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$b \to s + \gamma$ in SUSY without R-parity

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Inclusive $B \to X + Y$

- What is inclusive about it ?
  - B denotes $B^{\pm}, B_d$ or B_s. X is an inclusive hadronic state containing no charmed particle, Y is built out of leptons or photons $(l^+l^-, \gamma\gamma, \nu\bar{\nu})$.

- Inclusive $B \to X_s + \gamma$ particularly interesting !
  - These are FCNCs and occur in SM only at loop level.
  - The rate is order $G_F^2 \alpha_{QED}$ while most other FCNCs involving leptons or photons it is of order $G_F^2 \alpha_{QED}^2$
  - It is calculable ! Non-perturbative 'pathogens' under control.
  - It is measurable
  - Sensitive to exotic virtualities in the loop - compliments direct search of SUSY particles

Present Status: The SM

- Dramatic QCD enhancement by a factor of two.

- Substantial dependence at the LL on the choice of scale μ (25%). [Buras $et.al$ 94]. Must calculate at Next to Leading Log.

- The most difficult NLL calculations completed very recently [Misiak, Gambino, 2001, Buras $et.al$ 2002]

  $$Br[\bar{B} \to X_s + \gamma(E_\gamma > 1.6GeV)]_{SM} = (3.57 \pm 0.30) \times 10^{-4}$$

  Within 1σ it matches the experimental world-average

  $$Br[\bar{B} \to X_s + \gamma(E_\gamma > 1.6GeV)]_{EXP} = (3.12 \pm 0.41) \times 10^{-4}$$

- Not much room for new Physics contributions.

The MSSM

- Extra contributions from sparticles in the loop

- Complex interplay of various contributions

- NLL 2-loop matching coefficients in Constrained MSSM
[Ciuchini (98); Bobeth, Misiak, Urban (99)]

- Constrained MSSM with large $\tan\beta$
[Carena $et.al$ (00); Degrasi $et.al$ (00)]

- Gluino Contributions and QCD corrections
[Borzumati $et.al$ (00)]

SUSY without R-parity

- MSSM with *imposed* B and L conservation a bit *ad hoc*

- SM + SUSY $\Longrightarrow$ GSSM
  or the Generic Supersymmetric Standard Model popularly known as the SUSY without R-parity.

- GSSM - though theoretically well motivated, plethora of parameters make Phenomenology formidable.

- Need for an Optimal Parametrization.

The Single VEV Parametrization (SVP)

$$W = \mu_\alpha \hat{H}_u \hat{L}_\alpha + h^u_{ik} \hat{Q}_i \hat{H}_u \hat{U}^C_k + \lambda'_{\alpha,jk} \hat{L}_\alpha \hat{Q}_j \hat{D}^C_k$$
$$\frac{1}{2}\lambda_{\alpha\beta k} \hat{E}^C_k + \frac{1}{2}\lambda''_{ijk} \hat{U}^c_i \hat{D}^C_j D^C_k$$

- Freedom of $SU(4)$ rotation in $(\hat{L}_i, \hat{H}_d)$ space

  SVP $\Rightarrow$ Rotate away the three $< \tilde{\nu}_i >$

- Advantages:

  $\Rightarrow$ Simplifies the structure of mass matrices
  $\Rightarrow$ No extra contributions to sfermion masses $\propto < \tilde{\nu}_i >$.
  $\Rightarrow$ For small μ_i, (m_e, m_μ, m_τ) are the charged lepton masses.
  $\Rightarrow$ Considerable algebraic simplicity.

$B \to X_s\gamma$ in SUSY without R-parity

Motivation:

- Potential constraints on those combinations of trilinears that are only bound by perturbative unitarity

- A comprehensive study in the most general framework still lacking

  ⇒ [Carlos, White (00)] Consider only trilinears; incomplete operator basis
  ⇒ [Besmer, Steffan (00)] Treatment of bilinears not consistent

- We do a model independent numerical study in the most general framework under SVP at the Leading Log.

- Aim: Not a precision analysis but order of magnitude bounds.

The Calculation Strategy

$$\mathcal{H}_{eff} = \sum_i C_i(\mu)\, \mathcal{Q}_i(\mu)$$

- Matching: full theory $(M_W) \equiv$ effective theory (M_W)

  $\Rightarrow C_i(M_W)$.

- Resummation of the large logarithms:

$$\frac{dC_i}{dln\mu} = \gamma_{ji} C_j$$

  deriving γ_{ij}, the most difficult step.

- Calculation of matrix elements of operators at $\mu = m_b$

$$Br(B \to X) = Br(b \to q) + \mathcal{O}\left(\frac{1}{m_b}\right)$$

An Enlarged Operator Basis

- Naively speaking the relevant operator is the Magnetic Penguin

$$\mathcal{Q}_7 = \frac{e}{16\pi^2} m_b \, \bar{s}_{L\alpha} \, \sigma_{\mu\nu} \, b_{R\alpha} \, F^{\mu\nu}$$

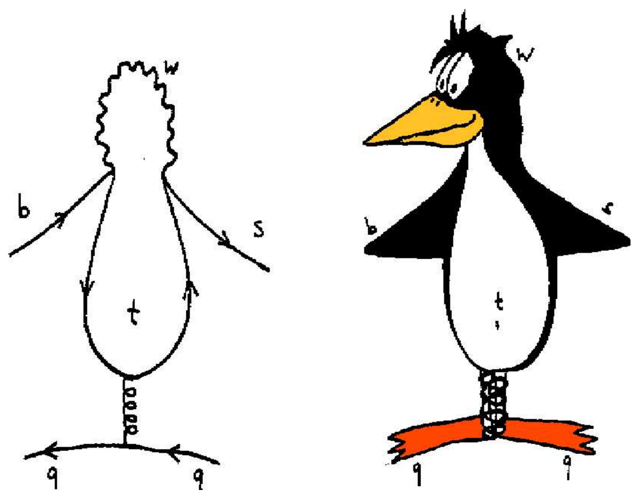


Figure 1: SUSY or not !!

- $\mathcal{Q}_7$ mixes with many other operators upon QCD renormalization. For the case of SM, we must write

$$\mathcal{H}_{eff} = \sum_{i=1,28} C_i \mathcal{Q}_i$$

- where

$$\begin{aligned}
\mathcal{Q}_{1,2} &= (\bar{s}_{L\alpha}\,\gamma^\mu\, b_{L\alpha,\beta})\ (\bar{c}_{L\beta}\,\gamma_\mu\, c_{L\beta,\alpha}) \\
\mathcal{Q}_{3,4} &= (\bar{s}_{L\alpha}\,\gamma^\mu\, b_{L\alpha,\beta}) \sum_i (\bar{q}_{Li\beta}\,\gamma_\mu\, q_{Li\beta\alpha}) \\
\mathcal{Q}_{5,6} &= (\bar{s}_{L\alpha}\,\gamma^\mu\, b_{L\alpha,\beta}) \sum_i (\bar{q}_{Ri\beta}\,\gamma_\mu\, q_{Ri\beta,\alpha}) \\
\mathcal{Q}_8 &= \frac{g_s}{16\pi^2} m_b\, \bar{s}_{L\alpha}\, \sigma_{\mu\nu}\, b_{R\beta}\, t^{a\alpha\beta} G^{a\mu\nu}
\end{aligned}$$

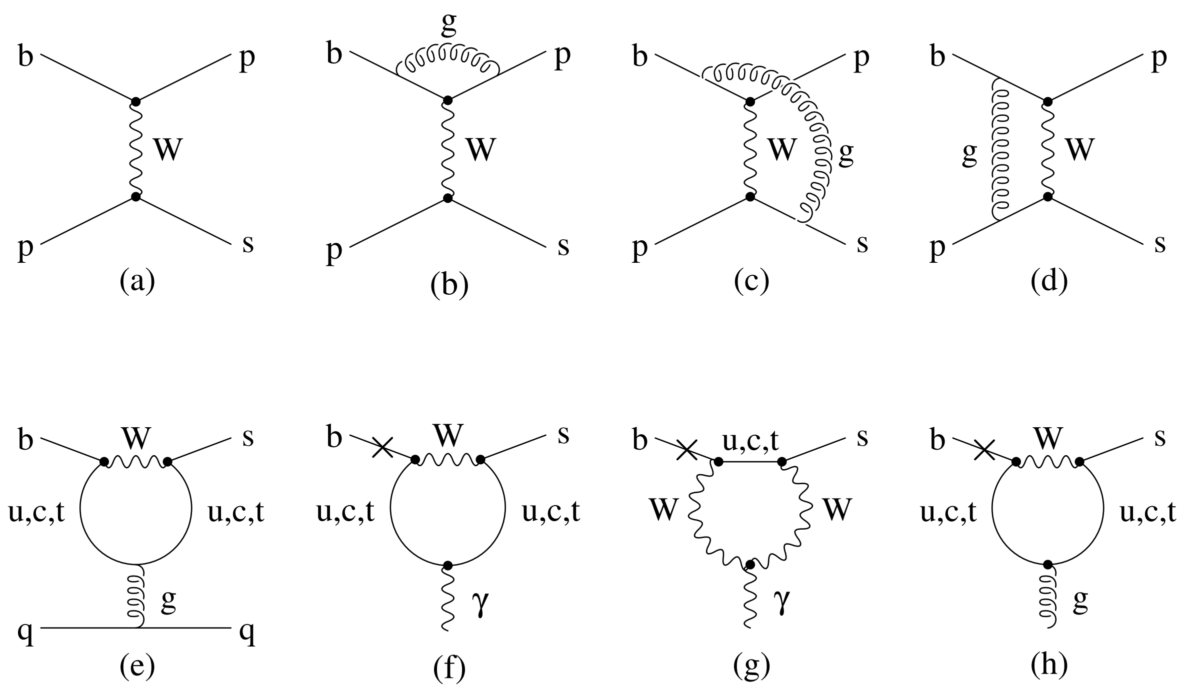


Figure 2: Diagrams a,b,c,d are the QCD corrected current-current operators. e is QCD penguin and f,g,h are the magnetic and chromo-magnetic penguins.

- MSSM: Gluino box diagrams generate new scalar and tensor type four-quark operators [Borzumati $et.al$'00]. Their Mixing with (Chromo)-magnetic operators, sub-dominant at LL and hence negligible [Okumura $et.al$ '03].

  $\Rightarrow$ Number of operators remain the same (8). Only C_7 and C_8 get modified.

- However, RPV introduces <u>20 extra operators</u> .

- From λ'_{ijk} we have 12 operators

$$
\begin{aligned}
\mathcal{Q}_6^q &= (\bar{s}_{L\alpha}\,\gamma^\mu\, b_{L\beta})\ (\bar{q}_{R\beta}\,\gamma_\mu\, q_{R\alpha})\ \ ;\ \ q = d, s, b \\
\tilde{\mathcal{Q}}_6^q &= (\bar{s}_{R\alpha}\,\gamma^\mu\, b_{R\beta})\ (\bar{q}_{L\beta}\,\gamma_\mu\, q_{L\alpha})\ \ ;\ \ q = u, c, d, s, b \\
\tilde{\mathcal{Q}}_{3,4} &= (\bar{s}_{R\alpha}\,\gamma^\mu b_{R\alpha,\beta})\sum_i (\bar{q}_{Ri\beta}\,\gamma^\mu\, q_{Ri\beta,\alpha}) \\
\tilde{\mathcal{Q}}_{5,6} &= (\bar{s}_{R\alpha}\,\gamma^\mu b_{R\alpha,\beta})\sum_i (\bar{q}_{Li\beta}\,\gamma^\mu\, q_{Li\beta,\alpha})
\end{aligned}
$$

- From λ''_{ijk} we get 6 operators

$$
\begin{aligned}
\tilde{\mathcal{Q}}_3^q &= (\bar{s}_{R\alpha}\,\gamma^\mu\,b_{R\alpha})\;(\bar{q}_{R\beta}\,\gamma_\mu\,q_{R\beta})\;\;;\;\;q = u, c, d \\
\tilde{\mathcal{Q}}_4^q &= (\bar{s}_{R\beta}\,\gamma^\mu\,b_{R\alpha})\;(\bar{q}_{R\beta}\,\gamma_\mu\,q_{R\alpha})\;\;;\;\;q = u, c, d
\end{aligned}
$$

A few important remarks

- These Operators cannot be transformed into each other by applying Eq. of motion. It is the only correct set if all external states are taken on-shell [Politzer '80, Simma'93].

- Additional operators have to be considered if an off-shell calculation is performed.

- There is yet another basis used in the literature, which allows to use fully anti-commuting γ_5 in the calculation [Chetyrkin,Misiak, Munz '97].

The Wilson Coefficients

- The non-zero effective Wilson coefficients at the scale M_W are given as:

$$
\begin{aligned}
C(\mathcal{Q}_2) &= \frac{G_F}{\sqrt{2}} V_{CKM}^{ts*} V_{CKM}^{tb} \\
C(\tilde{\mathcal{Q}}_6^u) &= \frac{-1}{8m^2_{\tilde{l}^-_m}} \lambda'_{i12} \lambda'^*_{j13} \mathcal{D}^l_{m,2+i} \mathcal{D}^{l*}_{2+j,m} \\
C(\tilde{\mathcal{Q}}_6^c) &= -\frac{1}{8m^2_{\tilde{l}^-_m}} \lambda'_{i22} \lambda'^*_{j23} \mathcal{D}^l_{m,2+i} \mathcal{D}^{l*}_{2+j,m} \\
&+ \frac{1}{8m^2_{\tilde{l}^-_m}} h_s h_b \mathcal{D}^l_{m,2} \mathcal{D}^{l*}_{2,m} V_{CKM}^{ts*} V_{CKM}^{tb} \\
&- \frac{1}{8m^2_{\tilde{l}^-_m}} \lambda'^*_{j23} V_{CKM}^{cs} h_s \mathcal{D}^l_{m,2+i} \mathcal{D}^{l*}_{2,m} \\
C(\tilde{\mathcal{Q}}_6^d) &= -\frac{1}{8m^2_{\tilde{\nu}_m}} \lambda'_{i12} \lambda'^*_{j13} \mathcal{D}^s
\end{aligned}
$$

$$
\begin{aligned}
C(\tilde{\mathcal{Q}}_6^s) &= -\frac{1}{8m_{\tilde{\nu}_m}^2}\lambda'^*_{j23}\left[\lambda'_{i22}\mathcal{D}^s + h_b\mathcal{D}^{s'}\right] \\
C(\tilde{\mathcal{Q}}_6^b) &= -\frac{1}{8m_{\tilde{\nu}_m}^2}\lambda'_{i32}\left[\lambda'^*_{j33}\mathcal{D}^s + h_b\mathcal{D}^{s'}\right] \\
C(\mathcal{Q}_6^d) &= -\frac{1}{8m_{\tilde{\nu}_m}^2}\lambda'_{i31}\lambda'^*_{j21}\mathcal{D}^s \\
C(\mathcal{Q}_6^s) &= -\frac{1}{8m_{\tilde{\nu}_m}^2}\lambda'_{i32}\left[\lambda'^*_{j22}\mathcal{D}^s + h_s\mathcal{D}^{s'}\right] \\
C(\mathcal{Q}_6^b) &= -\frac{1}{8m_{\tilde{\nu}_m}^2}\lambda'^*_{j23}\left[\lambda'_{i33}\mathcal{D}^s + h_b\mathcal{D}^{s'}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{D}^s &= \left(\mathcal{D}^s_{m,2+i} + i\mathcal{D}^s_{m,7+i}\right)\left(\mathcal{D}^s_{2+j,m} + i\mathcal{D}^s_{7+j,m}\right)^* \\
\mathcal{D}^{s'} &= \left(\mathcal{D}^s_{m,2+i} + i\mathcal{D}^s_{m,7+i}\right)\left(\mathcal{D}^s_{2,m} + i\mathcal{D}^s_{7,m}\right)^*
\end{aligned}
$$

$$
\begin{aligned}
C(\tilde{\mathcal{Q}}_3^u) &= -\frac{1}{8m_{\tilde{d}_m}^2}\lambda''_{1i2}\lambda''_{1j3}\mathcal{D}^d_{3+j,m}\mathcal{D}^{d\dagger}_{m,3+i} = -C(\tilde{\mathcal{Q}}_4^u) \\
C(\tilde{\mathcal{Q}}_3^c) &= -\frac{1}{8m_{\tilde{d}_m}^2}\lambda''_{2i2}\lambda''_{2j3}\mathcal{D}^d_{3+j,m}\mathcal{D}^{d\dagger}_{m,3+i} = -C(\tilde{\mathcal{Q}}_4^c) \\
C(\tilde{\mathcal{Q}}_3^d) &= -\frac{1}{8m_{\tilde{u}_m}^2}\lambda''_{i12}\lambda''_{j13}\mathcal{D}^u_{3+j,m}\mathcal{D}^{u\dagger}_{m,3+i} = -C(\tilde{\mathcal{Q}}_4^d)
\end{aligned}
$$

Wilson Coefficient for Magnetic Penguin

First we need to define relevant int. Lagrangian:

- For the Gluino-quark-squark interaction:

$$\mathcal{L}^{\tilde{g}} = g_s \overline{\Psi}(d_i) \left[\mathcal{G}^R_{im} \frac{1-\gamma_5}{2} + \mathcal{G}^L_{im} \frac{1+\gamma_5}{2} \right] \Psi(\tilde{g}) \phi(\tilde{d}_m) \; + \; \text{h.c.}$$

where

$$\mathcal{G}^L_{im} = -\sqrt{\frac{4}{3}} \mathcal{D}^d_{im} \, , \qquad \mathcal{G}^R_{im} = \sqrt{\frac{4}{3}} \mathcal{D}^d_{(i+3)m} \, .$$

- For the Chargino-quark-squark vertex we have

$$\mathcal{L}^{\chi^-} = g_2 \overline{\Psi}(d_i) \left[\mathcal{C}^R_{inm} \frac{1-\gamma_5}{2} + \mathcal{C}^L_{inm} \frac{1+\gamma_5}{2} \right] \Psi(\chi^-_n) \phi(\tilde{u}_m) \; + \text{h.c.}$$

where

$$\mathcal{C}^L_{inm} = -\boldsymbol{V}_{1n}\,\mathcal{D}^u_{im} + \frac{y_{u_k}}{g_2}\,V^{ki^*}_{\mathsf{CKM}}\,\boldsymbol{V}_{2n}\,\mathcal{D}^u_{(k+3)m}\,,$$

$$\mathcal{C}^R_{inm} = \frac{y_{d_i}}{g_2}\,V^{hi^*}_{\mathsf{CKM}}\,\boldsymbol{U}_{2n}\,\mathcal{D}^u_{hm} + \frac{\lambda'_{kji}}{g_2}\,V^{hj^*}_{\mathsf{CKM}}\,\boldsymbol{U}_{(k+2)n}\,\mathcal{D}^u_{hm}$$

- Similarly one can define $\mathcal{L}^{\chi^0,\phi^-,\phi^0}$ for the neutralino, charged scalar and neutral scalar interactions.

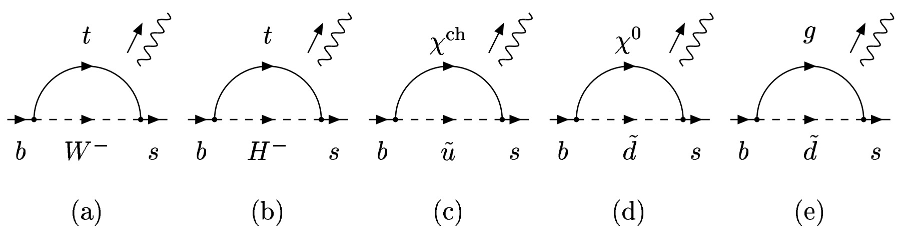


Figure 3: Diagrams that contribute to the matching of $\mathcal{Q}_{7/8}$ and $\tilde{\mathcal{Q}}_{7/8}$. The outgoing photon/gluon is attached at every possible position.

$$C(\mathcal{Q}_7) = A^L_W + A^L_{\tilde{g}} \; + \; A^L_{\chi^-} \; + \; A^L_{\chi^0} \; + \; A^L_{\phi^-} + A^L_{\phi^0}$$

where

$$\begin{aligned}
A^L_{\tilde{g}} \quad &= \quad \frac{2g_s^2}{3} \, \frac{1}{M^2_{\tilde{d}_m}} \mathcal{G}^{L^*}_{jm} \mathcal{G}^R_{im} \, \frac{M_{\tilde{g}}}{m_{d_j}} \, \mathcal{Q}_{\tilde{d}} \; F_3\!\left(\frac{M^2_{\tilde{g}}}{M^2_{\tilde{d}_m}}\right) \\
&+ \frac{2g_s^2}{3} \, \frac{1}{M^2_{\tilde{d}_m}} \mathcal{G}^{L^*}_{jnm} \mathcal{G}^L_{inm} \mathcal{Q}_{\tilde{d}} \; F_2\!\left(\frac{M^2_{\tilde{g}}}{M^2_{\tilde{d}_m}}\right) \\
&+ \frac{2g_s^2}{3} \, \frac{1}{M^2_{\tilde{d}_m}} \mathcal{G}^{R^*}_{jnm} \mathcal{G}^R_{inm} \frac{m_{d_i}}{m_{d_j}} \, \mathcal{Q}_{\tilde{d}} \; F_2\!\left(\frac{M^2_{\tilde{g}}}{M^2_{\tilde{d}_m}}\right) \\
A^L_{\chi^0} \quad &= \quad \frac{1}{4} \, \frac{1}{M^2_{\tilde{d}_m}} \mathcal{N}^{L^*}_{jnm} \mathcal{N}^R_{inm} \, \frac{M_{\chi^0_n}}{m_{d_j}} \, \mathcal{Q}_{\tilde{d}} \; F_3\!\left(\frac{M^2_{\chi^0_n}}{M^2_{\tilde{d}_m}}\right) \\
&+ \frac{1}{4} \, \frac{1}{M^2_{\tilde{d}_m}} \mathcal{N}^{L^*}_{jnm} \mathcal{N}^L_{inm} \mathcal{Q}_{\tilde{d}} \; F_2\!\left(\frac{M^2_{\chi^0_n}}{M^2_{\tilde{d}_m}}\right) \; ,
\end{aligned}$$

$$
\begin{aligned}
A^{L}_{\bar{\chi}} &= \frac{1}{4}\,\frac{1}{M^2_{\tilde{u}_m}}\mathcal{C}^{L^*}_{jnm}\mathcal{C}^{R}_{inm}\,\frac{M_{\bar{\chi}_n}}{m_{d_j}}\times \\
&\left[\mathcal{Q}_{\tilde{u}}\;F_3\!\left(\frac{M^2_{\bar{\chi}_n}}{M^2_{\tilde{u}_m}}\right)-\left(\mathcal{Q}_d-\mathcal{Q}_{\tilde{u}}\right)\,F_6\!\left(\frac{M^2_{\bar{\chi}_n}}{M^2_{\tilde{u}_m}}\right)\right] \\
&+\frac{1}{4}\,\frac{1}{M^2_{\tilde{u}_m}}\mathcal{C}^{L^*}_{jnm}\mathcal{C}^{L}_{inm}\times \\
&\left[\mathcal{Q}_{\tilde{u}}\;F_2\!\left(\frac{M^2_{\bar{\chi}_n}}{m^2_{\tilde{u}_m}}\right)-\left(\mathcal{Q}_d-\mathcal{Q}_{\tilde{u}}\right)\,F_5\!\left(\frac{M^2_{\bar{\chi}_n}}{M^2_{\tilde{u}_m}}\right)\right]\;, \\
A^{L}_{\phi^0} &= -\frac{1}{M^2_{S_m}}\,\widetilde{\mathcal{N}}^{L^*}_{jnm}\,\widetilde{\mathcal{N}}^{R}_{inm}\,\frac{m_{d_n}}{m_{d_j}}\;\mathcal{Q}_d\;F_6\!\left(\frac{m^2_{d_n}}{M^2_{S_m}}\right) \\
&-\frac{1}{M^2_{S_m}}\,\widetilde{\mathcal{N}}^{L^*}_{jnm}\,\widetilde{\mathcal{N}}^{L}_{inm}\;\mathcal{Q}_d\;F_5\!\left(\frac{m^2_{d_n}}{M^2_{S_m}}\right)
\end{aligned}
$$

$$
\begin{aligned}
A^L_{\bar{\phi}} &= \frac{1}{M^2_{\tilde{\ell}_m}} \, \widetilde{\mathcal{C}}^{L*}_{jnm} \, \widetilde{\mathcal{C}}^{R}_{inm} \, \frac{m_{u_n}}{m_{d_j}} \; \times \\
& \left[(\mathcal{Q}_d - \mathcal{Q}_u) \; F_3\!\left(\frac{m^2_{u_n}}{M^2_{\tilde{\ell}_m}}\right) - \mathcal{Q}_u \; F_6\!\left(\frac{m^2_{u_n}}{M^2_{\tilde{\ell}_m}}\right) \right] \\
& + \frac{1}{M^2_{\tilde{\ell}_m}} \, \widetilde{\mathcal{C}}^{L*}_{jnm} \, \widetilde{\mathcal{C}}^{L}_{inm} \; \times \\
& \left[(\mathcal{Q}_d - \mathcal{Q}_u) \; F_2\!\left(\frac{m^2_{u_n}}{M^2_{\tilde{\ell}_m}}\right) - \mathcal{Q}_u \; F_5\!\left(\frac{m^2_{u_n}}{M^2_{\tilde{\ell}_m}}\right) \right] \; , \\
A^L_W &= \frac{3 \, e \, g_2^2}{64\pi^2} \, \frac{m^2_{u_n}}{M^4_W} \, V^{nj}_{\mathsf{CKM}} \, V^{ni^*}_{\mathsf{CKM}} \; \times \\
& \left[(\mathcal{Q}_d - \mathcal{Q}_u) \; F_2\!\left(\frac{m^2_{u_n}}{M^2_W}\right) - \mathcal{Q}_u \; F_5\!\left(\frac{m^2_{u_n}}{M^2_W}\right) \right]
\end{aligned}
$$

where the Inami-Lim functions are given as:

$$F_2(x) = \frac{1}{6\,(1-x)^4}\,(1 - 6\,x + 3\,x^2 + 2\,x^3 - 6\,x^2\,\ln x)\;,$$

$$F_3(x) = \frac{1}{(1-x)^3}\,(1 - x^2 + 2\,x\,\ln x)\;,$$

$$F_5(x) = \frac{1}{6\,(1-x)^4}\,(2 + 3\,x - 6\,x^2 + x^3 + 6\,x\,\ln x)\;,$$

$$F_6(x) = \frac{1}{(1-x)^3}\,(-3 + 4\,x - x^2 - 2\,\ln x)\;,$$

Some Remarks

- Chirality of final quark is flipped from the initial quark through the dipole operator. This can be done in two ways.

  (A) Picking a mass from the external quark line

  (B) Picking a mass from the internal line

- While in SM only type (A) diagrams are possible, in SUSY Charginos and gluinos can induce a type (B) diagram also.

- Type (A) diagrams are proportional to $F_2(x), F_5(x)$, type (B) diagrams are proportional to $F_3(x), F_6(x)$.

Scheme Independence

- At the Leading Log, the anomalous dimension matrix γ_{ij} is known to be scheme dependent.

- This dependence gets canceled by finite one-loop contributions of some four-quark operators to the Amplitude A for $b \to s + \gamma$.

- Define

$$A = C_7^{eff} < s\gamma|\mathcal{Q}_7|b >_{tree} + \tilde{C}_7^{eff} < s\gamma|\tilde{\mathcal{Q}}_7|b >_{tree}$$

where,

$$\begin{aligned} < s\gamma|\mathcal{Q}_i|b >_{1-\text{loop}} &= y_i < s\gamma|\mathcal{Q}_7|b >_{\text{tree}} \\ < s\,\text{gluon}|\mathcal{Q}_i|b >_{1-\text{loop}} &= z_i < s\,\text{gluon}|\mathcal{Q}_8|b >_{\text{tree}} \\ < s\gamma|\tilde{\mathcal{Q}}_i|b >_{1-\text{loop}} &= \tilde{y}_i < s\gamma|\tilde{\mathcal{Q}}_7|b >_{\text{tree}} \\ < s\,\text{gluon}|\tilde{\mathcal{Q}}_i|b >_{1-\text{loop}} &= \tilde{z}_i < s\,\text{gluon}|\tilde{\mathcal{Q}}_8|b >_{\text{tree}} \end{aligned}$$

- The effective Wilson coefficients are defined as:

$$
\begin{aligned}
C_7^{\text{eff}}(\mu) &= C_7(\mu) + \sum_i y_i C_i(\mu) \\
C_8^{\text{eff}}(\mu) &= C_8(\mu) + \sum_i z_i C_i(\mu) \\
\tilde{C}_7^{\text{eff}}(\mu) &= \tilde{C}_7(\mu) + \sum_i \tilde{y}_i \tilde{C}_i(\mu) \\
\tilde{C}_8^{\text{eff}}(\mu) &= \tilde{C}_8(\mu) + \sum_i \tilde{z}_i C_i(\mu) \\
C_{\mathcal{Q}_i,\tilde{\mathcal{Q}}_i}^{\text{eff}}(\mu) &= C_{\mathcal{Q}_i,\tilde{\mathcal{Q}}_i}(\mu)
\end{aligned}
$$

- The RGE's for the effective coefficients:

$$
\frac{d}{\ln\mu} C_k^{\text{eff}}(\mu) = \frac{\alpha_s}{4\pi} \gamma_{jk}^{\text{eff}} C_j^{\text{eff}}(\mu)
$$

The Anomalous Dimension Matrix (AD)

- Since QCD does not know the sign of γ_5, there is no mixing among operators related by $L \leftrightarrow R$. The effective AD at Leading Log is obtained as:

$$\vec{\gamma}^{(0)eff} = \begin{pmatrix} \gamma_L & 0 \\ 0 & \gamma_R \end{pmatrix}$$

- γ_L represents QCD mixing of 11 operators whose chirality structure is similar to those of SM operators. γ_R represents the mixing of 17 operators whose chirality structure is obtained by $L \leftrightarrow R$ replacement with SM like operators.

$$
\gamma_L = \begin{pmatrix}
Q_3^c & Q_4^c & Q_3 & Q_4 & Q_5 & Q_6 & Q_6^d & Q_6^s & Q_6^b & Q_7 & Q_8 \\
-2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\
6 & -2 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & \frac{416}{81} & \frac{70}{27} \\
0 & 0 & -\frac{22}{9} & \frac{22}{3} & -\frac{4}{9} & \frac{4}{3} & 0 & 0 & 0 & -\frac{464}{81} & \frac{545}{27} \\
0 & 0 & \frac{44}{9} & \frac{4}{3} & -\frac{10}{9} & \frac{10}{3} & 0 & 0 & 0 & \frac{136}{81} & \frac{512}{27} \\
0 & 0 & 0 & 0 & 2 & -6 & 0 & 0 & 0 & \frac{32}{9} & -\frac{59}{3} \\
0 & 0 & -\frac{10}{9} & \frac{10}{3} & -\frac{10}{9} & -\frac{38}{3} & 0 & 0 & 0 & -\frac{296}{81} & \frac{703}{27} \\
0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & -16 & 0 & 0 & \frac{200}{81} & -\frac{119}{27} \\
0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & -16 & 0 & \frac{200}{81} & -\frac{119}{27} \\
0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & -16 & \frac{200}{81} & -\frac{227}{27} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{32}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{32}{9} & \frac{28}{3}
\end{pmatrix}
$$

$$
\gamma_R = \begin{pmatrix}
\tilde{Q}_3 & \tilde{Q}_4 & \tilde{Q}_5 & \tilde{Q}_6 & \tilde{Q}_3^u & \tilde{Q}_4^u & \tilde{Q}_3^c & \tilde{Q}_4^c & \tilde{Q}_3^d & \tilde{Q}_4^d & \tilde{Q}_6^u & \tilde{Q}_6^c & \tilde{Q}_6^d & \tilde{Q}_6^s & \tilde{Q}_6^b & \tilde{Q}_7 & \tilde{Q}_8 \\
-\frac{22}{9} & \frac{22}{3} & -\frac{4}{9} & \frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{464}{81} & \frac{545}{27} \\
\frac{44}{9} & \frac{4}{3} & -\frac{10}{9} & \frac{10}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{136}{81} & \frac{512}{27} \\
0 & 0 & 2 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{32}{9} & -\frac{59}{3} \\
-\frac{10}{9} & \frac{10}{3} & -\frac{10}{9} & -\frac{38}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{296}{81} & -\frac{703}{27} \\
0 & 0 & 0 & 0 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\
-\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{416}{81} & \frac{70}{27} \\
0 & 0 & 0 & 0 & 0 & 0 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\
-\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{416}{81} & \frac{70}{27} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\
-\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & -\frac{232}{81} & \frac{70}{27} \\
-\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -16 & 0 & 0 & 0 & 0 & -\frac{448}{81} & -\frac{119}{27} \\
-\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & 0 & 0 & 0 & -\frac{448}{81} & -\frac{119}{27} \\
-\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & 0 & 0 & \frac{200}{81} & -\frac{119}{27} \\
-\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & 0 & \frac{200}{81} & -\frac{119}{27} \\
-\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & \frac{200}{81} & -\frac{227}{27} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{32}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{32}{9} & \frac{28}{3}
\end{pmatrix}
$$

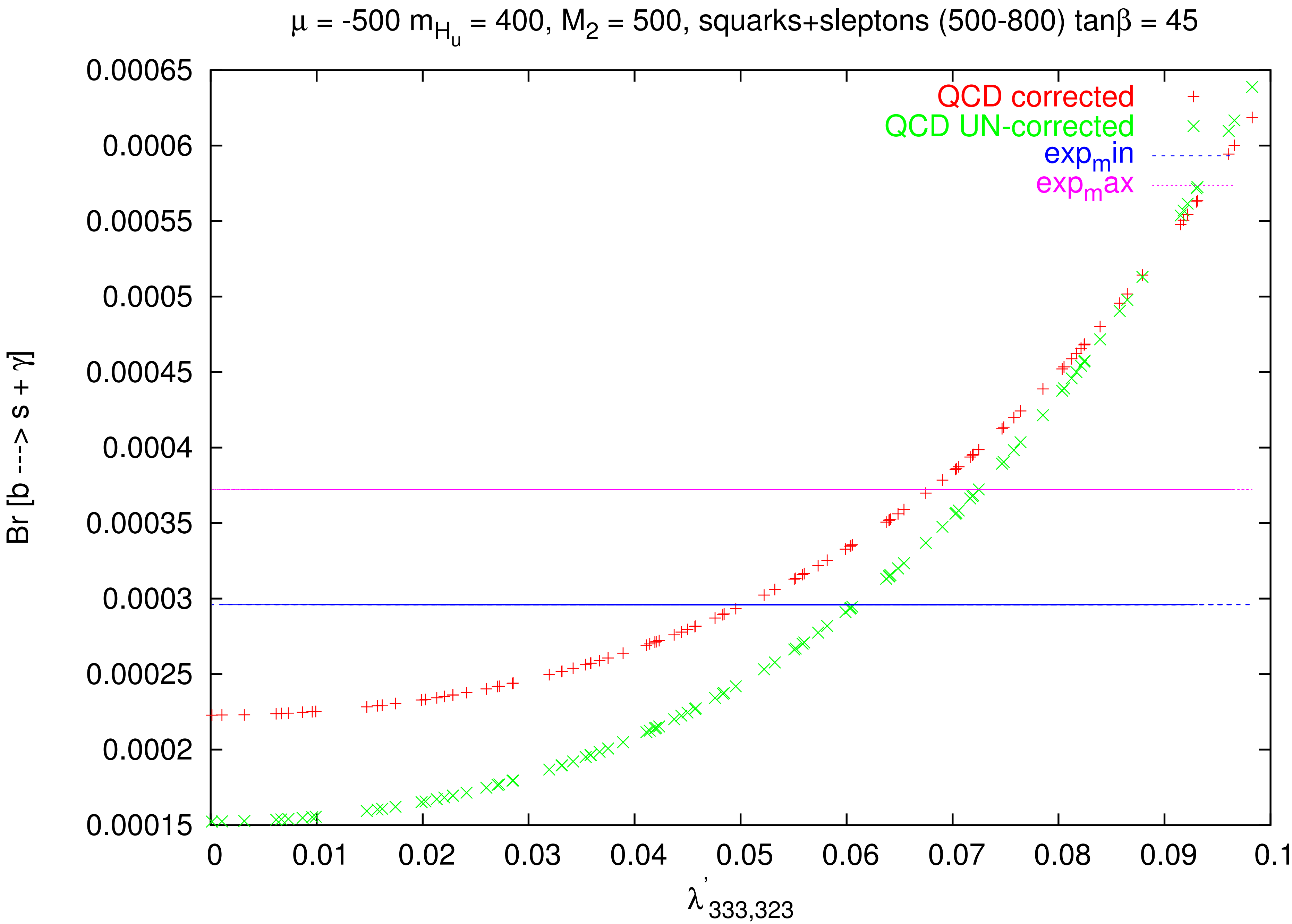
The decay rate

- The branching ratio $Br(b \to s + \gamma)$ is expressed through the the semi-leptonic decay $b \to u|ce\bar{\nu}$ so that the large bottom mass dependence ($\sim m_b^5$) and uncertainties in CKM elements cancel out.

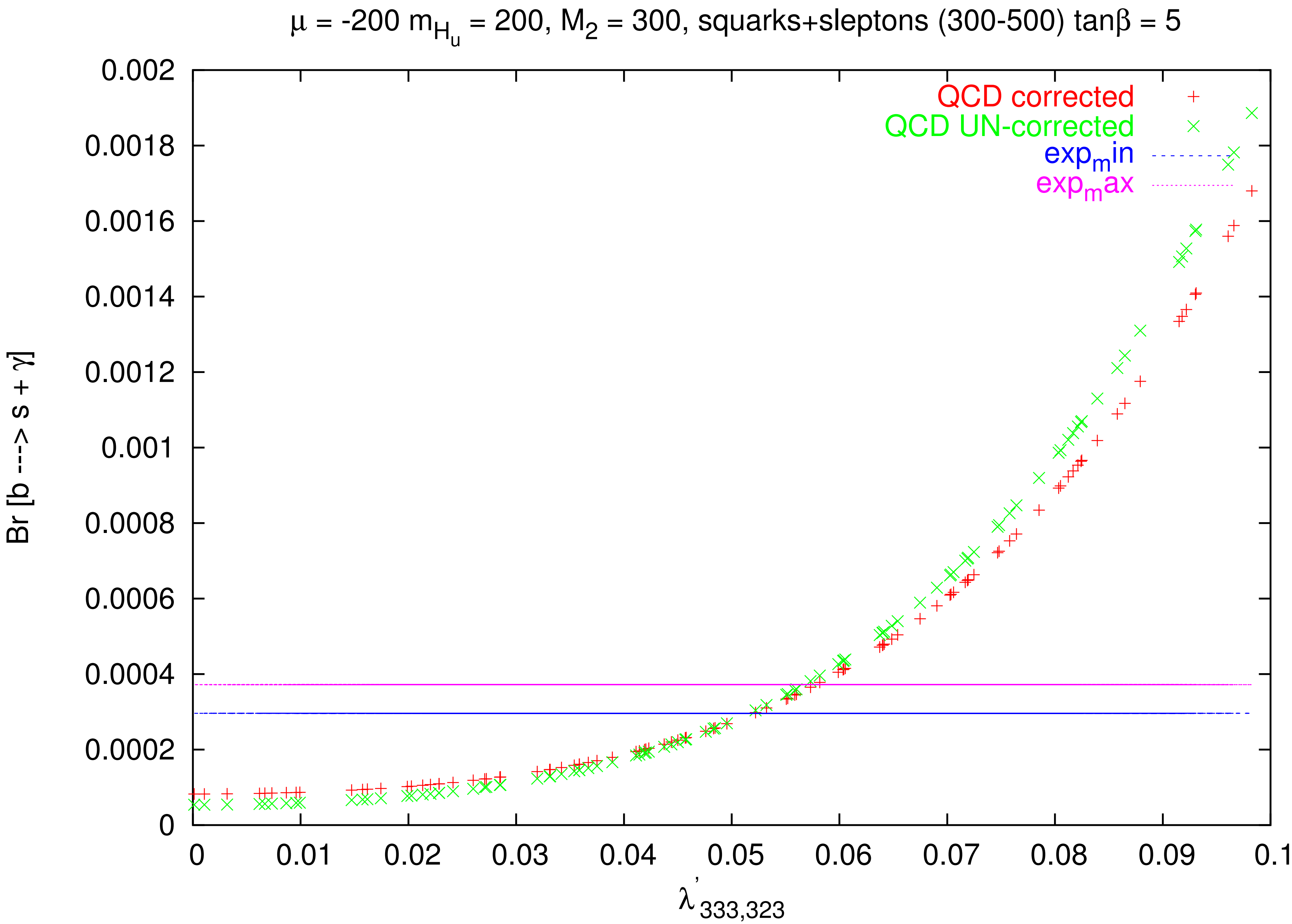
$$Br(b \to s + \gamma) = \frac{\Gamma(b \to s + \gamma)}{\Gamma(b \to u|c\,e\,\bar{\nu}_e)} Br_{exp}(b \to u|c\,e\,\bar{\nu}_e)$$

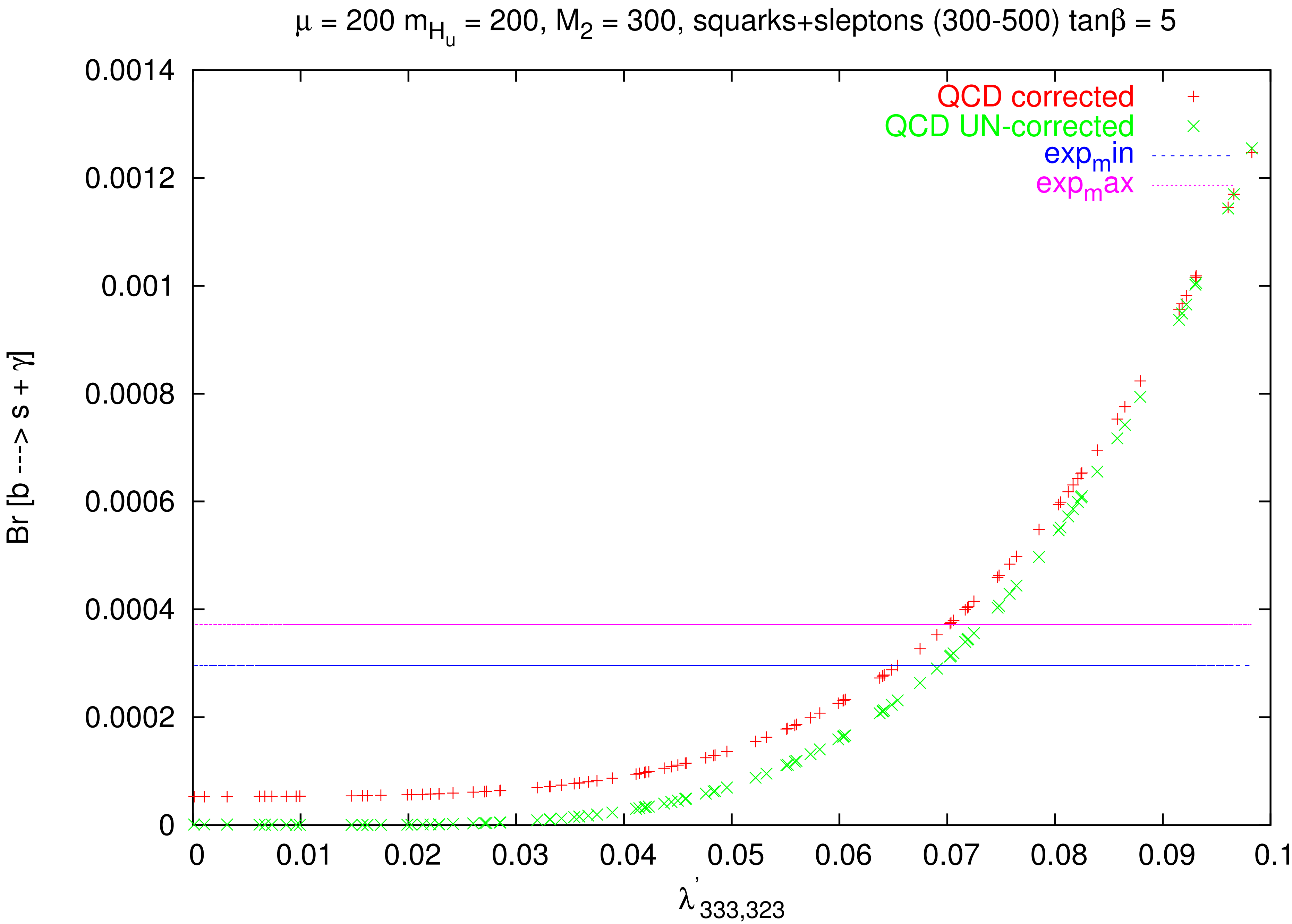
where $Br_{exp}(b \to u|c\,e\,\bar{\nu}_e) = 10.5\%$ [Caso $et.al$ '98] and,

$$\begin{aligned}
\Gamma(b \to \mathrm{s}\ \gamma) &= \frac{\alpha m_b^5}{64\pi^4}(|C_7(\mu_b)|^2 + |\tilde{C}_7(\mu_b)|^2) \\
\Gamma(b \to u|c\,e\,\bar{\nu}) &= \frac{mb^5}{192\pi^3}\left(f(\epsilon) f_1 + |B_1|^2 + |C_1|^2\right) \\
f(\epsilon) &= 1 - 8\epsilon^2 + 8\epsilon^6 - \epsilon^8 - 24\epsilon^4 \log\epsilon \\
f_1 &= G_F V_{ts} - \sum_{i=1}^{3} \frac{\lambda'_{i3k}\lambda'^{*}_{1mn}}{m^2_{\tilde{d}_l}} D^{d*}_{3+k,l} D^{d}_{l,3+n} V_{2m} \\
C_1 &= -\sum_{i=1}^{3} \frac{\lambda'_{i3k}\lambda'^{*}_{1mn}}{m^2_{\tilde{d}_l}} D^{l*}_{3+k,l} D^{l}_{l,3+n} V_{1n} \\
B_r &= -\sum_{i=1}^{3} \frac{\lambda_{ij1}\lambda'^{*}_{mn3}}{m^2_{\tilde{l}^-}} D^{l*}_{2+m,l} D^{l}_{l,2+j} V_{rn} \\
\epsilon &= \frac{m_c}{m_b}
\end{aligned}$$

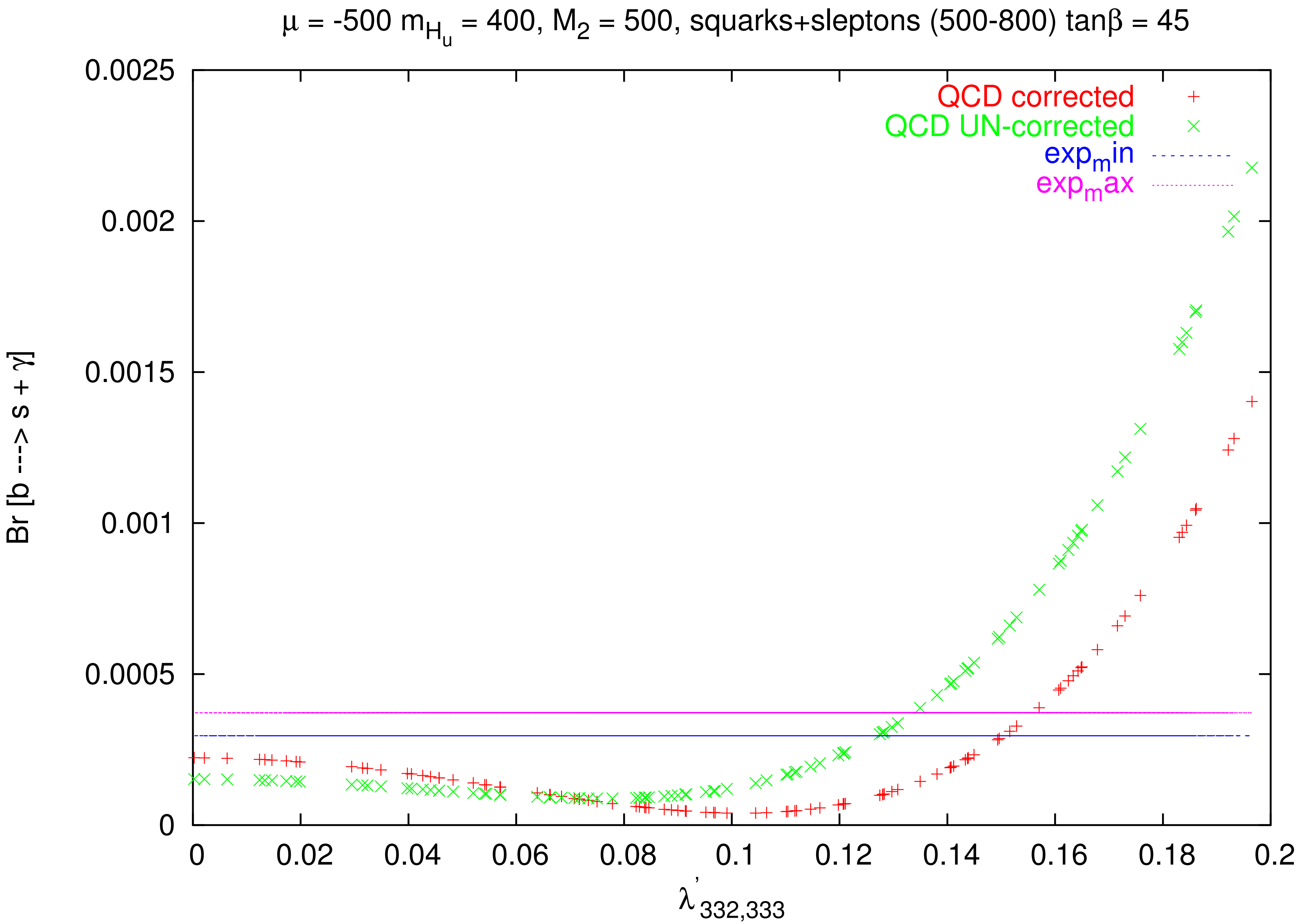


μ = -500 m_{H_u} = 400, M_2 = 500, squarks+sleptons (500-800) tanβ = 45
QCD corrected
QCD UN-corrected
exp_min
exp_max
Br [b ---> s + γ]
λ'_{333,323}
0.00065
0.0006
0.00055
0.0005
0.00045
0.0004
0.00035
0.0003
0.00025
0.0002
0.00015
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

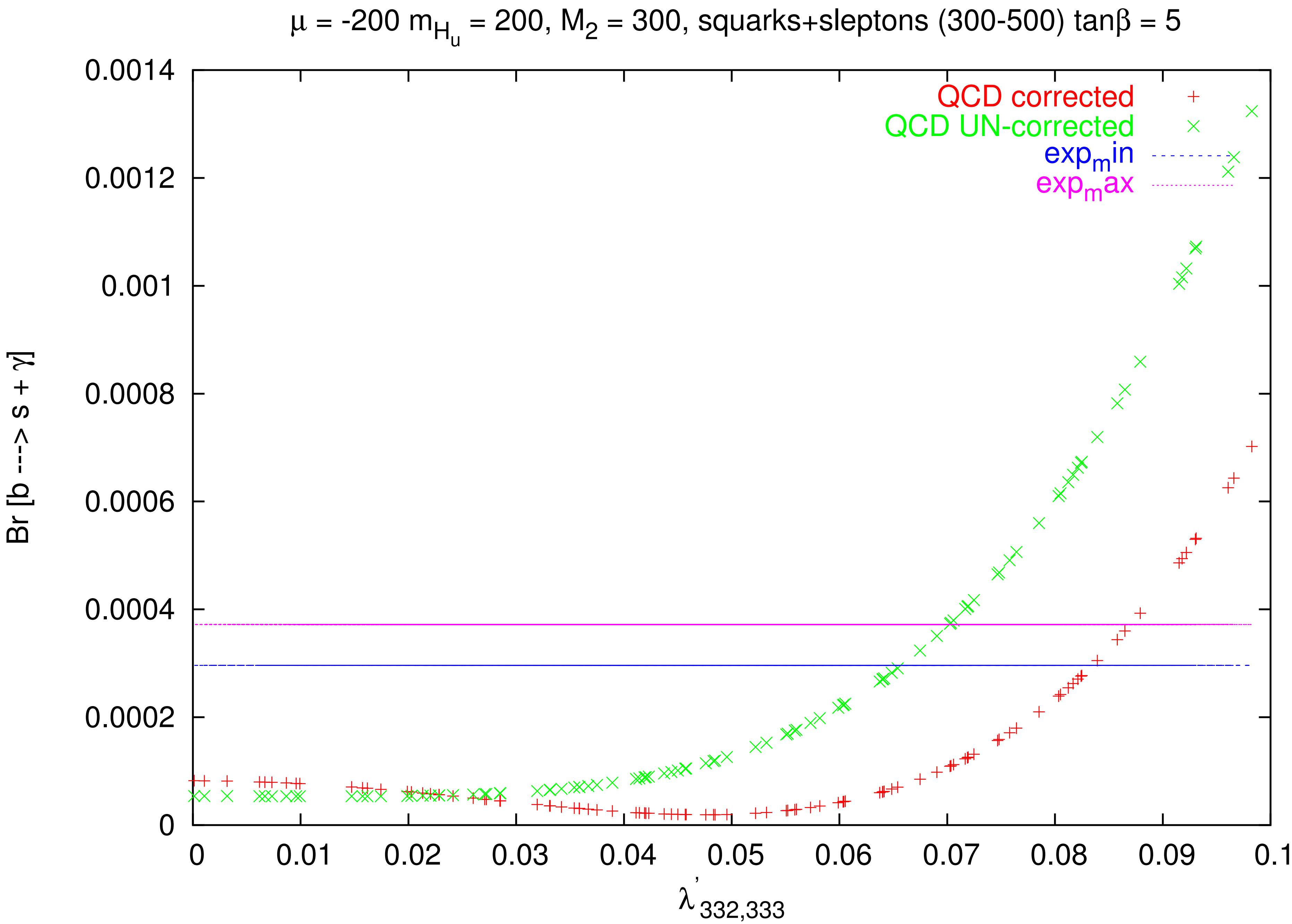

μ = -200 m_{H_u} = 200, M_2 = 300, squarks+sleptons (300-500) tanβ = 5
QCD corrected
QCD UN-corrected
exp_min
exp_max
Br [b ---> s + γ]
λ'_{333,323}
0.002
0.0018
0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1



μ = 200 m_Hu = 200, M2 = 300, squarks+sleptons (300-500) tanβ = 5
QCD corrected
QCD UN-corrected
exp_min
exp_max
Br [b ---> s + γ]
λ'333,323
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1



μ = -500 m$_{H_u}$ = 400, M$_2$ = 500, squarks+sleptons (500-800) tanβ = 45
QCD corrected
QCD UN-corrected
exp$_m$in
exp$_m$ax
Br [b ---> s + γ]
λ'$_{332,333}$
0.0025
0.002
0.0015
0.001
0.0005
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2



μ = -200 m_Hu = 200, M_2 = 300, squarks+sleptons (300-500) tanβ = 5
QCD corrected
QCD UN-corrected
exp_m in
exp_m ax
Br [b ---> s + γ]
λ'332,333
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

Preliminary Results

- For many of the RPV parameters relevant for the $b \to s + \gamma$ there are no bounds on the products available so far. The best bounds on most of the individual couplings are those coming from perturbative unitarity (quoted below in the bracket) which are typically of $O(1)$. For these products we obtain a few orders of magnitude improvement.

Product of λ'	From Individual bound	Our bound
$\lambda'_{333}\lambda'_{323}$	$0.45(1.04), 0.52\frac{m_{\tilde{b}_R}}{100}(1.12)$	3.6×10^{-3}
$\lambda'_{233}\lambda'_{223}$	$0.15\sqrt{\frac{m_{\tilde{b}}}{100}}, 0.21\frac{m_{\tilde{b}_R}}{100}(1.12)$	3.6×10^{-3}
$\lambda'_{232}\lambda'_{233}$	$0.56(1.04), 0.15\sqrt{\frac{m_{\tilde{b}}}{100}}$	8.1×10^{-3}
$\lambda'_{132}\lambda'_{133}$	$0.28\frac{m_{\tilde{t}_L}}{100}, 1.4 \times 10^{-3} sqrt \frac{m_{\tilde{b}}}{100}$	8.1×10^{-3}
$\lambda'_{332}\lambda'_{333}$	$0.45(1.04), 0.45(1.04)$	8.1×10^{-3}
$\lambda'_{222}\lambda'_{223}$	$0.21\frac{m_{\tilde{s}_R}}{100}, 0.21\frac{m_{\tilde{b}_R}}{100}$	5.3×10^{-2}
$\lambda'_{222}\lambda'_{123}$	$0.21\frac{m_{\tilde{s}_R}}{100}(1.12), 0.043\frac{m_{\tilde{b}_R}}{100}$	0.2
$\lambda'_{232}\lambda'_{333}$	$0.56(1.04), 0.45(1.04)$	0.25