

**A consistent resolution for
possible anomalies in $B \rightarrow \phi K_S$
and $B \rightarrow \eta' K$ decays**

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- Dutta, Kim, S.O., Phys. Rev. Lett. 90, 011801 (2003)

Outline

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Introduction

CP asymmetry in $B^0 \rightarrow \phi K_S$

- CP violation in B system has been confirmed in measurements of time-dependent CP asymmetries in $B^0 \rightarrow J/\Psi K_S$ decay.

World average (2003): $(\phi_1 \equiv \beta)$ $b \rightarrow c\bar{c}s$

$$\sin(2\phi_1)_{J/\Psi K_S} = +0.731 \pm 0.056$$

- Recent measurements in $B^0 \rightarrow \phi K_S$ (2003): $b \rightarrow s\bar{s}s$

$$\sin(2\phi_1)_{\phi K_S}^{Belle} = -0.96 \pm 0.50_{-0.11}^{+0.09} \quad (3.5\sigma \text{ off}) \quad [(2002) - 0.73 \pm 0.64 \pm 0.22]$$

$$\sin(2\phi_1)_{\phi K_S}^{BaBar} = +0.45 \pm 0.43 \pm 0.07 \quad [(2002) - 0.18 \pm 0.51 \pm 0.09]$$

\Rightarrow Average = -0.15 ± 0.33 (2.7σ off the SM)

$$\mathcal{B}(B^0 \rightarrow \phi K_S) = (8.0 \pm 1.3) \times 10^{-6}$$

$$\mathcal{B}(B^+ \rightarrow \phi K^+) = (10.9 \pm 1.0) \times 10^{-6}$$

$$\mathcal{A}_{CP}(B^+ \rightarrow \phi K^+) = (3.9 \pm 8.8 \pm 1.1)\%$$

- Any new physics effects in time-dependent CP asymmetries:
 - $B^0 - \bar{B}^0$ mixing amplitude (universal to all B_d^0 decays) and/or
 - decay amplitude of each mode
- ⇒ New physics effect from the decay amplitude of $B^0 \rightarrow \phi K_S$.

$\sin(2\tilde{\phi}_1)_{XY}$ for $B \rightarrow XY$

$$\sin(2\tilde{\phi}_1)_{XY} = -\frac{2 \operatorname{Im}\lambda_{XY}}{(1 + |\lambda_{XY}|^2)}$$

$$\lambda_{XY} = e^{-2i\phi_1} \frac{\bar{\mathcal{A}}_{XY}}{\mathcal{A}_{XY}} = e^{-i(2\phi_1 + \theta)} \left| \frac{\bar{\mathcal{A}}_{XY}}{\mathcal{A}_{XY}} \right|$$

$$\mathcal{A}_{XY} = \mathcal{A}_{XY}^{SM} + \mathcal{A}_{XY}^{Rp}$$

⇒ effective CP angle: $2\tilde{\phi}_1 = 2\phi_1 + \theta$

- What about other decay modes having the same internal quark level process $b \rightarrow s\bar{s}s$ (e.g., $B^0 \rightarrow \eta' K_S$)?
 \Rightarrow recent data

$$\begin{aligned}\sin(2\phi_1)_{\eta' K_S}^{Belle} &= +0.43 \pm 0.27 \pm 0.05 \quad (\text{hep-ex/0308035}) \\ \sin(2\phi_1)_{\eta' K_S}^{BaBar} &= +0.02 \pm 0.34 \pm 0.03 \quad (\text{hep-ex/0303046})\end{aligned}$$

BR of $B^+ \rightarrow \eta' K^+$

- Recent experimental data :

$$\begin{aligned}\mathcal{B}(B^+ \rightarrow \eta' K^+) &= (78 \pm 6 \pm 9) \times 10^{-6} \quad [\text{Belle}] \\ &= (76.9 \pm 3.5 \pm 4.4) \times 10^{-6} \quad [\text{BaBar}] \\ &= (80_{-9}^{+10} \pm 7) \times 10^{-6} \quad [\text{CLEO}]\end{aligned}$$

\Rightarrow still larger than that expected within the SM.

The goal

Try to find a *consistent* explanation for the recent data on $B \rightarrow \phi K_S$, $\eta' K_S$, and $\eta' K^+$ without disturbing all the other $B \rightarrow PP$ and $B \rightarrow VP$ decay modes within the framework of \mathcal{R}_p SUSY.

Effective Hamiltonian

- The effective Hamiltonian for charmless nonleptonic B decays can be written as

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{uq}^* \sum_{i=1,2} c_i O_i - V_{tb} V_{tq}^* \sum_{i=3}^{12} c_i O_i \right] + h.c.$$

- The \mathcal{R}_p part of the superpotential of MSSM

$$\mathcal{W}_{\mathcal{R}_p} = \kappa_i L_i H_2 + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c$$

For $b \rightarrow s\bar{s}$ transitions,

the sneutrino mediated diagrams: **Tree level effect!**

$$\frac{\lambda'_{i22}\lambda'^*_{i23}}{m_{\tilde{\nu}}^2}(\bar{s}_L^\alpha b_R^\alpha)(\bar{s}_R^\beta s_L^\beta)$$

$$\frac{\lambda'_{i32}\lambda'^*_{i22}}{m_{\tilde{\nu}}^2}(\bar{s}_R^\alpha b_L^\alpha)(\bar{s}_L^\beta s_R^\beta)$$

\Rightarrow after the Fierz rearrangement:

$$\frac{\lambda'_{i22}\lambda'^*_{i23}}{8m_{\tilde{\nu}}^2}(\bar{s}^\alpha \gamma_L^\mu s^\beta)(\bar{s}^\beta \gamma_{\mu R} b^\alpha)$$

$$\frac{\lambda'_{i32}\lambda'^*_{i22}}{8m_{\tilde{\nu}}^2}(\bar{s}^\alpha \gamma_R^\mu s^\beta)(\bar{s}^\beta \gamma_{\mu L} b^\alpha)$$

$$\gamma_{R,L}^\mu = \gamma^\mu(1 \pm \gamma_5)$$

$$\sum_{i=1}^3 \frac{\lambda'_{i22}\lambda'^*_{i23}}{8m_{\tilde{\nu}_i L}^2} \equiv d_{222}^R, \quad \sum_{i=1}^3 \frac{\lambda'_{i32}\lambda'^*_{i22}}{8m_{\tilde{\nu}_i L}^2} \equiv d_{222}^L$$

\mathbb{R}_p part

$$\begin{aligned}
 H_{eff}^{\lambda'}(b \rightarrow \bar{d}_j d_k d_n) &= d_{jkn}^R [\bar{d}_{n\alpha} \gamma_L^\mu d_{j\beta} \bar{d}_{k\beta} \gamma_{\mu R} b_\alpha] \\
 &+ d_{jkn}^L [\bar{d}_{n\alpha} \gamma_L^\mu b_\beta \bar{d}_{k\beta} \gamma_{\mu R} d_{j\alpha}] , \\
 H_{eff}^{\lambda'}(b \rightarrow \bar{u}_j u_k d_n) &= u_{jkn}^R [\bar{u}_{k\alpha} \gamma_L^\mu u_{j\beta} \bar{d}_{n\beta} \gamma_{\mu R} b_\alpha] , \\
 H_{eff}^{\lambda''}(b \rightarrow \bar{d}_j d_k d_n) &= \frac{1}{2} d_{jkn}'' [\bar{d}_{k\alpha} \gamma_R^\mu d_{j\beta} \cdot \bar{d}_{n\beta} \gamma_{\mu R} b_\alpha \\
 &- \bar{d}_{k\alpha} \gamma_R^\mu d_{j\alpha} \cdot \bar{d}_{n\beta} \gamma_{\mu R} b_\beta] \\
 H_{eff}^{\lambda''}(b \rightarrow \bar{u}_j u_k d_n) &= u_{jkn}'' [\bar{u}_{k\alpha} \gamma_R^\mu u_{j\beta} \cdot \bar{d}_{n\beta} \gamma_{\mu R} b_\alpha \\
 &- \bar{u}_{k\alpha} \gamma_R^\mu u_{j\alpha} \cdot \bar{d}_{n\beta} \gamma_{\mu R} b_\beta]
 \end{aligned}$$

with

$$\begin{aligned}
 d_{jkn}^R &= \sum_{i=1}^3 \frac{\lambda'_{ijk} \lambda'^*_{in3}}{8m_{\tilde{\nu}_i^L}^2} \\
 d_{jkn}^L &= \sum_{i=1}^3 \frac{\lambda'_{i3k} \lambda'^*_{inj}}{8m_{\tilde{\nu}_i^L}^2} \quad (j, k, n = 1, 2) \\
 u_{jkn}^R &= \sum_{i=1}^3 \frac{\lambda'_{ijn} \lambda'^*_{ik3}}{8m_{\tilde{e}_i^L}^2} \quad (j, k = 1; n = 2) \\
 d_{jkn}'' &= \sum_{i=1}^3 \frac{\lambda''_{ij3} \lambda''^*_{ikn}}{4m_{\tilde{u}_i^R}^2} \\
 u_{jkn}'' &= \sum_{i=1}^2 \frac{\lambda''_{ji3} \lambda''^*_{kin}}{4m_{\tilde{d}_i^R}^2} \quad (j = 1, 2; k = 1; n = 2)
 \end{aligned}$$

α, β : color indices

QCD Factorization

Beneke, Buchalla, Neubert, Sachrajda

- allows calculations of non-factorizable contributions (dominated by hard gluon exchange)
- Decay amplitude

$$\mathcal{A}(B \rightarrow PV) = \mathcal{A}^f(B \rightarrow PV) + \mathcal{A}^a(B \rightarrow PV) ,$$

where

$$\mathcal{A}^f(B \rightarrow PV) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{i=1}^{10} V_{pb} V_{pq}^* a_i^p \langle PV | O_i | B \rangle_{\text{NF}} ,$$

$$(q = d, s)$$

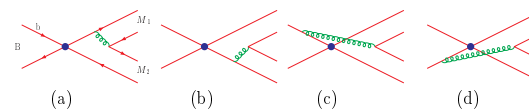
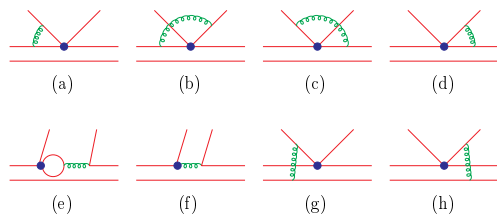
including vertex corrections, penguin corrections, and hard spectator scattering contributions.

$$\mathcal{A}^a(B \rightarrow PV) \propto f_B f_P f_V \sum V_{pb} V_{pq}^* b_i ,$$

including weak annihilation contributions

Order of α_s corrections

vertex, penguin, hard spectator scattering, and weak annihilation



$B \rightarrow \phi K$ and $B \rightarrow \eta^{(\prime)} K$

Decay amplitude of $B \rightarrow \phi K$

$$\mathcal{A}(B \rightarrow \phi K) \equiv \mathcal{A}_{\phi K} = \mathcal{A}_{\phi K}^{SM} + \mathcal{A}_{\phi K}^{R_p} ,$$

$$\mathcal{A}_{\phi K}^{SM} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pq}^* \left(a_3 + a_4^p + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9 - \frac{1}{2} a_{10}^p \right) A_\phi + \mathcal{A}_{\phi K}^a$$

$$\mathcal{A}_{\phi K}^{R_p} \propto \left(d_{222}^L + d_{222}^R \right) A_\phi \quad \left(d_{222}^L \sim \frac{\lambda'_{323} \lambda'_{322}^*}{8m_{\tilde{\nu}}^2} \right)$$

$$A_\phi = \langle K | \bar{s} \gamma^\mu (1 - \gamma_5) b | B \rangle \langle \phi | \bar{s} \gamma_\mu s | 0 \rangle , \quad a_i = a_i^L + a_i^V + a_i^P + a_i^H$$

Decay amplitude of $B \rightarrow \eta' K$

$$\mathcal{A}_{\eta' K}^{R_p} \sim \left(d_{222}^L - d_{222}^R \right) \left[\frac{\bar{m}}{m_s} \left(A_{\eta'}^s - A_{\eta'}^u \right) \left(\tilde{a}_6 + \frac{f_{\eta'}^u}{f_{\eta'}^s} \tilde{a}'_6 \right) + A_{\eta'}^s (\tilde{a}_4 - \tilde{a}_5) + A_{\eta'}^u \tilde{a}_4 \right]$$

$$\bar{m} \equiv m_{\eta'}^2 / (m_b - m_s), \quad A_{\eta'}^{u(s)} = f_{\eta'}^{u(s)} F^{B \rightarrow K} (m_B^2 - m_K^2)$$

- $\eta - \eta'$ mixing :

$$|\eta\rangle = \cos \theta_8 |\eta_8\rangle - \sin \theta_0 |\eta_0\rangle$$

$$|\eta'\rangle = \sin \theta_8 |\eta_8\rangle + \cos \theta_0 |\eta_0\rangle \quad (\theta_8 \approx -22.2^\circ, \theta_0 \approx -9.1^\circ)$$

NOTE

$$d_{222}^R [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) s_\beta] [\bar{s}_\beta \gamma_\mu (1 + \gamma_5) b_\alpha] \\ + d_{222}^L [\bar{s}_\alpha \gamma^\mu (1 + \gamma_5) s_\beta] [\bar{s}_\beta \gamma_\mu (1 - \gamma_5) b_\alpha]$$

$$\Rightarrow d_{222}^R \langle \phi(\eta') | \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) s_\alpha | 0 \rangle \langle K | \bar{s}_\beta \gamma_\mu (1 + \gamma_5) b_\beta | B \rangle \\ + d_{222}^L \langle \phi(\eta') | \bar{s}_\alpha \gamma^\mu (1 + \gamma_5) s_\alpha | 0 \rangle \langle K | \bar{s}_\beta \gamma_\mu (1 - \gamma_5) b_\beta | B \rangle$$

SM & QCD Factorization

New parameters

- End point divergent integrals in hard spectator scattering & weak annihilation contributions
 \Rightarrow phenomenological parameters (8 new parameters for $B \rightarrow PP$ & PV)

$$X_{H,A} \equiv \int_0^1 \frac{dx}{x} = \left(1 + \rho_{H,A} e^{i\phi_{H,A}}\right) \ln \frac{m_B}{\Lambda_h}$$

$$\rho_{H,A} \leq 1, \quad 0^0 \leq \phi_{H,A} \leq 360^0, \quad \Lambda_h = 0.5 \text{ GeV}$$

Global fit for $B \rightarrow PP$ & $B \rightarrow PV$: to determine $\rho_{H,A}$ & $\phi_{H,A}$ (Du, Gong, Sun, Yang, Zhu)

(12 modes: $B \rightarrow \pi\pi, \pi K, \pi\rho, K\rho, \dots$, except $B \rightarrow \phi K, \eta^{(\prime)} X$)

- Large X_A & X_H case (unphysical)

$$\rho_H = \rho_A = 1, \quad \phi_H^{PP} = -22^0, \quad \phi_H^{PV} = 198^0, \quad \phi_A^{PP} = 54^0, \quad \phi_A^{PV} = -55^0$$

\Rightarrow BR for $B \rightarrow \phi K$ and $B \rightarrow \eta' K$: fit the exp. data ($\chi^2 = 7.6$)

\Rightarrow No solution for $\sin(2\phi_1)_{\phi K_S}$

- Small X_A & X_H case

$$\rho_A^{PP} = 0, \quad \rho_A^{PV} = 0.5, \quad \rho_H^{PP} = 1, \quad \rho_H^{PV} = 0.746,$$

$$\phi_H^{PP} = \phi_H^{PV} = 180^0, \quad \phi_A^{PV} = -6^0 \quad (\chi^2 = 18.3)$$

$\Rightarrow \mathcal{B}(B^+ \rightarrow \phi K^+) = 7.35 \times 10^{-6}$ somewhat small

$\mathcal{B}(B^+ \rightarrow \eta' K^+) = 48.5 \times 10^{-6}$ quite small

Strategy for finding possible solutions

- Need new amplitude(s) to explain the large $\mathcal{B}(B^\pm \rightarrow \eta' K^\pm)$, but do not affect (much) the BRs of $B^{\pm(0)} \rightarrow \phi K^{\pm(0)}$, $B \rightarrow \pi\pi$, $K\pi$, $\rho\pi$, etc.
- Need new phase(s) to understand $\sin(2\phi_1)_{\phi K_S}$ [Belle, or Average of Belle & BaBar], but do not affect $\sin(2\phi_1)_{\eta' K_S}$ [Belle, or Average of Belle & BaBar]
- Concentrate on d_{222}^L and d_{222}^R :
less constrained & $b \rightarrow s\bar{s}s$ only!
 \Rightarrow No contribution to most $B \rightarrow PP$ and
 $B \rightarrow PV$ modes (e.g., $B \rightarrow \pi\pi$, $K\pi$, ρK , etc),
except $B \rightarrow \eta^{(\prime)} K$, $B \rightarrow \eta^{(\prime)} K^*$, $B \rightarrow \phi K$
- $\bar{\mathcal{A}}_{\phi K}^{R_p} \propto (d_{222}^L + d_{222}^R)$ only
 $\bar{\mathcal{A}}_{\eta' K}^{R_p} \propto (d_{222}^L - d_{222}^R)$

Solution

In QCD factorization,

δ' : possible strong phase from $O(\Lambda_{\text{QCD}}/m_b) \sim O(\alpha_s)$

For $\delta' = 0 \Rightarrow$ unlikely to find a solution

For $\delta' = 30^\circ \Rightarrow$ Solution !!

$$d_{222}^L \propto |\lambda'_{i32} \lambda'^*_{i22}| e^{i\theta_L}, \quad d_{222}^R \propto |\lambda'_{i22} \lambda'^*_{i23}| e^{i\theta_R}$$

$$|\lambda'_{322}| = 0.086, \quad |\lambda'_{332}| = 0.089, \quad |\lambda'_{323}| = 0.030, \\ \theta_L = 0.66, \quad \theta_R = -2.25, \quad m_{\text{SUSY}} = 200 \text{ GeV}$$

In Generalized Factorization

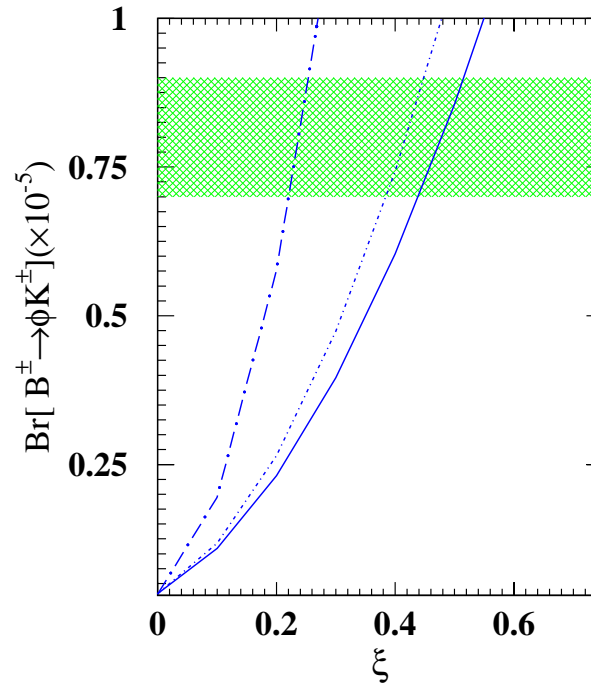


Figure 1: BR vs ξ . The dotted and dot-dashed lines correspond to Case 1 and Case 2 respectively. The solid line corresponds to the SM (The SM BR is same for both cases). The shaded region is allowed by the experimental data. ($\xi \equiv \frac{1}{N_c}$)

In Generalized Factorization

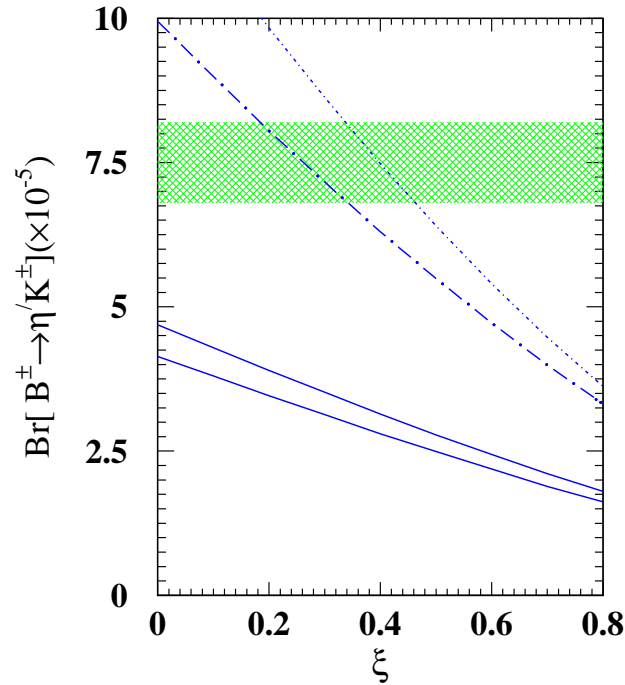


Figure 2: BR vs ξ . The dotted and dot-dashed lines correspond to Case 1 and Case 2 respectively. The solid lines correspond to the SM. The upper solid line is for Case 1 and the lower solid line is for Case 2. The shaded region is allowed by the experimental data.

Table 1: CP asymmetries in $B^0 \rightarrow \phi K_S$ and $B^0 \rightarrow \eta' K_S$.

$\sin(2\tilde{\phi}_1)$	Our result	experimental data
$\sin(2\tilde{\phi}_1)_{\phi K_S}$	-0.64	$-0.96 \pm 0.50^{+0.09}_{-0.11}$ [Belle] $+0.45 \pm 0.43 \pm 0.07$ [BaBar]
$\sin(2\tilde{\phi}_1)_{\eta' K_S}$	+0.55	$+0.43 \pm 0.27 \pm 0.05$ [Belle] $+0.02 \pm 0.34 \pm 0.03$ [BaBar]

Table 2: The BRs (\mathcal{B}) and CP rate asymmetries (\mathcal{A}_{CP}) for $B \rightarrow \eta' K$ and $B \rightarrow \phi K$.

mode	Our result		Exp. data	
	$\mathcal{B} \times 10^6$	\mathcal{A}_{CP}	$\mathcal{B} \times 10^6$	\mathcal{A}_{CP}
$B^- \rightarrow \phi K^-$	10.5	-4%	(10.9 ± 1.0)	$(3.9 \pm 8.8 \pm 1.1)\%$
$B^0 \rightarrow \phi K_S$	9.8	-2%	(8.0 ± 1.3)	$(-19 \pm 30)\%$
$B^- \rightarrow \eta' K^-$	71.1	8%	(77.6 ± 4.6)	$(2 \pm 4)\%$
$B^0 \rightarrow \eta' K_S$	65.9	8%	(60.6 ± 7.0)	$(8 \pm 18)\%$

In Generalized Factorization

Table 3: CP asymmetries in $B^0 \rightarrow \phi K_S$ and $B^0 \rightarrow \eta' K_S$.

$\sin(2\tilde{\phi}_1)$	Case 1 ($N_c \approx 2$)	Case 2 ($N_c \approx 4$)	experimental data
$\sin(2\tilde{\phi}_1)_{\phi K_S}$	0	-0.82	$-0.73 \pm 0.64 \pm 0.22$ [(NEW) $-0.96 \pm 0.50^{+0.09}_{-0.11}$] [Belle] $-0.19^{+0.52}_{-0.50} \pm 0.09$ [(NEW) $+0.45 \pm 0.43 \pm 0.07$] [BaBar]
$\sin(2\tilde{\phi}_1)_{\eta' K_S}$	0.73	0.72	$+0.71 \pm 0.37^{+0.05}_{-0.06}$ [(NEW) $+0.43 \pm 0.27 \pm 0.05$] [Belle] $+0.02 \pm 0.34 \pm 0.03$ [BaBar]

Table 4: The BRs (\mathcal{B}) and CP rate asymmetries (\mathcal{A}_{CP}) for $B \rightarrow \eta^{(\prime)} K^{(*)}$ and $B \rightarrow \phi K$.

mode	Case 1		Case 2	
	$\mathcal{B} \times 10^6$	\mathcal{A}_{CP}	$\mathcal{B} \times 10^6$	\mathcal{A}_{CP}
$B^+ \rightarrow \eta' K^+$	69.3	0.01	76.1	0.01
$B^+ \rightarrow \eta K^{*+}$	27.9	0.04	35.2	0.03
$B^0 \rightarrow \eta' K^0$	107.4	0.00	98.9	0.00
$B^0 \rightarrow \eta K^{*0}$	20.5	-0.71	11.7	-0.15
$B^+ \rightarrow \phi K^+$	8.99	0.21	8.52	0.25

Summary

- In R_p violating SUSY & QCDF, possible to consistently understand:
the *anomalous* $\sin(2\phi_1)_{\phi K_S}$ as well as the *normal* $\sin(2\phi_1)_{\eta' K_S}$
& the large $\mathcal{B}(B \rightarrow \eta' K)$.
 \Rightarrow Need a sizable strong phase from $O(\Lambda_{QCD}/m_b)$ contributions.
- All the observed data can be accommodated for certain values of R_p couplings.
- RPV with (QCDF vs. pQCD) would be interesting
- future measurement:
 - $b \rightarrow s\bar{q}q$ penguin processes ($q = s, u, d$):
precise measurement of direct CP asymmetries for $B \rightarrow \phi K, \eta' K$
 - ηK decay channels: small BR
- R_p conserving SUSY
(minimal extension of mSUGRA : non-universal soft breaking A terms)
 \Rightarrow interesting solutions (**coming soon**)
Arnowitt, Dutta, Hu
Khalil, Kou