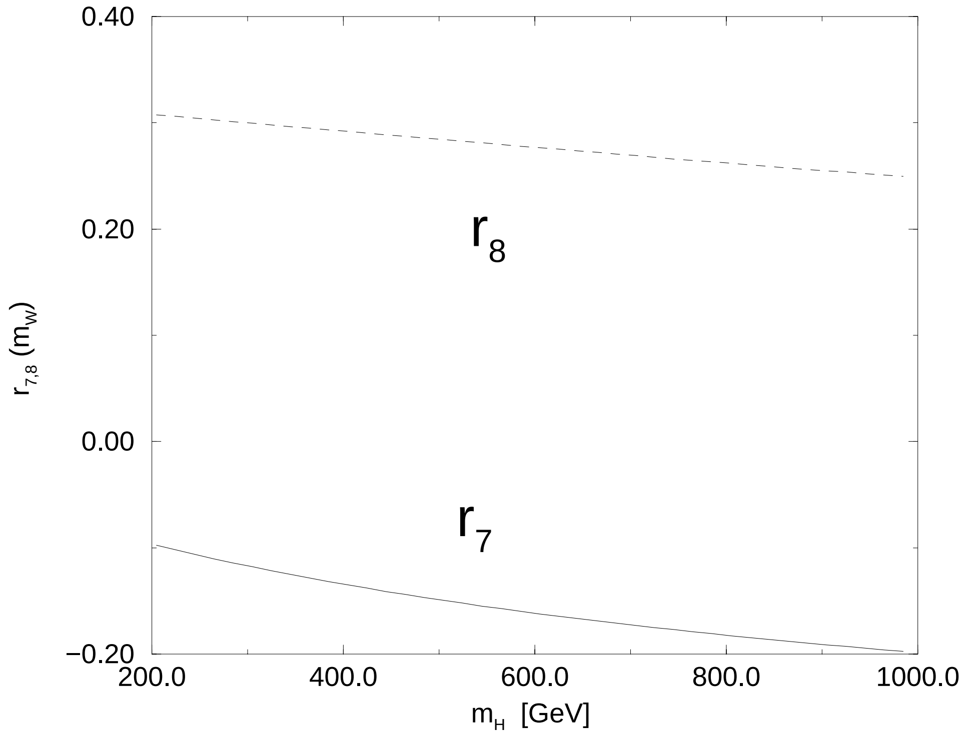
Goodness of the nondecoupling approx.

(1) Heavy SUSY $[(m_{\tilde{s}_L}, m_{\tilde{t}_1}, m_{\tilde{t}_2}) = (700, 500, 450)$ GeV, $\cos\theta_t = 0.8$, $\tan\beta = 30$, $m_{\tilde{g}} = 600$ GeV, $\mu = 550$ GeV]



Very small deviation from exact results:
Deviations do not so increase for $m_H > M_{\text{SUSY}}$

(2) Light SUSY ($M_{\rm SUSY} \sim m_{weak}$) $[(m_{\tilde{s}_L}, m_{\tilde{t}_1}, m_{\tilde{t}_2}) = (350, 400, 320)$ GeV, $\cos\theta_t = 0.8$, $\tan\beta = 30$, $m_{\tilde{g}} = 300$ GeV, $\mu = 450$ GeV]

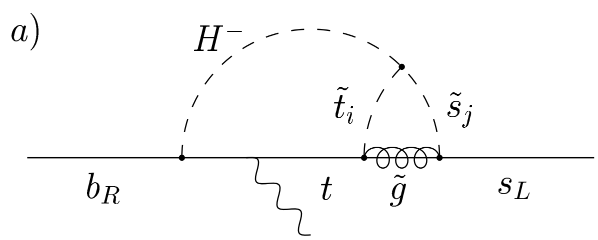


Larger deviation: $\mathcal{O}(m^2_{\rm weak}/M^2_{\rm SUSY})$
But again no significant increase for $m_H > M_{\rm SUSY}$

Why the nondecoupling approximation works so well?

(no $\mathcal{O}(m_H^2/M_{\text{SUSY}}^2)$ deviation from exact result)

$*$ γ/g emission from top:



$$= \int \frac{d^4k}{(2\pi)^4} \frac{m_t^2 k^2}{\left[k^2 - m_t^2\right]^3 \left[k^2 - M_H^2\right]} Y_{tR}(k^2)$$

$Y_{tR}(k^2)$: form factor for the effective $H^- \bar{s}_L t_R$ vertex

$$\sim \begin{cases} Y_{tR}(\text{NonDec}) + \mathcal{O}(m_t^2, k^2/M_{\text{SUSY}}^2) & (k \ll M_{\text{SUSY}}) \\ \mathcal{O}(M_{\text{SUSY}}^2 \ln k^2/k^2) & (k \gg M_{\text{SUSY}}) \end{cases}$$

nondecoupling approx.: replace $Y_{tR}(k^2)$ by $Y_{tR}(\text{NonDec})$

k-integration: dominated by the $k \sim \mathcal{O}(m_t)$ region.

(similar result for other diagrams)

$\Rightarrow$ the nondecoupling approximation works well, even if $m_H > M_{\text{SUSY}}$.

Improvement by Heavy Mass Expansion

What about the improvement of the nondecoupling approximation by, instead of full two-loop integrals, including higher-dimensional interactions into the effective 2HD lagrangian, order by order?

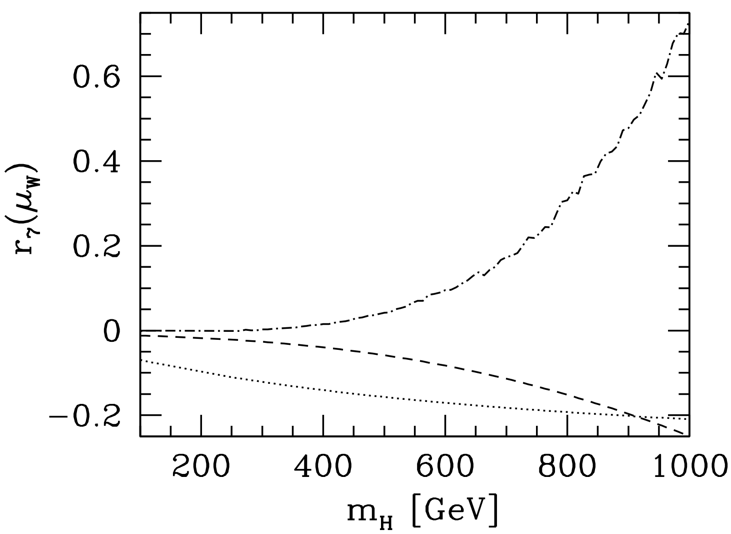
A systematic procedure for this:
Heavy Mass Expansion of the diagrams
in $(m^2_{\text{weak}}, m^2_H)/M^2_{\text{SUSY}}$

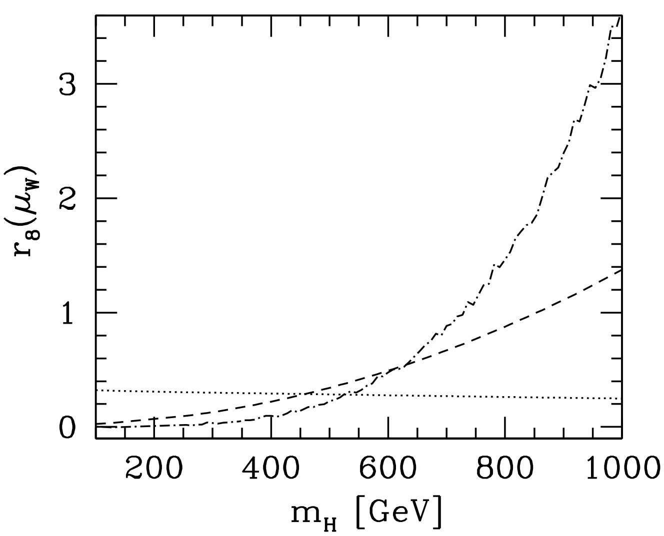
We compare
(1) Expansion to $1/M^0_{\text{SUSY}} \equiv$ nondecoupling approx.
(2) Expansion to $1/M^2_{\text{SUSY}}$
(3) Expansion to $1/M^4_{\text{SUSY}}$
to exact calculation

Result:
$m_H < M_{\text{SUSY}}$:
HME improves the approximation
$m_H > M_{\text{SUSY}}$:
HME only worsens the approximation

Goodness of HME to higher-order :

Light SUSY case [$M_{\mathsf{SUSY}} = 300 - 400$ GeV, $\tan\beta = 30$]





Dotted lines: Expansion to $1/M^0_{\mathsf{SUSY}}$ (= nondecoupling approx.)

Dashed lines: Expansion to $1/M^2_{\mathsf{SUSY}}$

Dot-dashed lines: Expansion to $1/M^4_{\mathsf{SUSY}}$

Conclusion

- In the MSSM with large $\tan\beta$, the $H^{\pm}$ loop contribution to the $b \to (s\gamma, sg)$ decay receives large $\mathcal{O}(\alpha_s \tan\beta)$ two-loop corrections.

- They have been calculated in the "non-decoupling" approximation using effective 2HD lagrangian. However, large deviation from this approx. was expected when $(m_{\text{weak}}, m_{H^+}) < M_{\text{SUSY}}$ is not satisfied.

- We performed the exact evaluation of the relevant two-loop diagrams for $b \to (s\gamma, sg)$. The deviation from the nondecouplings approximation was shown to be small, even for $m_{H^+} > M_{\text{SUSY}}$, unless $M_{\text{SUSY}} \sim m_{weak}$. This follows from the structure of the relevant Feynman integrals.